System Analysis of Educational Process

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Abstract
One of the most important issues of the social sphere is the effective management of complex educational systems. The relevance and importance of solving this problem are determined by the modernization of Russian education, the process of its digitalization, and the introduction of new innovative technologies. Various methods and models are used for the mathematical description of educational organizations and the study of their functioning. None of these methods separately can solve all the problems in this area completely successfully. Therefore, the most convenient and effective method to solve them is system analysis. This article considers the application of Markovian chains for the system analysis of the educational process. The structural scheme of the educational process is developed and the educational system processes are described using discrete and continuous Markovian chains and network modeling. A method for analyzing the stability of the system of Kolmogorov equations, which corresponds to the considered model of the educational process, is formulated. The study resulted in the developed mathematical model of the educational system based on the system analysis method. The method proposed in the article can be used not only for managing educational organizations but also in other areas, for example, in economics and agriculture

Keywords [1]
Markovian chain, model, the system of differential equations

1. Introduction

The administration of effective management of educational organizations is one of the key issues in the educational process [12]. This issue is becoming particularly relevant in our country in the context of Russian education modernization. Various methods and models are used to describe various processes in educational institutions. The main research area is the generalization of complex [8], system, process, competence [10] approaches, as well as the use of modern innovative technologies [5], including intelligent databases and knowledge [9].

For complex educational systems, the tool techniques of the Markovian chain theory are often used. In the articles [1, 3, 6, 7, 11] this mathematical tool technique is used to assess the indicators of the functioning of an educational institution and the creation of an educational environment. The article [4] takes up the use of Markovian chains for managing student learning. The use of Markovian processes for the study of the educational process should be considered from the system analysis perspective. In our opinion, insufficient attention has been paid to this topical issue in scientific research.

The purpose of this article is the application of Markovian chains for the system analysis of the educational process.

To achieve it, you need to address the following challenges:

1. To develop a structural scheme of the educational process.
2. To describe the processes in the system using discrete and continuous Markovian chains.
3. To examine the stability of the educational system.

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2. Scheme of the Educational Process System

The educational process takes place in a complex social system. The overall scheme of such a system is shown in Figure 1.

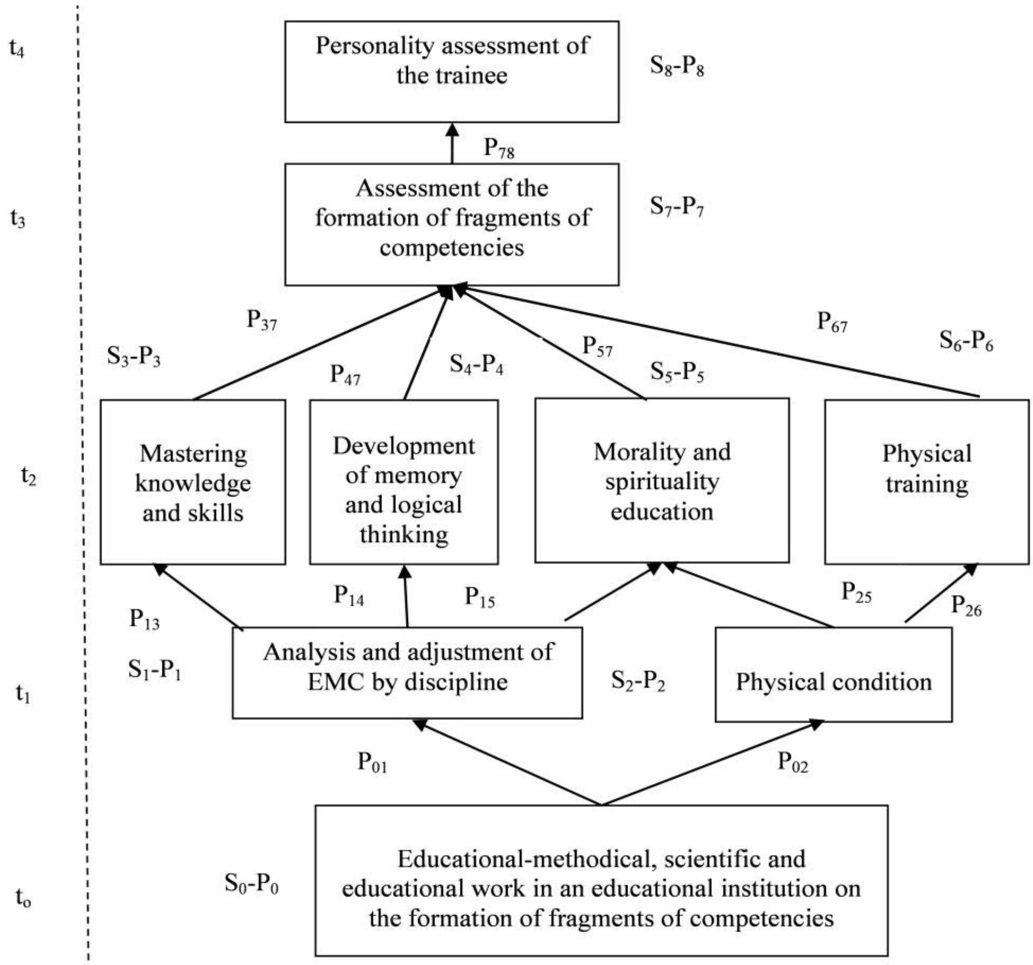

Figure 1: Structural scheme of the educational process

The following key shall apply for the scheme: S_i - system status; P_i -probability of finding the system in S_i; P_{ij} - probabilities of transition from S_i to S_j; $t_i\,(i=\overline{0,4})$ - system levels; EMC - educational and methodological complex. The numbering of the levels is made from the zero (initial) one. Transition probabilities P_{ij} have the meaning of a fraction of efforts aimed at solving problems of this structural level.

3. Discrete Markovian Chain

The conditions under which the learning process can be considered as Markovian are specified in the work [2].

For fixed stages, we t_i have a discrete Markovian chain, the matrix of transition probabilities of which is as follows:

$$\begin{pmatrix} P_{01} & P_{02} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{13} & P_{14} & P_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{25} & P_{26} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{37} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{47} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{57} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{78} \end{pmatrix},$$

provided that $P_0 = 1,\ P_1(\mathrm{t}_1) = P_{01},\ P_2(\mathrm{t}_1) = P_{02}$. (1)

The remaining probabilities of the states are obtained by applying the formula of total probability:

$$\begin{cases} P_3(t_2) = P_{13} \cdot P_1(t_1) = P_{13} \cdot P_{01}; P_4(t_2) = P_{14} \cdot P_1(t_1) = P_{14} \cdot P_{01}; \\ P_5(t_2) = P_{15} \cdot P_1(t_1) + P_{25} \cdot P_2(t_1) = P_{15} \cdot P_{01} + P_{25} \cdot P_{02}; \\ P_6(t_2) = P_{26} \cdot P_2(t_1) = P_{26} \cdot P_{02}; \\ P_7(t_3) = P_{37} \cdot P_3(t_2) + P_{47} \cdot P_4(t_2) + P_{57} \cdot P_5(t_2) + P_{67} \cdot P_6(t_2) = \\ = P_{37} \cdot P_{13} \cdot P_{01} + P_{47} \cdot P_{14} \cdot P_{01} + P_{57} \cdot (P_{15} \cdot P_{01} + P_{25} \cdot P_{02}); \\ P_8(t_4) = P_{78} \cdot P_7(t_3) = P_{78} \cdot P_{37} \cdot P_{13} \cdot P_{01} + \\ P_{78} \cdot P_{47} \cdot P_{14} \cdot P_{01} + P_{78} \cdot P_{57} \cdot P_{15} \cdot P_{01} + P_{78} \cdot P_{57} \cdot P_{25} \cdot P_{02}. \end{cases} \tag{2}$$

Let's write down the normalizing conditions:

$$\begin{cases} P_1(t_1) + P_2(t_1) = 1, \text{ т.е. } P_{01} + P_{02} = 1; \\ P_3(t_2) + P_4(t_2) + P_5(t_2) + P_6(t_2) = 1, \text{т.е,} P_{13}P_{01} + P_{14}P_{01} + P_{15}P_{01} + P_{24}P_{02} = 1; \\ P_7(t_3) = 1, \text{ т.е. } P_{37} \cdot P_{13} \cdot P_{01} + P_{47} \cdot P_{14} \cdot P_{01} + \\ +P_{57} \cdot P_{15} \cdot P_{01} + P_{57} \cdot P_{25} \cdot P_{02} \cdot P_{25} \cdot P_{02} = 1; \\ P_8(t_4) = 1, \text{ т.е. } P_{78} \cdot P_{37} \cdot P_{13} \cdot P_{01} + P_{78} \cdot P_{47} \cdot P_{14} \cdot P_{01} + \\ +P_{78} \cdot P_{57} \cdot P_{15} \cdot P_{01} + P_{78} \cdot P_{57} \cdot P_{25} \cdot P_{02} = 1. \end{cases} \tag{3}$$

The transition probabilities can be estimated based on the experimental data. Then, based on conditions (1), systems (2), and (3), the probabilities of the system states can be found.

For the final state, we S_8 specify the minimum threshold P_8^*, its probability is $P_8\,(P_8^* \le P_8)$. If it follows from (1), (2), and (3) that $P_8^* < P_8$, this means disorganization of functioning of the system under consideration, and it follows that certain conditions shall be created to increase the transition probabilities.

In this case, the probabilities of processes in the system shall be considered.

4. Continuous Markovian Chain

For a continuous Markovian chain, the time interval from the system in the state S_0 to the state S_8 is considered continuous. Instead, the P_{ij} flow densities λ_{ij} are considered. The flows can be directed from S_i to S_j (at *j* > *i*) and from S_j to S_i (at *j* > *i*). The density of the reverse flow is specified as μ_{ij}. E.g., control commands of higher levels of the system can act as reverse flows.

For the Markovian process, we can write a system of Kolmogorov equations.

For a sufficiently large interval of observation time *t*, all the flow densities λ_{ij}, μ_{ij} and probabilities P_i $(i = \overline{0,8})$ can be considered constant values, and all probabilities $P_i'(t)$ $(i = \overline{0,8})$ equal to zero.

This mode is the system's stationary mode. In this case, we have a homogeneous system of algebraic equations.

Thus, the functioning of the lower block associated with the state S_0 in the stationary mode is described by the algebraic equation

$$-P_0(\lambda_{01}+\lambda_{02})+P_1\mu_{10}+P_2\mu_{20}=0.$$

The functioning of the block «Analysis and Correction of the educational and methodological complex for the Discipline» is described by the equation

$$P_0\lambda_{01}-P_1\lambda_{13}-P_1\lambda_{14}-P_1\lambda_{15}-P_1\mu_{10}+P_3\mu_{31}+P_4\mu_{41}+P_5\mu_{51}=0.$$

The other blocks are described in the same way.

We P_i $(i=\overline{0,8})$ find the sought probabilities by solving a system of algebraic equations for given values λ_{ij} and μ_{ji} $(i,j=\overline{0,8})$. The value is P_8 compared with the threshold value P_8^*. When doing the inequality, it is $P_8^*>P$ necessary to change the densities λ_{ij} and μ_{ji} of transferring flows.

5. Network Representation of the Educational Process Acknowledgements

The system shown in Fig. 1 can be represented as a graph (Figure 2), the points of which are the states S_i $(i=\overline{0,8})$, and the arcs are the connections between the structural elements of the system indicated in the figure by arrows. Such a graph that has a single initial and a single final point is called a network. The points of the network are called "events", the path passing through these points is called "work".

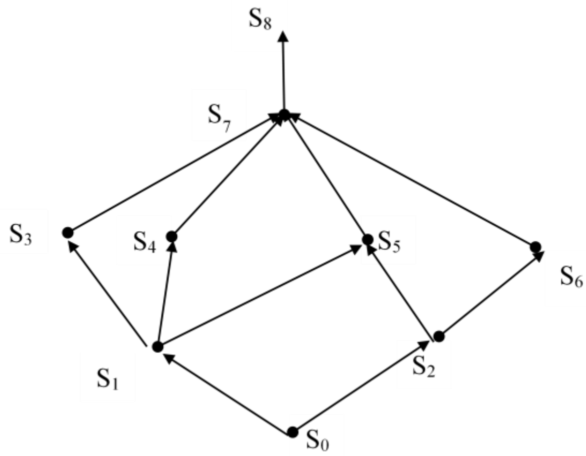

Figure 2: Educational process curve

Each state of the S_i zero, first and second levels, respectively, $i=\overline{0,8}$ also represents a system that is shown as a network. Either conditional transition probabilities or densities of transferring flows can be considered as the "loads" in the network.

Any network can be described using certain parameters related to the time characteristics of the process:

1. The early (late) dates of the occurrence of each event; are connected to the transition to this point of the graph (this state) after the completion of all previous works and events.
2. Early (late) dates of the start and end of each work.
3. Time reserves of events and works.
4. Full path (the path connecting the initial point of the network to the final one).
5. The critical path is the longest of the complete paths.

The following probabilistic characteristics of the network can be considered:

1. The length of the event path S_i is the sum of all the transition probabilities P_{kl} (or densities λ_{kl}) that specify the "loads" on the arcs that determine the path from the initial point to the point S_i.
2. The minimum (maximum) probability of the occurrence of the *i*-th event (reaching the point S_i) is equal to the sum of the transition probabilities related to all the paths leading from the initial point to the point S_i.

3. Directive probability is the probability of reaching the final state; it is equal to the minimum probability of reaching the final point.
4. A full path is any path connecting the start and endpoints of the network.
5. The critical path on the graph is the path to which the highest total probability is attributed.
6. Reserve probability of occurrence of an event is the maximum probability by which the probability of occurrence of such event can be reduced without reducing the directive probability.

The proper functioning of an educational institution should be stable. Stability is understood as the preservation of the main characteristics of the system under external impacts.

For an educational system, sustainability is determined by its ability to function under changing conditions of the internal and external environment.

Factors of the external environment are, for example, the state of the economy, the development of technology, the socio-cultural sphere, the legal system, the demographic situation, the standard of living of the region.

The internal environment includes finance, personnel, organizational culture, image, number of students and quality indicators of students, specialties, the level of qualification of professors and teachers and scientific potential, the organization of a system for monitoring the quality of training, the state of educational laboratory, instrumental, library and sports bases.

The system is structurally stable when in case of sufficiently small structural changes, its functioning does not change significantly.

Suppose the process is described by a system of equations of the type

$$\frac{dx_i}{dt} = a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n,\ i = \overline{1,n}, \tag{4}$$

where all x_i - time-varying functions *t*, a_{ij} $\left(i, j = \overline{1,n}\right)$ - general characteristics of the functioning of the element S_{ij}.

In this case, the method developed in [2] can be used to study its stability.

Using the matrix $A = \{a_{ij}\}$, we make up the determinant of the following form:

$$\begin{vmatrix} (a_{11}-\delta) & a_{12} & ... & a_{1n} \\ a_{21} & (a_{22}-\delta) & ... & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & ... & (a_{nn}-\delta) \end{vmatrix} = 0, \tag{5}$$

which is set to zero, and an algebraic equation of the *n-th* order is obtained:

$$\delta^n + b_1\delta^{n-1} + ... + b_{n-1}\delta + b_n = 0. \tag{6}$$

A square Hurwitz square matrix is formed based on the coefficients of this equation:

$$\Gamma = \begin{pmatrix} b_1 & 1 & 0 & 0 & ... & 0 \\ b_3 & b_2 & b_1 & 1 & ... & 0 \\ b_5 & b_4 & b_3 & b_2 & ... & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & ... & b_n \end{pmatrix},$$

where $b_m = 0$ at $m > n.$

For the asymptotic stability of the system (4), all the principal diagonal minors of the corresponding Hurwitz matrix are:

$$\Delta_1 = b_1,\ \Delta_2 = \begin{vmatrix} b_1 & 1 \\ b_3 & b_2 \end{vmatrix},\ \Delta_3 = \begin{vmatrix} b_1 & 1 & 0 \\ b_3 & b_2 & 1 \\ b_5 & b_4 & b_3 \end{vmatrix}, ...,\ \Delta_n = b_n \cdot \Delta_{n-1},$$

are positive, i.e. $\Delta_i > 0$ when $i = 1,2,...,n$.

Thus, it is possible to determine the stability of the management system of an educational organization.

6. Conclusion

In this study, a detailed structural scheme of the educational process has been developed. To describe the processes occurring in the system, the mathematical tool techniques of discrete and continuous Markov chains are used. Formula expressions for calculating the probabilities of the system states are obtained. It is shown how the scheme of the educational process can be represented in the form of a network graph. The study determines the parameters associated with the time characteristics of the process. A new approach to the analysis of the stability of the educational system under consideration has been proposed.

The novelty of the proposed work is in the fact that a comprehensive analysis of the educational system (structure, functioning, stability) has been carried out using the method of system analysis.

A systematic description of the educational process can be widely used to find control flows and calculate indicators that determine the stable functioning of not only educational systems but also other social and economic systems for arranging effective management.

7. References

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