Simulation of gas consumption process based on the mathematical model in the form of cyclic random process considering the scale factors

Iaroslav Lytvynenko^a, Serhii Lupenko^a, Nataliia Kunanets^b, Oleg Nazarevych^a, Grigorii Shymchuk^a, Volodymyr Hotovych^a

^aTernopil Ivan Puluj National Technical University, 56, Ruska Street, Ternopil, 46001, Ukraine ^bLviv Polytechnic National University, 12, Bandera Street, Lviv, 79013, Ukraine

Abstract

The article deals with the problem of constructing a new mathematical model of gas consumption process in which, in contrast to the known mathematical model, in the form of an additive combination of three components: cyclic random process, trend component, and stochastic residue, the component in the form of a cyclic random process with taking into account the scale factors, is introduced. Based on segmentation using the Caterpillar method, ten components of singular decomposition are obtained. The sum of nine components of singular decomposition forms the cyclic component, the cyclic random process. This component takes into account the scale factors of gas consumption range on every segment cycle. The trend component of mathematical model is the second component of singular decomposition and stochastic residue, which is formed on the basis difference of values of the studied gas consumption process and the sum of cyclic and trend components. In the research, computer simulation of gas consumption process on the basis of the known model and the new one is carried out and simulation errors with the real gas consumption process are estimated. The computer simulation results are compared in accordance with the proposed mathematical model and the known one. The use of the proposed mathematical model considering the cyclic component as a random cyclic process with cyclic structure and scale factors allowed to increase the accuracy of computer simulation which is evidenced by the obtained error estimation results.

Keywords

Cyclic process, gas consumption process, statistical processing, segmentation, cyclic random process.

1. Introduction

Gas consumption management is of particular urgency nowadays. Analysis, simulation and prediction of gas consumption process are the vital problems set by gas companies which, in turn, requires efficient hardware and software for gas consumption processing. Development of new software complexes demands the improving of mathematical software, which involves the development of new mathematical models and methods of gas consumption processing. This will ultimately allow to form the forecast results based on the information obtained.

ORCID: 0000-0001-7311-4103 (I. Lytvynenko); 0000-0002-6559-0721 (S. Lupenko); 0000-0003-3007-2462 (N. Kunanets); 0000-0002-8883-6157 (O. Nazarevych), 0000-0003-2362-7386 (G. Shymchuk), 0000-0003-2143-6818 (V. Hotovych)

ITTAP'2021: 1nd International Workshop on Information Technologies: Theoretical and Applied Problems, November 16-18, 2021, Ternopil, Ukraine

EMAIL: iaroslav.lytvynenko@gmail.com (I. Lytvynenko); lupenko.san@gmail.com (S. Lupenko); nek.lviv@gmail.com (N. Kunanets); taltek.te@gmail.com (O. Nazarevych), gorych@gmail.com (G. Shymchuk), gotovych@gmail.com (V. Hotovych)

^{© 2021} Copyright for this paper by its authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0).

CEUR Workshop Proceedings (CEUR-WS.org)

This research deals with creating a new mathematical model of gas consumption process using stochastic approach in the form of additive combination of three components, one of which is a cyclic random process that takes into account scale factors, other components are trend and stochastic residue.

2. Analysis of recent research

In the field of energy consumption, in particular, electricity and gas, various mathematical models are used to describe them, and there are two approaches to their representation - deterministic and stochastic one [1-13]. When studying gas consumption process, not only presenting it objectively in the form of mathematical model is essential, but also the ability to draw a gas consumption forecast based on it. In [14], a short-term forecast using neural networks and the ARMA model is considered. In [15], a neural network algorithm is used to draw an electricity consumption model in a gas transmission system. Monthly gas consumption for household consumers was studied in [16], where a multivariate regression analysis was considered, which made it possible to show the dependence of gas consumption on the average monthly temperature. The model based on machine forecast is presented in [17], which enables the prediction of future gas consumption using a statistical sample. In [18], the description of mathematical model of energy loads based on a linear periodic random process (stochastic approach) is given, which allows to take into account the main reasons that cause the rhythmicity of the process. In [20], a mathematical model of loads on gas transmission system in the form of a conditional linear periodic random process is presented. An interesting approach to drawing mathematical model is described in [21] where the additive model is considered as a sum of the deterministic annual trend and stochastic balance as a stochastic-periodic process, which allows to take into account the periodic and random nature of gas consumption and variable topology of consumers in the annual observation interval. In this approach, a singular decomposition based on the use of the Caterpillar method was used to obtain the components. The emphasis in this paper is on the use of stochastic residue for segmentation of gas consumption process in the season features. However, this mathematical model did not suggest simulation of gas consumption process, and therefore to draw a forecast.

3. Main part

In [21], a mathematical model of gas consumption process was considered, one of the components of which was a stochastic-periodic random process. In [22] it was shown that the model in the form of a cyclic random process, as a partial case, includes a model in the form of stochastic-periodic random process.

We present the known mathematical model of the random cyclic gas consumption process $\xi'(\omega,t)$ as an additive model (1) which consists of three components. As it was mentioned above, the similar approach was used in [21], however, we specify the components of the proposed mathematical model:

$$\xi'(\omega,t) = \xi(\omega,t) + f_{tr}(t) + f_{rem}(\omega'',t), t \in \mathbf{W}, \omega \in \Omega, \omega'' \in \Omega'',$$
(1)

where $\xi(\omega, t)$ is a cyclic component, $f_{tr}(t)$ is a trend function, $f_{rem}(\omega'', t)$ is a stochatic residue function, **W** is a domain of determining, ω is a elementary event, Ω is a space of elementary events, Ω'' and ω'' are another space of elementary events and elementary event from that space respectively.

Since in practice we are dealing with discrete data, we present mathematical model (1) as follows:

$$\xi'_{\omega}(l) = \xi_{\omega}(l) + f_{tr}(l) + f_{rem\omega'}(l), l \in \mathbf{W} = \mathbf{D},$$
(2)

where $\xi_{\omega}(l)$ is an implementation of cyclic component of gas consumption process, $f_{tr}(l)$ is a trend function, $f_{rem\omega}(l)$ is a function of stochatic residue, l stands for discrete samples of gas consumption process, **D** is a discrete domain of determining.

For obtaining the components of mathematical model (2) during processing of the real cyclic gas consumption process $\xi'_{\omega}(l), l = \overline{1, L}$ we apply the SSA-Caterpillar method. This method is given in [23] and describes the transformation of a one-dimensional time series into a multidimensional one, which

makes it possible to obtain components of a singular segmentation. Comparison of time series processing methods using the Caterpillar-SSA method is presented in [24].

When applying the Caterpillar method, we obtain k implementations of components $\{\bar{f}_k(l), k = \overline{0, K-1}, l = \overline{1, L}\}$, where K = 10, l stands for parts of gas consumption process during 2006-2019 years, L is the number of discrete implementation samples.

The cyclic component is obtained by summing the components obtained on the basis of the Caterpillar method, in particular, components: 0-1,3-9, component 2 is a component of the trend $\bar{f}_2(l)$:

$$\xi_{\omega}(l) = \sum_{k=0}^{1} \bar{f}_{k}(l) + \sum_{k=3}^{9} \bar{f}_{k}(l), l = \overline{1, L}, \qquad (3)$$

$$f_{tr}(l) = \bar{f}_2(l) , \ l = \overline{1, L} . \tag{4}$$

The stochastic residue is obtained on the basis of the relation:

$$f_{rem\omega'}(l) = \xi'_{\omega}(l) - (\xi_{\omega}(l) + f_{tr}(l)), \ l = 1, L.$$
(5)

Consider $\xi_{\omega}(l)$, the cyclic component of the mathematical model (1), which carries information about the process of gas consumption in more detail, we present it as:

$$\xi_{\omega}(l) = \sum_{i=1}^{C} f_i(l), l \in \mathbf{W},$$
(6)

where *C* is the number of segments-cycles of the cyclic process of gas consumption, **W** is the domain of determining the cyclic process of gas consumption, and the domain of its values, for the case of the stochastic approach is the Hilbert space of random variables given on one probabilistic space $(\xi_{\omega}(l) \in \Psi = \mathbf{L}_2(\Omega, \mathbf{P}))$. In the design (6), the segments-cycles $f_i(l)$ of the cyclic gas consumption process are determined by indicator functions, i.e.:

$$f_i(l) = \xi_{\omega}(l) \cdot I_{\mathbf{W}_i}(l), i = \overline{1, C}, \ l \in \mathbf{W}.$$
(7)

The indicator functions, which allocate segments-cycles, are defined as:

$$I_{\mathbf{W}_{i}}\left(l\right) = \begin{cases} 1, l \in \mathbf{W}_{i}, \\ 0, l \notin \mathbf{W}_{i}, \end{cases} \quad i = \overline{1, C} ,$$

$$\tag{8}$$

where \mathbf{W}_i is the domain of determining the indicator function, which in the case of a discrete signal, i.e. $\mathbf{W} = \mathbf{D}$, is equal to a discrete set of samples:

$$\mathbf{W}_{i} = \left\{ l_{i,j}, \, j = \overline{\mathbf{1}, J} \right\}, \, i = \overline{\mathbf{1}, C} \,. \tag{9}$$

The segmental cyclic structure $\hat{\mathbf{D}}_c$ is taken into account by the set of time samples $\{l_i\}$ or $\{l_{i,j}\}$, $i = \overline{1, C}$, $j = \overline{1, J}$, where *J* is the number of discrete samples in the cycle. This notation of the mathematical model (9) takes into account the rhythm of the cyclic gas consumption process due to the continuous function of the rhythm T(l, n) namely:

$$I_{\mathbf{W}_{i}}(l) = I_{\mathbf{W}_{i+n}}(l+T(l,n)), \ i = \overline{1,C}, \ n = 1, \ l \in \mathbf{W}.$$
(10)

In order to estimate the rhythm function T(l, n), the segment structure of gas consumption process (in this case the segment cyclic structure) was first determined as $\hat{\mathbf{D}}_c = \{l_i, i = \overline{1, C}\}$ which is a set of time moments that correspond to the boundaries of the segments-cycles of gas consumption process. In this case, the estimation of the segmental cyclic structure of gas consumption process can be performed using the segmentation method presented in [25]. It has been shown before that segmentation of gas consumption process is better not to carry out on the vertices, but on the depressions, which does not allow "blurring" of statistical estimates after processing the studied implementation.

Consider the block diagram of the method of computer simulation of gas consumption process based on the known mathematical model (1) described in [26] for the process of relief formations (Fig. 1).

After applying the Caterpillar method, the trend component and the stochastic residue enter the computer simulation unit, except for the cyclic component. The cyclic component enters the block of statistical processing where the estimation of mathematical expectation and variance is carried out on the basis of the received information on the estimated rhythm function which is obtained on the basis of information on segmental and rhythmic structure. The estimated rhythm function, statistical estimates, trend component and stochastic balance are used for computer simulation of gas consumption process implementation.



Figure 1: Block diagram of the method of computer simulation of gas consumption process (known approach)

An example of result of gas consumption process computer simulation based on the block diagram (see Fig. 1) is given in Fig. 2.



Figure 2: Block diagram of the proposed method of computer simulation of gas consumption process

To adequately describe the real gas consumption process, it is also necessary to consider changes in the load amplitude on the segments-cycles, which are caused by various factors, such as climate (temperature, pressure, wind force and direction, humidity), changes in consumer topology [21]. For this reason, it is considered in the proposed mathematical model.

In the new design of mathematical model (1), the cyclic component (6) takes into account the segments-cycles of cyclic gas consumption process as multiplicative components considering the indicator functions and scale factors of gas consumption amplitude, i.e.

$$f_i(l) = \xi_{\omega}(l) \cdot \alpha_{\mathbf{W}_i}(l) \cdot I_{\mathbf{W}_i}(l), i = 1, C, \ l \in \mathbf{W}.$$

$$(11)$$

In formula (11), an additional component $\alpha_{\mathbf{W}_i}(l)$ that reflects the scale factors of gas consumption amplitude in each segment-cycle of the cyclic process, are introduced:

$$\alpha_{\mathbf{W}_{i}}(l) = \begin{cases} \alpha_{i}, l \in \mathbf{W}_{i}, \\ 0, l \notin \mathbf{W}_{i}. \end{cases} \quad i = \overline{1, C} ,$$
(12)

where α_i stands for the scale factors of gas consumption amplitude at every *i*-segment-cycle, are determined as follows:

$$\alpha_i = \frac{\alpha_{i\max}}{\alpha_{aver}}, \ i = \overline{1, C} , \tag{13}$$

where $\alpha_{i\max}$ is the maximum value of gas consumption range at *i*-segment-cycle (determined at the stage of segmentation of cyclic gas consumption process), α_{aver} is the average value of gas consumption range (the maximum value of estimation range of mathematical expectation, is determined at the stage of statistical processing of cyclic gas consumption). The block diagram of the proposed approach to gas consumption process simulation will look as in Fig. 2.

Consider and compare the results of computer simulation based on two approaches (two mathematical models, known and the proposed one).



Figure 3: Fragment of input implementation of cyclic gas consumption process



Figure 4: Estimated components of the mathematical model: a) trend component; b) stochastic residue



Figure 5: Fragments of the studied implementation of gas consumption process for the case of segmentation by depressions: a) cyclic component; b) results of segmentation of the cyclic component into segments-cycles (on the abscissa axis the data are given in conventional units, the specified number of samples)



Figure 6: Fragments of the result of estimated rhythm function (piecewise linear interpolation) of cyclic component of gas consumption process (based on segmentation into cycles by depressions)

Having obtained the segment structure $\hat{\mathbf{D}}_c$ and estimating the rhythmic structure (discrete rhythm function T(l,n)) by the methods proposed in [27], the methods of statistical processing were applied taking into account the rhythm function [27], while the estimation of mathematical expectation was determined:

$$\hat{m}_{\xi_{T(l,n)}}(l) = \frac{1}{M} \sum_{n=1}^{M} \xi_{\omega} (l + T(l,n)), l \in \mathbf{W}_{1} = [l_{1}, l_{2}),$$
(13)

where $l_1 \neq 0$ in the general case, l_1, l_2 are the discrete time samples which correspond to the beginning and end of the first segment-cycle, M is the number of cycles.

And the estimation of variance was determined as follows:

$$\hat{d}_{\xi_{T(l,n)}}(l) = \frac{1}{M} \cdot \sum_{n=1}^{M} \left[\xi_{\omega}(l+T(l,n)) - \hat{m}_{\xi_{T(l,n)}}(l+T(l,n)) \right]^2, \ l \in \mathbf{W}_1 = [l_1, l_2).$$
(14)

Applying the methods of statistical processing, we obtained statistical estimates of probabilistic characteristics (mathematical expectation $\hat{m}_{\xi_{T(l,n)}}(l), l \in \mathbf{W}_1$ and variance $\hat{d}_{\xi_{T(l,n)}}(l), l \in \mathbf{W}_1$ based on the rhythm function T(l,n) of the cyclic component of gas consumption process. Examples of the obtained estimates are given in Fig. 7.



Figure 7: Estimation of mathematical expectation and variance based on the estimated rhythm function of cyclic component of gas consumption process (segmentation into cycles by depressions): a) estimation of mathematical expectation; b) estimation of variance

Taking into account the obtained statistical estimates, carry out the computer simulation of cyclic components of gas consumption process implementations on the basis of two mathematical models (see Fig. 8).



Figure 8: Results of computer simulation of the cyclic component of gas consumption process based on two mathematical models: a) implementation of the cyclic component of gas consumption process of real data and simulated on the basis of a known model; b) implementation of the cyclic component of gas consumption process of real data and simulated on the basis of a proposed model



Figure 9: Results of computer simulation of the cyclic component of gas consumption process and the trend component based on two mathematical models: a) implementation of the simulated cyclic component of gas consumption process based on the known model and trend component; b) implementation of the simulated cyclic component of gas consumption process based on the proposed model and trend component



Figure 10: Results of computer simulation of gas consumption process based on two mathematical models: a) implementation of simulated cyclic component of gas consumption process based on a known model, trend component and stochastic residue; b) implementation of simulated cyclic component of gas consumption process based on the proposed model, trend component and stochastic balance

The root mean square absolute and relative errors of computer simulation of gas consumption

process were determined by the formulas:

$$\Delta_{q}(k) = \sqrt{\frac{1}{L} \sum_{l=1}^{L} \left(\xi_{\omega}'(l) - \xi_{\text{mod}\,q\omega}(l) \right)^{2}}; \ \delta_{q}(k) = \frac{\Delta_{q}(k)}{\sqrt{\frac{1}{L} \sum_{l=1}^{L} \xi_{\text{mod}\,q\omega}(l)^{2}}}, \ k = \overline{1, L}, q = \overline{1, 2},$$
(15)

where $\xi_{\text{mod }q\omega}(l)$ is the value of simulated gas consumption process implementation based on two mathematical models, $q = \overline{1,2}$; $\xi_{\omega}'(l)$ is the value of actual gas consumption implementation process; L is the number of implementations samples, $l = \overline{1,L}$; k is the sample for absolute and relative errors, respectively, $k = \overline{1,L}$. The results of the obtained absolute and relative errors are shown in the Fig. 11.



Figure 11: Results of determining the absolute error of computer simulation of gas consumption process on the basis of two mathematical models: a) on the basis of a known model; b) based on the proposed model



Figure 12: Results of determining the relative error of computer simulation of gas consumption process on the basis of two mathematical models: a) on the basis of a known model; b) based on the proposed model

4. Discussion of obtained results

Based on the obtained statistical estimates of the cyclic component (mathematical expectation and variance), computer simulation of cyclic components of gas consumption process implementations on the basis of two mathematical models was performed. After that, the trend component (see Fig. 9) and the stochastic residue (see Fig. 10) were added to the simulated cyclic components. The obtained results of computer simulation of gas consumption process implementations on the basis of the proposed mathematical model (see Fig. 10, b) taking into account scale factors allow to obtain a smaller relative modeling error (see Fig. 12, b) in comparison with the results on the basis of known mathematical

model (see Fig. 10, a), which allows more accurate computer simulation, and hence making a forecast based on this approach.

5. Conclusions

In the research, a new mathematical model in the form of an additive combination of three components, a cyclic random process taking into account the scale factors, a trend component and a stochastic residue, are developed. A new method of computer simulation of gas consumption process based on a proved mathematical model is proposed. The application of the proposed model taking into account the scale factors allowed to increase the accuracy of computer modeling, as evidenced by the results of the estimated errors in comparison with the known mathematical model.

In further research the values of scale factors obtained on the basis of aggregated data of climatic indicators in the developed mathematical model will be taken into account and comparative analysis of computer simulation of gas consumption based on a three-component mathematical model is going to be conducted.

6. References

- W. A. Gardner, A. Napolitano, L. Paura, Cyclostationarity: Half a century of research, Signal Processing. – 2005. – № 86 (2006). – P. 639-697. https://doi.org/10.1016/j.sigpro.2005.06.016.
- [2] H. L. Hurd, Periodically Correlated Random Sequences : Spectral Theory and Practice, The University of North Carolina at Chapel Hill Hampton University.
- [3] P. Kochel, Periodically stationary Markovian decision models, Elektron. Informationsverarb. Kybernet. – 1980. – No. 16. – P. 553-567 (in German).
- [4] A. R. Nematollahi, A. R. Soltani, Discrete time periodically correlated Markov processes, Probability and Mathematical Statistics. 2000. No. 20 (1). P. 127-140.
- [5] E. Ghysels, R. E. McCulloch, R. S. Tsay, Bayesian Inference for a General Class of Periodic Markov Switching Models, 1993.
- [6] I. Javorskyj, R. Yuzefovych, I. Kravets, I. Matsko, Methods of periodically correlated random processes and their generalizations. In: Cyclostationarity: Theory and Methods, 73–93 (2014).
- [7] E. Ghysels, On the Periodic Structure of the Business Cycle, Cowles Foundation, Yale Universiti. 1992. No. 1028. https://doi.org/10.2307/1392085.
- [8] Bittanti S. Markovian representations of cyclostationary processes, in: L. Gerencser, P.E. Caines (Eds.) / S. Bittanti, F. Lorito, S. Strada // Topics in Stochastic Systems : Modelling, Estimation and Adaptive Control. – Springer, Berlin, Germany. – 1991. – Vol. 161. – P. 31-46.
- [9] H.L. Hurd, A.G. Miamee, Periodically Correlated Random Sequences: Spectral Theory and Practice, Wiley, New York, 2006.
- [10] A. Napolitano, Asymptotic Normality of statisticalfunction estimators for generalized almostcyclostationary processes, in: Proceedings of XIII European Signal Processing Conference (EUSIPCO 2005), Antalya, Turkey, 2005.
- [11] E. Broszkiewicz-Suwaj, A. Makagon, R. Weron, A. Wy"oman'ska, On detecting and modeling periodic correlation in financial data, Physica A Statist. Mech. Appl. 336 (1–2) (2004) 196–205.
- [12] H.L. Hurd, A. Miamee, Periodically Correlated Random Sequences, Spectral Theory and Practice. Wiley, New Jersey (2007).
- [13] I. Javorskyi, Mathematical models and analyss of stochastic oscillations. Lviv, Karpenko Physico-Mechanical Institute (2013). (in Ukrainian)
- [14] R. Gutiérrez, A. Nafidi, R. Gutiérrez Sánchez, Forecasting total natural-gas consumption in Spain by using the stochastic Gompertz innovation diffusion model, Applied Energy 80(2005) 115–124
- [15] G. A. Darbellay, M. Slama, Forecasting the short-term demand for electricity. Do neural networks stand a better chance?, International Journal of Forecasting 16(2000) 71–83.
- [16] M. Thaler, I. Grabec, A. Poredoš, Prediction of energy consumption and risk of excess demand in a distribution system, Physica A 355(2005) 46–53.
- [17] T. Gelo, Econometric modelling of gas demand (In Croatian), Economic Review, 57(2006) 80– 96.

- [18] P. Potočnik, M. Thaler, E. Govekar, I. Grabec, A. Poredos, Forecasting risks of natural gas consumption in Slovenia, Energy Policy 35(2007) 4271–4282.
- [19] M. Pryimak Analiz enerhonavantazhen iz vykorystanniam liniinykh vypadkovykh protsesiv. Scientific Journal of the Ternopil state technical university 1999, Vol 4, No 1, pp. 84-87. [in Ukrainian].
- [20] B. Marchenko, N. Mulyk, M. Fryz, Kharakterystychna funktsiia umovnoho liniinoho vypadkovoho protsesu yak matematychnoi modeli hazospozhyvannia Electronics and control systems, 2006, No 3 (9), pp. 40–46. [in Ukrainian].
- [21] O. Nazarevych Vydilennia richnoho trendu yak adytyvnoi skladovoi chasovoho riadu hazospozhyvannia Scientific Journal of the Ternopil national technical university 2011, Vol 16, No 4, pp. 201-209. [in Ukrainian].
- [22] S. Lupenko, Cyclic functions and their classification in the tasks of modeling cyclic signals and oscillation system, Measuring and computing method in technological processes. - Khmelnytskyi, 2005. - #1. - P. 277-185.
- [23] Nina Golyandina, Anatoly Zhigljavsky, Singular Spectrum Analysis for Time Series, SpringerBriefs in Statistics. Springer-Verlag Berlin Heidelberg. 2020. P. 146.
- [24] N.E. Golyandina, V.V Nekrutkin, A.A. Zhigljavsky, Analysis of Time Series Structure: SSA and Related Techniques /. Boca Raton: Chapman&Hall / CRC, 2000. 305 p.
- [25] I.V. Lytvynenko, The method of segmentation of stochastic cyclic signals for the problems of their processing and modeling. Journal of Hydrocarbon Power Engineering, Oil and Gas Measurement and Testing. 2017, Vol. 4, No. 2, pp. 93–103.
- [26] Volodymyr Hutsaylyuk, Iaroslav Lytvynenko, Pavlo Maruschak, Volodymyr Dzyura, Georg Schnell, and Hermann Seitz, A new method for modeling the cyclic structure of the surface microrelief of titanium alloy ti6al4v after processing with femtosecond pulses Materials (Basel). 2020 Nov; 13(21): 4983. doi:10.3390/ma13214983.
- [27] I. Lytvynenko, A. Horkunenko, O. Kuchvara, Y. Palaniza, Methods of processing cyclic signals in automated cardiodiagnostic complexes, in: Proceedings of the 1st International Workshop on Information-Communication Technologies and Embedded Systems, Mykolaiv, Ukraine, 2019, pp. 116-127.