

# PWK Agent Belief Suspension in Argumentation

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## Abstract

In this article, we consider argumentation as an epistemic process used by an agent to improve beliefs and gain knowledge according to the information provided by the environment. While producing an argument an agent needs to revise her/his beliefs based on some new information. Such a process can generate a suspension in the argumentative process. There might be two kinds of suspensions of information flow: critical suspension and non-critical suspension. In this short paper, we distinguish these two kinds of suspensions and we sketch a formalization of them that consists in considering an expansion of AGM with Paraconsistent Weak Kleene Logic (PWK) – where the third value of PWK means *off-topic*.

## Keywords

AGM, Paraconsistent Weak Kleene Logic, Suspension, Critical and Non-critical Error

## 1. Introduction

According to [1, 2, 3], argumentation theory, the logical study of non-monotonic reasoning, and the dialogical study of inter-agent communication are closely connected in the field of Artificial intelligence. The connection and influence between belief revision and argumentation are also largely discussed (see [4, 5, 6]). Here we consider one specific aspect where argumentation is viewed as an epistemic process of an agent to improve her/his beliefs and gain knowledge through some new information from her/his external environment. Moreover, we also consider the topic of an argumentation process as the subject matter of some question; e.g., an argumentation on “How many stars are there?” consists of arguments about the number of stars instead of arguments about the number of seats in a library.

While producing an argument an agent needs to revise her/his beliefs based on some new information from the environment. Such a process can generate a suspension. In [7], we propose to consider two kinds of suspensions in an epistemic process: non-critical suspension and critical suspension. When an agent neither believes nor disbelieves certain information, such a suspension is non-critical. It is non-critical because the agent can form a judgment or continue to process an argument as long as she/he gains more information from her/his environment. A non-critical suspension can be modeled through the AGM paradigm [8]. A critical suspension happens when an agent gains some irrelevant and even malicious information from the environment. Critical suspension cannot be fixed in the subsequent epistemic process

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and should be prevented and filtered. These two kinds of suspensions correspond to what in the computation is taken as two kinds of errors: 1) critical errors and 2) non-critical errors. A critical error globally stops the program, which means the error cannot be fixed in the subsequent computational process. A non-critical error, instead, partially stops the computation program, and the error can be fixed in the subsequent computational process. In this short paper, we sketch a proposal that consists in considering an expansion of AGM with Paraconsistent Weak Kleene Logic PWK, where the third value of PWK means *off-topic*. According to this new interpretation, if a proposition obtains the third value **u**, it means the proposition is off-topic. A PWK belief revision theory is sketched accordingly. Within our framework of PWK belief revision theory, we characterize a non-critical suspension and a critical suspension and distinguish one from the other.

## 2. PWK and Off-topic

In the field of many-valued logics, weak Kleene systems are a greatly underdeveloped subject compared to their strong counterparts (on these systems see, for example, [9]). The language of paraconsistent weak Kleene (PWK) is the standard Backus-Naur Form (BNF):

$$\Phi_L ::= p \mid \neg\alpha \mid \alpha \vee \beta \mid \alpha \wedge \beta \mid \alpha \supset \beta.$$

**Definition 2.1** (Valuation). *A valuation  $V : \Phi_L \mapsto \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$  is induced by Table 1.*

**Table 1**

Weak tables for logical connectives in  $L$

	$\neg\alpha$		$\alpha \vee \beta$	<b>t</b>	<b>u</b>	<b>f</b>		
<b>t</b>	<b>f</b>	<b>t</b>	<b>t</b>	<b>t</b>	<b>u</b>	<b>t</b>		
<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>		
<b>f</b>	<b>t</b>	<b>f</b>	<b>f</b>	<b>t</b>	<b>u</b>	<b>f</b>		
	$\alpha \wedge \beta$	<b>t</b>	<b>u</b>	<b>f</b>	$\alpha \supset \beta$	<b>t</b>	<b>u</b>	<b>f</b>
<b>t</b>	<b>t</b>	<b>u</b>	<b>u</b>	<b>f</b>	<b>t</b>	<b>t</b>	<b>u</b>	<b>f</b>
<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>	<b>f</b>	<b>u</b>	<b>u</b>	<b>u</b>	<b>u</b>
<b>f</b>	<b>f</b>	<b>u</b>	<b>f</b>	<b>f</b>	<b>f</b>	<b>t</b>	<b>u</b>	<b>t</b>

Table 1 shows the complete *weak tables* by supplying the third value throughout the row and column headed by the third value. The way **u** transmits is usually called *contamination* (or *infection*), as the value propagates from any  $\alpha \in \Phi_L$  to any construction  $\otimes(\alpha, \beta)$  independently from the value of  $\beta$  (here,  $\otimes$  is any connective defined in terms of  $\neg, \vee, \wedge, \supset$ ).

The following is a straightforward and intuitive expression of *contamination*:

**Fact 2.1** (Contamination). *For all formulas  $\alpha$  in  $L$  and any valuation  $V$ :*

$$V(\alpha) = \mathbf{u} \Leftrightarrow V(p) = \mathbf{u} \text{ for some component } p \text{ of } \alpha$$

It is interesting to observe that negation in PWK works like in strong Kleene, but conjunction and disjunction in PWK work differently from strong Kleene. In particular, the interpretation of disjunction is not *max* and the interpretation of conjunction is not *min*. The logical consequence is defined as the preservation of non-false value:

**Definition 2.2.**  $\Gamma \models_{\text{pwk}} \Delta$  iff there is no valuation  $V$  such that:

$$V(\alpha) \neq \mathbf{f} \text{ for all } \alpha \in \Gamma \text{ and } V(\beta) = \mathbf{f} \text{ for all } \beta \in \Delta.$$

PWK is reflexive, transitive and monotonic. Although PWK has this last property in the sense that if  $\Gamma \models_{\text{pwk}} \Delta$  then  $\Gamma \cup \{\alpha\} \models_{\text{pwk}} \Delta$ , given the behaviour of conjunction in the premise side PWK has a “non-monotonic flavour” in the sense that, for example,  $p \models_{\text{pwk}} p$  but  $p \wedge q \not\models_{\text{pwk}} p$ . Observe that the inclusion of all the atoms of a premise set  $\Gamma$  in a conclusion set  $\Delta$  guarantees that if  $\Gamma \models_{\text{cl}} \Delta$  then  $\Gamma \models_{\text{pwk}} \Delta$ .  $\models_{\text{cl}}$  is the classical consequence relation.

### 2.1. Off-topic Interpretation and Computational Errors

Recently, the third value **u**—initially understood as *nonsense*, *meaninglessness* or *undefined*—has been studied in more depth. A new proposal by [10] suggests reading it as *off-topic*. Thus, a proposition that obtains the third value should be regarded as being *off-topic*. Through this new interpretation, we can consider a correspondence between the computational errors and suspension [7]. In a computational program, there are two kinds of computational errors: 1) critical error and 2) non-critical error: a critical error stops the program in a global way, which means the error cannot be fixed in the subsequent computational process; a non-critical error partially stops the computation program, and the error can be fixed in the subsequent computational process. In the next section, we propose a framework of PWK belief revision theory, in which a non-critical error corresponds to a non-critical suspension and that a critical error corresponds to a critical suspension.

## 3. A PWK belief revision theory

### 3.1. Topic

We consider a PWK belief revision theory as an expansion of the AGM belief revision theory. In the AGM paradigm, an agent’s belief state is formalized as a belief set  $\Theta = Cn(\Theta)$  in which  $Cn$  is a consequence operation.

As for a certain proposition, an AGM agent might believe it, disbelieve it, or keep it in suspension. Three basic kinds of operators model the belief changes of an agent: – expansion  $+_{AGM}$ , contraction  $-_{AGM}$ , and revision  $*_{AGM}$ . Suppose that  $\Theta$  is a belief set and  $\alpha$  is a proposition. Belief expansion  $\Theta +_{AGM} \alpha$  means the agent expands her beliefs with a new proposition  $\alpha$ ; belief contraction  $\Theta -_{AGM} \alpha$  means the agent has to contract  $\alpha$  from her beliefs in a way that  $\alpha$  will not be derived again after the contraction; belief revision  $\Theta *_{AGM} \alpha$  means the agent has to accommodate  $\alpha$  into  $\Theta$  in a way that a possible contradiction brought by  $\alpha$  can be removed at the lowest cost.

A PWK agent's belief state – differently from the AGM belief set – concerns a topic, which corresponds to a set of propositions as answers to a question provoked by certain argumentation. For example, for a question “How many stars are there?” the topic set can be {“No stars are there.”, “One star is there.”, “Two stars are there.”, “One star is there or two stars is there.”...}. Whether a proposition  $\alpha$  is on-topic or off-topic also depends on whether  $\alpha$  contains an off-topic component atomic proposition: this is due to the contamination feature of PWK. Taken the same example above, an argumentation process concerning the number of stars would regard an argument such as “There are five empty seats in the library” as being off-topic.

### 3.2. PWK Belief State

A PWK agent's epistemic attitude toward a given proposition  $\alpha$  from the environment depends on whether  $\alpha$  is on-topic or off-topic. If  $\alpha$  is on-topic, the agent would believe, disbelieve it or keep it in non-critical suspension. If  $\alpha$  is off-topic, the agent would keep it in critical suspension. Non-critical suspension and critical suspension are two exclusive attitudes:

- 1) If  $\alpha$  is in non-critical suspension,  $\alpha$  is still available to be believed or disbelieved by the agent later. Therefore, an on-topic sentence  $\alpha$  can be released from non-critical suspension. We still consider this kind of suspension an error because it might be problematic at the moment when the agent has to decide about the epistemic status of  $\alpha$ . In this case, the non-critical suspension of  $\alpha$  also suspends the agent from doing something else. The agent may collect more information or require more reliable information sources to release  $\alpha$  from non-critical suspension. Computationally speaking, it is a non-critical error.
- 2) If  $\alpha$  is off-topic, then  $\alpha$  should be isolated from the current belief change process and be kept in critical suspension.  $\alpha$ 's being off-topic could be the result of erroneous information, which reflects some aspects of the environment. Computationally speaking, it is a critical error.

**Definition 3.1** (PWK Belief State). *A PWK agent's belief state is  $\langle \Theta, \Delta, \Sigma \rangle$ .  $\Theta$ ,  $\Delta$  and  $\Sigma$  are all sets of PWK propositions, i.e.  $\Theta, \Delta, \Sigma \subseteq \Phi_L$ :*

$\Theta$  is a belief set if  $\Theta = Cn(\Theta) \setminus \{\alpha \in \Phi_L \mid \alpha \text{ is off-topic}\}$ ;

$\Delta$  is a non-critical suspension set if  $\Delta = \Delta \cup \{\neg\alpha \mid \alpha \in \Delta\}$ , for any  $\alpha \in \Delta$ ,  $\alpha$  is on-topic;

$\Sigma$  is a critical suspension set if for any  $\alpha \in \Sigma$ ,  $\alpha$  is off-topic;

$\Theta \cap \Delta = \Theta \cap \Sigma = \Delta \cap \Sigma = \emptyset$ ,  $\Theta \cup \Delta \cup \Sigma \subseteq \Phi_L$ .

### 3.3. Three Belief State Change Operators

**Definition 3.2** (PWK Belief State Change  $\phi$ ). *We denote three PWK belief state change operators, expansion, contraction and revision, as  $\phi$ .  $\phi$  takes a PWK belief state  $\langle \Theta, \Delta, \Sigma \rangle$  and a*

propositional input  $\phi$  as two variables. It can be defined from  $\langle \langle \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L) \rangle, \Phi_L \rangle$  to  $\langle \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L) \rangle$ :

$$\mathcal{P}(\langle \Theta, \Delta, \Sigma \rangle, \phi) = \begin{cases} \mathcal{P}(\langle \Theta, \Delta, \Sigma \rangle, \phi) & \text{if } \phi \text{ is on-topic,} \\ \mathcal{P}(\langle \Theta, \Delta, \Sigma \rangle, \phi) & \text{if } \phi \text{ is off-topic.} \end{cases}$$

### 3.4. Specific Operators

In this subsection, we propose a concrete model for PWK belief change operators to show how these ideas work and how AGM operators could also be preserved within the framework.

**Definition 3.3** (PWK Plain Expansion  $\mathcal{P}^+$ , Contraction  $\mathcal{P}^-$ , Revision  $\mathcal{P}^*$ ). *The expansion, contraction and revision of a belief state  $\langle \Theta, \Delta, \Sigma \rangle$  with respect to a new proposition  $\phi$  can be seen as operators defined from  $\langle \langle \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L) \rangle, \Phi_L \rangle$  to  $\langle \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L), \mathcal{P}(\Phi_L) \rangle$ :*

$$\mathcal{P}^+(\langle \Theta, \Delta, \Sigma \rangle, \phi) = \begin{cases} \langle \Theta +_{AGM} \phi, \Delta, \Sigma \rangle & \text{if } \phi \text{ is on-topic,} \\ \langle \Theta, \Delta, \Sigma \cup \{\phi\} \rangle & \text{if } \phi \text{ is off-topic.} \end{cases}$$

$$\mathcal{P}^-(\langle \Theta, \Delta, \Sigma \rangle, \phi) = \begin{cases} \langle \langle \Theta, \Delta \rangle -_{AGM} \phi, \Sigma \rangle & \text{if } \phi \text{ is on-topic,} \\ \langle \Theta, \Delta, \Sigma \rangle & \text{if } \phi \text{ is off-topic.} \end{cases}$$

$$\mathcal{P}^*(\langle \Theta, \Delta, \Sigma \rangle, \phi) = \begin{cases} \langle \langle \Theta, \Delta \rangle *_{AGM} \phi, \Sigma \rangle & \text{if } \phi \text{ is on-topic,} \\ \langle \Theta, \Delta, \Sigma \cup \{\phi\} \rangle & \text{if } \phi \text{ is off-topic.} \end{cases}$$

Three AGM operators,  $+_{AGM}$ ,  $-_{AGM}$  and  $*_{AGM}$  have been embedded into sub-operators  $\mathcal{P}^+$ ,  $\mathcal{P}^-$ , and  $\mathcal{P}^*$  to deal with the belief change concerning an on-topic input.  $\mathcal{P}^+$ ,  $\mathcal{P}^-$ , and  $\mathcal{P}^*$  are independent from them. They collect and preserve the off-topic inputs. In this way, we can show how the on-topic part of a belief state changes concerning an on-topic proposition; at least, we can see clearly that the PWK belief change is not merely expansion. The above definitions of belief state change operators also show the insulation feature of a PWK epistemic agent's mind.

**Theorem 3.4.** *AGM postulates agree with a PWK belief change framework.*

*Proof.* According to the definition 3.3, AGM operators are adopted to deal with the on-topic part of PWK belief change.  $+_{AGM}$ ,  $-_{AGM}$ , and  $*_{AGM}$  are embedded into  $\{\mathcal{P}^+, \mathcal{P}^-, \mathcal{P}^*\}$ . As long as AGM operators follow AGM postulates,  $\{\mathcal{P}^+, \mathcal{P}^-, \mathcal{P}^*\}$  do as well. Therefore, AGM postulates, which regulate  $\{\mathcal{P}^+, \mathcal{P}^-, \mathcal{P}^*\}$ , also support this PWK belief change framework based on  $\{\mathcal{P}^+, \mathcal{P}^-, \mathcal{P}^*\}$ .  $\square$

## 4. Concluding Remarks

In this article we briefly sketch the basic elements of a PWK belief revision theory, a theory which can accommodate two kinds of agent belief suspensions, a distinction useful in an argumentation process.

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