

Intelligent Decision-Making in Conditions of Uncertainty in Games with Nature

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Abstract

Paper represents games with nature. The theory of the game with nature and its concept is described. Artificial intelligence methods are applied to develop methods of decision-making in conditions of uncertainty. The Wald criterion, the optimism criterion, the pessimism criterion and the Savage criterions are described. The comparison of developed methods has been done. Denoting the behavior of the game functions depends on the winnings a corresponds to the first icon in the name of the criterion.

Keywords 1

Game with nature, artificial intelligence, decision-making, conditions of uncertainty.

1. Introduction

Games with nature are mathematical models for which the choice of a decision depends on objective reality. For example, customer demand, the state of nature, etc. [1-2].

"Nature" is a generalized concept of an adversary who does not pursue his own goals in a given conflict [3].

To apply this theory, it is necessary to be able to represent conflicts in the form of games [4]. A characteristic feature of any conflict is that none of the parties involved knows in advance exactly and completely all their possible solutions, as well as the other parties, their future behavior and, therefore, each is forced to act in conditions of uncertainty [5]. The uncertainty of the outcome can be due to both the conscious actions of active opponents and unconscious, passive manifestations, for example, of the elemental forces of nature: rain, sun, wind, avalanche, etc. In such cases, the possibility of an accurate prediction of the outcome is excluded [6]. In some conflicts, the opposite side is a consciously and purposefully acting active adversary who is interested in our defeat, who deliberately prevents success, and achieves victory by any means [7-8]. In other conflicts, there is no such a conscious enemy, but only the so-called "blind forces of nature" operate [9]: weather conditions [10], the state of trade equipment at the enterprise [11], illness of employees [12], the instability of the economic situation [13], market conditions [14], the dynamics of exchange rates [15], the level of inflation [16], tax policy [17], changing purchasing demand, etc. With global pandemic of COVID-19 [18] games with nature can be used for decision-making in preventing and control measures to eliminate the epidemic dynamics [19]. In such cases, nature is not malicious and acts passively, sometimes to the detriment of man, and sometimes to his benefit, but her state and manifestation can significantly affect the result of the activity [20].

In such games, a person tries to act prudently, for example, using a strategy that allows you to get the least loss. The second player (nature) acts unintentionally, completely by accident, his possible strategies are known (nature's strategies). Such situations are investigated using the theory of statistical decisions [21]. Although there may well be situations in which nature can really act as a player. For example, circumstances associated with weather conditions or with natural elemental forces. Man's play with nature also reflects a conflict situation that arises when interests clash in

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choosing a solution. But “the elemental forces of nature” cannot be attributed to reasonable actions directed against a person, and even more so any “malicious intent” [22]. Thus, it is more correct to talk about a conflict situation caused by a clash of human interests and the uncertainty of nature's actions, but without an obvious antagonistic coloration [23]. Situations in which the risk is associated not with the conscious opposition of the opposite side (environment), but with insufficient awareness of its behavior or the state of the decision-maker, are investigated using the theory of statistical decisions.

The **aim** of research is to investigate methods of decision-making in conditions of uncertainty in games with nature.

2. Materials and methods

Matrix of playing with nature:

$$A = || a_{ij} || \quad (1)$$

where a_{ij} is the payoff of player 1 in the implementation of his pure strategy i and pure strategy j of player 2 (nature) ($i = 1, \dots, m; j = 1, \dots, n$).

All possible states are considered as P_1, P_2, \dots, P_n of nature P , which it calls randomly regardless of the actions of player A without malicious opposition to the strategies of player A . Nature can be in only one of the noted states, but in which one it is unknown, although in some cases only the probabilities of these states may be known.

$$q_1 = p(P_1), q_2 = p(P_2), \dots, q_n = p(P_n) \quad (2)$$

$$\sum_{j=1}^n q_j = 1 \quad (3)$$

Possible strategies A_1, A_2, \dots, A_m of player A and his payoffs $a_{ij} \geq 0$ for each of the strategies and each of the states of nature P_j are also known. These winnings can be shown in the form of a payoff matrix (Table 1).

Table 1

Matrix of winnings

$(a_{ij}) =$	$A_i \backslash P_j$	P_1	P_2	\dots	P_n
	A_1	a_{11}	a_{12}	\dots	a_{1n}
	A_2	a_{21}	a_{22}	\dots	a_{2n}
	\dots	\dots	\dots	\dots	\dots
	A_m	a_{m1}	a_{m2}	\dots	a_{mn}
	q_j	q_1	q_2	\dots	q_n

The bottom row of the matrix shows the probabilities q_j of the states of nature $P_j, j = 1, \dots, n$.

Imagine that player A , not knowing the state of nature, chose strategy A_i . If nature has assumed the state P_j , then the payoff of player A will be a_{ij} . But if player A knew in advance that nature would take the state P_j , then he would choose the strategy A_{i0} , which achieves the greatest payoff a_{i0j} .

$$\beta_j = \max_{1 \leq i \leq m} a_{ij} = a_{i0j} \quad (4)$$

Difference

$$r_{ij} = \beta_j - a_{ij} \quad (5)$$

between the payoff β_j of player A under the known state of nature P_j and the payoff a_{ij} if the player A does not know the state of nature, it is called the risk under the strategy A_i and the state of nature P_j . Thus, the risk r_{ij} is that part of the greatest payoff β_j in the state of nature P_j which player A did not win by applying strategy A_i through ignorance of the state of nature.

Table 2
Risk matrix

(r_{ij}) =

	P_j	P_1	P_2	...	P_n
A_i					
A_1		r_{11}	r_{12}	...	r_{1n}
A_2		r_{21}	r_{22}	...	r_{2n}
...	
A_m		r_{m1}	r_{m2}	...	r_{mn}
q_j		q_1	q_2	...	q_n

The last line shows the probabilities of the states of nature $q_j, j = 1, \dots, n$. Since $0 \leq a_{ij} \leq \beta_j$ (the right inequality follows from (4)), then from (5) we obtain that $0 \leq r_{ij} \leq \beta_j$.

The probability q_j of the state of nature P_j is obviously the probability of winning a_{ij} and risk r_{ij} for each strategy $A_i, i = 1, \dots, m$. Therefore, each strategy A_i can be interpreted as a discrete random variable, which can take values equal to the winnings a_{i1}, \dots, a_{in} or risks r_{i1}, \dots, r_{in} with the corresponding probabilities q_1, \dots, q_n .

Player A's task is to choose the optimal strategy from the possible strategies A_1, \dots, A_m . The optimality of a strategy is understood in various senses and is chosen according to various criteria.

The result of the game generally depends on three numerical parameters: the payoffs a of player A, the risks r that appear when player A chooses a particular strategy, and the probabilities q of states of nature. The desire to "fold" these three parameters into one indicator leads to some numerical function depending on these three parameters. Let's call it $G(a, r, q)$ and call it the game function. The nature of the dependence of the game function G on a, r and q is motivated by the logic of the applied criterion. The values

$$G_{ij} = G(a_{ij}, r_{ij}, q_j), i = 1, \dots, m; j = 1, \dots, n \quad (6)$$

of the functions of the game will be called the indicators of the game. These indicators form the matrix of the game (Table 3).

Table 2
Game matrix

(G_{ij}) =

	P_j	P_1	P_2	...	P_n
A_i					
A_1		G_{11}	G_{12}	...	G_{1n}
A_2		G_{21}	G_{22}	...	G_{2n}
...	
A_m		G_{m1}	G_{m2}	...	G_{mn}

The vector argument criterion φ assumes the assignment of some numerical function

$$(G_{i1}, \dots, G_{in}), i = 1, \dots, m, \quad (7)$$

whose value

$$G_i = \varphi(G_{i1}, \dots, G_{in}), i = 1, \dots, m \quad (8)$$

will be called the indicator of the strategy A_i .

Then, among the indicators G_i of strategies A_i , an extreme one is selected. For some criteria, this is the maximum value: $Ext = max$, and for others, the minimum: $Ext = min$. If $Ext = max$, then the indicator G_i is called the indicator of the optimality of the strategy A_i ; if $Ext = min$, then G_i is called the non-optimality indicator of the strategy A_i .

$$\underset{1 \leq i \leq n}{Ext} G_i = G_j \quad (9)$$

Applying the described scheme, we will form some classes of criteria.

3. Results

For maximum criteria (extreme pessimism).

$$\underset{1 \leq i \leq m}{Ext} G_i = \max_{1 \leq i \leq m} G_i, \quad (10)$$

and indicators of strategies A_i are determined as follows:

$$G_i = \varphi(G_{i1}, \dots, G_{in}) \max_{1 \leq j \leq n} G_{ij} \quad (11)$$

and are (10) indicators of the optimality of strategies.

Thus, G_i is the worst indicator of the game under the strategy A_i . Hence it follows that the function of the game $G(a, r, q)$ should be non-decreasing in the payoff a and non-increasing in the risk r .

The game performance is also influenced by the probabilities of states of nature q . So, for example, if the worst smallest payoff a_{ij} for strategy A_i has a sufficiently small probability q_j , then it is no longer advisable to consider it as the smallest one. For this gain to remain practically the smallest, it should have a sufficiently high probability. With risks, the opposite is true: for the worst, greatest risk r_{ij} with strategy A_i to remain practically the greatest, its probability should also be large enough. This suggests that the game function should not increase in probability q .

So, the logic of the maximin criterion determines the behavior of the game function depending on the payoff a , risk r and probability q :

$$G(a, r, q) \begin{matrix} \uparrow & \text{by } a; \\ \downarrow & \text{by } r; \\ \downarrow & \text{by } q \end{matrix} \quad (12)$$

For convenience, in what follows, for the maximin criterion, we denote the game function G by W , the indicators of the game G_{ij} by W_{ij} , and the optimality indicators G_i of strategies A_i by W_i .

Thus, for the maximin criterion, the game function

$$W(a, r, q) \begin{matrix} \uparrow & \text{by } a; \\ \downarrow & \text{by } r; \\ \downarrow & \text{by } q, \end{matrix} \quad (13)$$

Game performance is:

$$W_{ij} = W(a_{ij}, r_{ij}, q_j) \quad i = 1, \dots, m; \quad j = 1, \dots, n \quad (14)$$

Strategy optimality indicators are

$$W_i = \min_{1 \leq j \leq n} W_{ij}, \quad i = 1, \dots, m. \quad (15)$$

Optimal according to the maximin criterion is considered the strategy A_{i_0} , for which

$$\max_{1 \leq i \leq m} W_i = W_{i_0} \quad (16)$$

The maximin criterion is a criterion for the extreme pessimism of a person who chooses a strategy, since it orients him to the worst manifestation of the state of nature for him and, as a consequence, to very careful behavior when making a decision.

The specific function of the game $W(a, r, q)$ can be chosen in different ways, but with the indispensable requirement of possessing properties (13).

Examples of maximin criteria with specific functions of the game $W(a, r, q)$ are the following criteria:

$$W(a, r, q) = a; \quad (17)$$

$$W(a, r, q) = (1-q)a; \quad (18)$$

$$W(a, r, q) = a-r; \quad (19)$$

$$W(a, r, q) = (1-q)a-qr. \quad (20)$$

Each of these functions possesses properties (13), can be checked by the sign of the partial derivatives.

In criterion (17), the indicators of the game are the winnings: $W_{ij}=a_{ij}$, and therefore it does not take into account either the risks or the probabilities of the states of nature. Criterion (17) is Wald's criterion allowing to justify the choice of a solution in conditions of complete uncertainty, in conditions of ignorance of the probabilities of states of nature [24]. Criterion (18) takes into account the gains and probabilities of states of nature, but does not take into account the risks. Criterion (19) takes into account the gains and risks without considering the probabilities of states of nature. Criterion (20) takes into account the gains, risks, and probabilities of states of nature.

For the minimax criterion (extreme pessimism), we denote the game function by $S(a, r, q)$. It should be non-increasing in the payoff a and non-decreasing in the risk r and the probability q of the states of nature:

$$S(a, r, q) \begin{cases} \text{decreases} & \text{by } a; \\ \text{increases} & \text{by } r; \\ \text{increases} & \text{by } q \end{cases} \quad (21)$$

Then $S_{ij} = S(a_{ij}, r_{ij}, q_j)$ are the indicators of the game. Strategy indicators are defined as follows:

$$S_i = \varphi(S_{i1}, \dots, S_{in}) = \max_{1 \leq j \leq n} S_{ij}, \quad i = 1, \dots, m. \quad (22)$$

Then $S_{ij} = S(a_{ij}, r_{ij}, q_j)$ are the indicators of the game. Strategy indicators are defined as follows:

$$\min_{1 \leq j \leq m} S_j = S_{j_0} \quad (23)$$

By virtue of (23), the indicators S_i are indicators of the non-optimality of the strategies A_i . The game function $S(a, r, q)$ should have properties (21) in view of (22) and (23).

Let us present some minimax criteria with specific functions of the game $S(a, r, q)$ satisfying conditions (21):

$$S(a, r, q) = r; \quad (24)$$

$$S(a, r, q) = qr; \quad (25)$$

$$S(a, r, q) = r - a; \quad (26)$$

$$S(a, r, q) = qr - (1 - q)a. \quad (27)$$

Criterion (24), in which the indicators of the game are risks, does not take into account either the gains or the probabilities of the states of nature. This is the Savage criterion.

Comparing the maximin and minimax criteria, we can say the following.

Statement 1. The maximin criteria (19) and (20) are equivalent to the minimax criteria (26) and (27), respectively.

The first of these equivalents means that strategy A_i is optimal according to criterion (19) if and only if it is optimal according to criterion (26). A similar explanation applies to the second equivalent.

Evidence. Let us first prove the equivalence (19) \hat{U} (26). Since the game functions W and S , respectively, of criteria (19) and (26) satisfy the equality $S = -W$, then the game indicators also satisfy the analogous equality $S_{ij} = -W_{ij}$. Then

$$S_i = \max_{1 \leq j \leq n} S_{ij} = \max_{1 \leq j \leq n} (-W_{ij}) = \min_{1 \leq j \leq n} W_{ij} = -W_i \quad (28)$$

from where

$$\min_{1 \leq i \leq m} S_i = \min_{1 \leq j \leq m} (W_j) = - \max_{1 \leq i \leq m} W_i \quad (29)$$

Thus, S_i is minimal for the number i , for which W_i is maximal, and the equivalence (19) \hat{U} (26) is proved. Then the equivalence (20) \hat{U} (27) is also proved.

In case of maximax criteria (extreme optimism), the game function, which we denote by $M(a, r, q)$, should not decrease with respect to the payoff a and the probability q of states of nature and not increase with respect to the risk r :

$$M(a, r, q) \hat{U} a; \emptyset \text{ by } r; \text{ by } \hat{U} q. \quad (30)$$

Indicators of the game $M_{ij} = M(a_{ij}, r_{ij}, q_j)$. Optimality indicators of strategies

$$M_i = \varphi(M_{i1}, \dots, M_{in}) = \max_{1 \leq j \leq n} M_{ij}, i = 1, \dots, m \quad (31)$$

An optimal strategy is a strategy A_{i_0} for which

$$\max_{1 \leq i \leq m} M_i = M_{i_0} \quad (32)$$

The maximax criteria are criteria of extreme optimism, since they assume that nature will be in the most favorable state for player A, and therefore the strategy is chosen as the optimal one, in which the maximum indicator of the game – the indicator of optimality is maximum among the maximum indicators of all strategies.

As maximax criteria with specific functions of the game $M(a, r, q)$ possessing properties (30), we can take, for example, the following:

$$M(a, r, q) = a; \quad (33)$$

$$M(a, r, q) = qa; \quad (34)$$

$$M(a, r, q) = a-r; \quad (35)$$

$$M(a, r, q) = qa-(1-q)r. \quad (36)$$

In criterion (33), the indicators of the game are winnings $M_{ij} = a_{ij}$.

The function of the game in case of minimum criteria (extreme optimism), we define it through $E(a, r, q)$, is chosen non-increasing in terms of payoff, and also in terms of the probability q of states of nature and non-decreasing in terms of risk r :

$$E(a, r, q) \text{ } \emptyset \text{ by } a; \text{ } \dot{U} \text{ by } r; \text{ } \emptyset \text{ by } q. \quad (37)$$

As indicators of non-optimal strategies A_i , we take

$$E_i = \varphi(E_{i1}, \dots, E_{in}) = \min_{1 \leq j \leq n} E_{ij}, i = 1, \dots, m, \quad (38)$$

where $E_{ij} = E(a_{ij}, r_{ij}, q_i)$ are the indicators of the game.

The optimal strategy is assigned to the strategy A_{i_0} , which minimizes the non-optimal index E_i .

$$\min_{1 \leq i \leq m} E_i = E_{i_0} \quad (39)$$

Minimum criteria are also criteria of extreme optimism, since an optimal strategy is understood as a strategy in which the non-optimal indicator is the minimum among the non-optimal indicators of all strategies.

Examples of minimin criteria with functions of the game $E(a, r, q)$ with properties (37) can be:

$$E(a, r, q) = r; \quad (40)$$

$$E(a, r, q) = (1-q)r; \quad (41)$$

$$E(a, r, q) = r-a; \quad (42)$$

$$E(a, r, q) = (1-q)r-qa. \quad (43)$$

The indicators of play in criterion (40) are risks, and thus it turns into a minimum criterion for risks.

Statement 2. The maximax criteria (35) and (36) are equivalent to the minimum criterion (42) and (43), respectively.

The proof is similar to that of Statement 1, namely, for criteria (35) and (42) we have: $E = -M$ and, therefore, $E_{ij} = -M_{ij}$, whence

$$E_i = \min_{1 \leq j \leq m} (-M_{ij}) = - \max_{1 \leq j \leq n} M_{ij} = -M_i \quad (44)$$

therefore

$$\min_{1 \leq i \leq m} E_i = \min_{1 \leq i \leq n} (-M_i) = - \max_{1 \leq i \leq m} M_i \quad (45)$$

the equivalence of (35) \hat{U} (42) is proved.

For better visibility (13), (21), (30), and (37) to the non-increasing or non-decreasing of the game functions depending on the payoffs a , risks r , and states of nature q , let us summarize them in the following table 4.

Table 4
Comparison of methods

Arguments	Game functions and criteria			
Game functions	W(a, r, q)	S(a, r, q)	M(a, r, q)	E(a, r, q)
	max min	min max	max max	min min
a	\hat{U}	\emptyset	\hat{U}	\emptyset
r	\emptyset	\hat{U}	\emptyset	\hat{U}
q	\emptyset	\hat{U}	\hat{U}	\emptyset

It can be seen from this table that denoting the behavior of the game functions depending on the winnings a correspond to the first icon in the name of the criterion: $max - \hat{U}$, $min - \emptyset$, $max - \hat{U}$, $min - \emptyset$. And in the second line, indicating the behavior of the game functions depending on the risks r , are opposite to the arrows in the first line.

4. Conclusion

In this paper, the theory of the game and its types were described, the concept of games with nature was also described. Methods of decision-making in conditions of uncertainty were described, such as the Wald criterion, the optimism criterion, the pessimism criterion, the Savage criterion. For each criterion, its exceptional feature was described.

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