Information support of controlling influences formation using fractional order controllers

Yaroslav Marushchak[1] and Bohdan Kopchak[1]

[1] *Lviv Polytechnic National University, 12 Bandera Street, Lviv, 79013, Ukraine*

Abstract
Computer research of algorithm and programs realization of differentiating and integrating fractional order parts, as components of PI$^{\lambda}$D$^{\mu}$ controllers, has shown the application efficiency of fractional order transfer function approximation. The application of the decomposition theorem of rational fractions allowed to construct structural schemes from parallel connected aperiodic parts for the realization of approximated arbitrary order transfer functions. It has been experimentally proved that the implementation of fractional order controllers based on approximated transfer functions can work in real time as the integral part of highly dynamic automatic control systems. Tests of the frequency converter MFC 710 option with the PI$^{\lambda}$D$^{\mu}$ fractional order controller in the speed control system using the Twerd experimental stand have confirmed its efficiency in terms of expanding the regulatory capabilities of such automatic control systems.

Keywords
Fractional order, PI$^{\lambda}$D$^{\mu}$ controller, Oustaloup transformation, electromechanical systems

1. Introduction

Construction of automatic control systems (ACS) using fractional order controllers significantly expands the possibilities compared to conventional controllers. Control influences, which are formed by controllers, provide the specified indicators of control of electric power and technological processes. In [1-9] the advantages of ACS for the use of fractional PI$^{\lambda}$D$^{\mu}$ controller with transfer function are shown

$$W_C(s) = k_p + k_i s^{-\lambda} + k_d s^{\mu}.$$

In such controller I- and D-fractional order components give a wider range of settings. Naturally, in addition to the values of proportional, differential and integral components k_p, k_i and k_d, the fractional order controller has two more parameters: fractional powers λ and μ of the Laplace operator s in the integrator and differentiator, respectively.

2. Research Analysis

In order to find possible ways to implement fractional order controllers, an analysis of the use of Riemann, Riemann-Liouville and Grunwald-Letnikov representations for the construction of such controllers was carried out. To calculate the transition functions for the fractional order integrating controller in the Riemann representation.

$$D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) dt, \tag{1}$$

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EMAIL: yaroslav.y.marushchak@lpnu.ua (Y. Marushchak); bohdan.l.kopchak@lpnu.ua (B. Kopchak)
ORCID: 0000-0002-7901-3343 (Y. Marushchak); 0000-0002-2705-8240 (B. Kopchak)

and a differential fractional order controller in the Riemann-Liouville representation

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(\mathrm{n}-\alpha)} (\frac{d}{dt})^n \int_{a}^{t} \frac{f(\tau)dt}{(t-\tau)^{\alpha-n+1}}, \tag{2}$$

appropriate programs have been developed. Based on the analysis of the obtained results, the following problems were revealed:

- the right integration limit for expressions (1) and (2) gives a division by zero;
- the calculation of each subsequent point of the transition process of the integral fractional part requires the presence of all previous values of the sub integral function (process input signal) starting from zero, and therefore each subsequent point requires a larger amount of calculations. To calculate the current value of the function, it is necessary to remember the value of the function at all previous points in time. Thus, the CPU load increases and thus significantly complicates the work of such controllers in the ACS, where there are transition processes.

In the literature on the implementation of fractional order controllers, there is a reference to the integral-differential fractional order controller model with TF $s^{\pm\alpha}$ in the representation of Grunwald-Letnikov [10-11]:

$$D_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \frac{\Gamma(\alpha+1)\mathrm{f}(\mathrm{t}-\mathrm{jh})}{\Gamma(\mathrm{j}+1)\Gamma(\alpha-\mathrm{j}+1)}.$$

The advantages of this presentation are:

- the formula is easy to use because it is written on the basis of a finite sum, not an integral;
- the model for the representation of integrative-differentiating controllers by the Grunwald-Letnikov formula provides a higher, compared to the above models, the speed of calculations;
- the same formula is used to represent the integrating or differentiating fractional order controller, only the sign of fractional order is changed ("+" for differentiator, "-" for integrator).

The main disadvantages of this model are that the calculation of the transition process of the integrating or differentiating of fractional order controller is complicated by the presence in the formula of gamma functions, attempts to increase the accuracy of determining which significantly increases the calculation time of the transition process.

Thus, the application of these methods for the physical implementation of fractional order controllers actually makes it impossible to use them in high-speed systems with long-term operating conditions. Such shortcomings have been partially eliminated in [12-13], where fractional order controllers are used in indoor climate automation and supercapacitor charge systems. It should be noted that such systems are characterized by low speed.

If we consider ACS with fractional order controllers that form control effects for various electric power and electromechanical systems, the main problem to be solved is the operation of such controllers in real time, when dynamic processes are characterized by high speed.

3. Problem Solving

Given that the implementation of integer-order controllers is well developed in both analog and digital execution, the problem of technical implementation of synthesized fractional order controllers in such systems can be solved by equivalent replacement (approximation) of their transfer functions (TF) to integer order TF. Equivalence implies the provision of the same transition functions and frequency characteristics in the appropriate frequency range for both TF representations. Such an approximation can be performed using the known formulas of Oustaloup (Oustaloup A.) transformation [14]. According to this method, given the lower and upper levels of the frequency range ω_l, ω_h, for which the equivalence of the frequency characteristics of fractional controllers both representations, we can write the following expression of the approximation of the integrating and differentiating fractional order parts α

$$s^{\pm\alpha} = \left(\frac{\omega_u}{\omega_h}\right)^{\alpha} \prod_{k=-N}^{k=N} \frac{1 + s/\omega'_k}{1 + s/\omega_k}, \tag{3}$$

where $\omega_u = \sqrt{\omega_l \omega_h}$; N – order of approximation to be specified; ω'_k, ω_k, − zeros and poles of equivalent integer order TF, respectively.

The calculation of zeros and poles of the approximate integer order TF is carried out according to the following expressions:

$$\omega'_k = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(k+N+0{,}5-0{,}5\cdot\alpha)/(2N+1)}, \tag{4}$$

$$\omega_k = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(k+N+0{,}5+0{,}5\cdot\alpha)/(2N+1)}. \tag{5}$$

Let us denote $k_{п} = \left(\frac{\omega_u}{\omega_h}\right)^{\alpha}$ the gain of the approximating TF.

In the general case, the order of approximation is possible at (2N + 1) levels. The idea is that to replace of the fractional order TF with the whole order TF, the coefficient, zeros and poles of the expected TF are first calculated. In the next step, using the found zeros and poles, the TF is written in the form

$$W(s) = k\frac{(s-\omega'_1)(s-\omega'_2)\dots(s-\omega'_{2N+1})}{(s-\omega_1)(s-\omega_2)\dots(s-\omega_{2N+1})}, \tag{6}$$

where $\omega'_1, \omega'_2 \dots \omega'_{2N+1}$ calculated according to (4) the values of the zeros of the integer order TF; $\omega_1, \omega_2 \dots \omega_{2N+1}$ − calculated according to (5) the poles of the integer order TF.

We present TF (6) as the ratio of polynomials

$$W(s) = k\frac{b_{2N+1}s^{2N+1} + b_{2N}s^{2N} + \dots + b_1 s + b_0}{a_{2N+1}s^{2N+1} + a_{2N}s^{2N} + \dots + a_1 s + a_0} = \frac{P(s)}{Q(s)}. \tag{7}$$

To verify the adequacy of the replacement of fractional order TF by the integer order TF, a comparative analysis of their logarithmic frequency characteristics was performed. For this purpose, given the order of approximation (2N+1), the frequency characteristics of both TFs were calculated.

The expression $s^{\pm\alpha}$ can be interpreted as the expression of fractional differentiator (+α) or integrator (-α), by means of which the total TF of fractional $PI^{\lambda}D^{\mu}$ controllers is formed. Therefore, the following analysis is devoted to such TF. To implement it in the MATLAB environment, a program was developed that implements the Oustaloup method according to (3) to approximate $s^{\pm\alpha}$ by the integer order TF.

Using the developed program, it is possible to transform differential-integral parts of fractional order TF with different degrees α in a certain frequency range provided that the order of approximation changes within N = 1÷5. The frequency range for the differential part is selected within (0.01÷100)s^{-1}, and for the integral - the range (0.001 ÷ 1000) s^{-1}.

Let us represent the differential and integral fractional order parts $s^{\pm\alpha}$ with powers: α = -1; -0.75; -0.5; -0.25; 0; 0.25; 0.5; 0.75; 1 by the integer order TF parts.

Using the developed program, we found approximating TF of differentiation and integrating fractional order parts in the frequency range 0.01-100 s^{-1} for different orders of approximation N. Some of them are shown in Table 1.

Logarithmic frequency characteristics were constructed for the TF approximation expressions obtained in this way.

As shown in [14], the frequency characteristics obtained for the corresponding components of the integer order controller (α = ± 1.0) based on the Oustaloup A. approximation completely coincide with the well-known results for integer TFs. This confirms the correctness of using the Oustaloup A. approximation for the partial case of $PI^{\lambda}D^{\mu}$ controllers when λ and μ are integers. In addition, it is shown that the value of N does not affect these frequency characteristics.

In the case of fractional differential components of the controllers, the logarithmic frequency characteristics are shown in Figure 1a and Figure 1b.

Figure 1a and Figure 1b show the Bode diagrams of differential (α = 1.0) and integral (α = -1.0) parts when N = 1 (curve 1), N = 2 (curve 2), N = 3 (curve 3), N = 4 (curve 4).

Table 1
Approximating integer order TF obtained using the Oustaloup transformation for N=1 and N=2

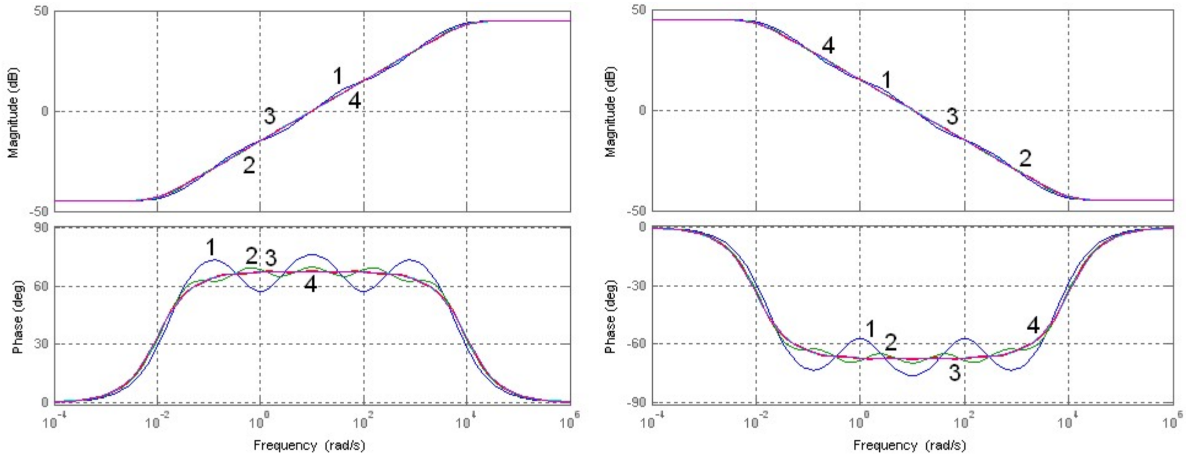
α	N	W(s)
-1.0	N=1	$\frac{0.01s^3+1.049s^2+4.867s+1}{s^3+4.867s^2+1.049s+0.01}$
	N=2	$\frac{0.01s^5+1.188s^4+19.31s^3+48.49s^2+18.83s+1}{s^5+18.83s^4+48.49s^3+19.31s^2+1.188+0.01}$
-0.75	N=1	$\frac{0.03162s^3+2.259s^2+7.144s+1}{s^3+7.144s^2+2.259s+0.03162}$
	N=2	$\frac{0.03162s^5+2.985s^4+38.52s^3+76.85s^2+23.71s+1}{s^5+23.71s^4+76.85s^3+38.52s^2+2.985+0.03162}$
-0.5	N=1	$\frac{0.1s^3+4.867s^2+10.49s+1}{s^3+10.49s^2+4.867s+0.1}$
	N=2	$\frac{0.1s^5+7.497s^4+76.85s^3+121.8s^2+29.85s+1}{s^5+29.85s^4+121.8s^3+76.85s^2+7.497+0.1}$
-0.25	N=1	$\frac{0.3162s^3+10.49s^2+15.39s+1}{s^3+15.39s^2+10.49s+0.3162}$
	N=2	$\frac{0.3162s^5+18.83s^4+153.3s^3+193.1s^2+37.57s+1}{s^5+37.57s^4+193.1s^3+153.3s^2+18.83+0.3162}$
0	N=1	$\frac{s^3+22.59s^2+22.59s+1}{s^3+22.59s^2+22.59s+1}$
	N=2	$\frac{s^5+47.3s^4+306s^3+306s^2+47.3s+1}{s^5+47.3s^4+306s^3+306s^2+47.3s+1}$
0.25	N=1	$\frac{3.162s^3+48.67s^2+33.16s+1}{s^3+33.16s^2+48.67s+3.162}$
	N=2	$\frac{3.162s^5+118.8s^4+610.5s^3+484.9s^2+59.55s+1}{s^5+59.55s^4+484.9s^3+610.5s^2+118.8+3.162}$
0.5	N=1	$\frac{10s^3+104.9s^2+48.67s+1}{s^3+48.67s^2+104.9s+10}$
	N=2	$\frac{10s^5+298.5s^4+1218s^3+768.5s^2+74.97s+1}{s^5+74.97s^4+768.5s^3+1218s^2+298.5+10}$
0.75	N=1	$\frac{31.62s^3+225.9s^2+71.44s+1}{s^3+71.44s^2+225.9s+31.62}$
	N=2	$\frac{31.62s^5+749.7s^4+2430s^3+1218s^2+94.38s+1}{s^5+94.38s^4+1218s^3+2430s^2+749.7s+31.62}$
1.0	N=1	$\frac{100s^3+486.7s^2+104.9s+1}{s^3+104.9s^2+486.7s+100}$
	N=2	$\frac{100s^5+1883s^4+4849s^3+1931s^2+118.8s+1}{s^5+118.8s^4+1931s^3+4849s^2+1883s+100}$

As can be seen from the above characteristics, the phase of each of the parts approaches the value of $\varphi = \pm\alpha\pi/2$. This is the result obtained analytically for fractional differential parts.

Analysing the obtained results, we can say that the accuracy of the approximation depends on the value of N, and already at N = 4 the curves almost coincide with the real frequency characteristics of the respective parts.

Similar results were obtained for other fractional values of α.

Thus, applying the Oustaloup approximation to fractional order controllers of ACS under the condition N≥4, the approximating TF is described by polynomials P(s) and Q(s) not lower than the 9th order (n≥9). Reducing the value of N simplifies the expression of the approximating TF and facilitates its practical implementation. Given the need to ensure the highest adequacy of the approximation, it seems that there is, at first glance, almost impossible to implement controllers with such high order TF.



a b

Figure 1: Bode diagrams of fractional differential (a) and integral (b) (α = 0.75) parts

This problem can be solved by applying to the expression TF (7) the theorem on the decomposition of a rational fraction into elementary ones. Then expression (6), and hence (7), can be represented as follows:

$$W(s) = A_1 \frac{1}{s-\omega_1} + A_2 \frac{1}{s-\omega_2} + \cdots + A_n \frac{1}{s-\omega_n}. \tag{8}$$

The coefficients $A_1, A_2 \ldots A_n$ according to the theorem are by expression.

$$A_i = \frac{P(s)}{Q'(s)} \; for \; s = \omega_i.$$

Thus, the integral and differential fractional order parts on the basis of (8) can be represented by a block diagram, which is shown in Figure 2. In this form, the TF expressions of the components of the fractional order controller can be easily implemented in any software environment (C, C#, C ++, Assembler, etc.), or in analog.

Below, as an example, are approximating expressions of TF obtained by applying the Oustaloup transformation with N = 2 with respect to the differential and integrating fractional order parts with TF $W(s) = s^{\pm 0.5}$ [14].

1. Integral fractional order parts $W(s) = s^{-0.5}$

$$s^{-0.5} = \frac{0.1s^5 + 7.497s^4 + 76.85s^3 + 121.8s^2 + 29.85s + 1}{s^5 + 29.85s^4 + 121.8s^3 + 76.85s^2 + 7.497 + 0.1} \Rightarrow$$

$$\Rightarrow W(s) = 0.1 + \frac{0.1082}{s + 0.0158} + \frac{0.1942}{s + 0.1} + \frac{0.4678}{s + 0.6310} + \frac{1.1501}{s + 3.9811} + \frac{2.5922}{s + 25.1189}.$$

2. Differential fractional order parts $W(s) = s^{0.5}$

$$s^{0.5} = \frac{10s^5 + 298.5s^4 + 1218s^3 + 768.5s^2 + 74.97s + 1}{s^5 + 74.97s^4 + 768.5s^3 + 1218s^2 + 298.5 + 10} \Rightarrow$$

$$\Rightarrow W(s) = 10 + \frac{0.0041}{s + 0.0398} + \frac{0.0726}{s + 0.2512} + \frac{1.1750}{s + 1.5849} + \frac{19.4241}{s + 10} + \frac{430.573}{s + 63.0957}.$$

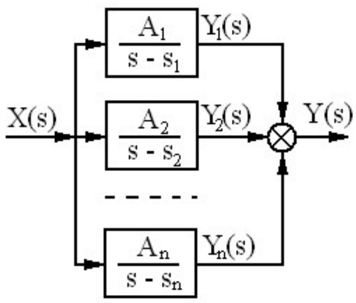


Figure 2: Block diagram of the integral or differential fractional order parts in the integer order TF form

To implement practical programming of a fractional $PI^{\lambda}D^{\mu}$ controller on a microcontroller or signal processor, a requirement is set that it is not possible to use ready-built functions already built into MATLAB or another programming language, for example, step, etc. That is, the solution must:

- be as simple as possible;
- provide a minimum of computational operations (maximum performance of the processor);
- provide high accuracy;
- provide the ability to choose the calculation step in a wide range;
- do not limit the time range of the calculation.

Based on a pre-designed and debugged program in MATLAB, the fractional order controller at the stage of development and improvement of the algorithm is implemented using the C programming language and Arduino Mega 2560 and Arduino DUE due to the possibility of such boards working with a computer. The Arduino Mega 2560 board is built using the Atmel ATMega2560 microcontroller and has the following main technical characteristics: operating voltage - 5V; clock frequency 16 MHz. The Arduino DUE board is built using the Atmel ATSAM3X8E ARM microcontroller and its main differences from the Arduino Mega 2560 board are a higher clock frequency of 84 MHz and the presence of two 12-bit DACs, i.e. analog outputs.

Physical implementation of fractional order controller is possible in two ways.

The first method involves the possibility of implementing the integrated and differential parts of the fractional order controller using the complexes "computer - board Arduino Mega 2560" and "computer - board Arduino DUE". In this case, all calculations are performed by the computer, and synchronized with the external board "I/O" inputs a hopping input signal x = 1V to the input of the controller and output the calculated signal "y" to the specified board terminals, which are the output controller voltage .

The second method involves the possibility of implementing the integrated and differential parts of the fractional order controller by adapting to the software environment Arduino (programming language C). After debugging the programs, they were written to the memory of the Arduino Mega 2560 and Arduino DUE, respectively. At that time, their research was conducted autonomous without the use of computer calculations. In this case, the computer was used only to power the board and to register the transition processes (output of the calculated analog output signal according to the specified conversion option). If you provide another power source (battery or original power supply) and recording devices, you may not use the computer.

In Figure 3a shows the results of the study of transition processes (functions) of integral fractional order parts with TF $W_1(s) = s^{-0.1}-$ curve 1, $W_2(s) = s^{-0.3}-$ curve 2, $W_3(s) = s^{-0.5}-$ curve 3, $W_4(s) = s^{-0.7}-$ curve 4 and $W_5(s) = s^{-0.9}-$ curve 5, and Figure 3b – differential fractional units with TF $W_1(s) = s^{0.1}$ curve 1, $W_2(s) = s^{0.3}-$ curve 2, $W_3(s) = s^{0.5}-$ curve 3, $W_4(s) = s^{0.7}-$ curve 4 and $W_5(s) = s^{0.9}-$ curve 5. These dynamic processes are obtained under the condition of autonomous operation of the external board Arduino DUE, programmed according to the application of the Oustaloup transformation. The points of the transition functions of the parts are obtained in the calculation cycle at the output of the board before writing to the output port.

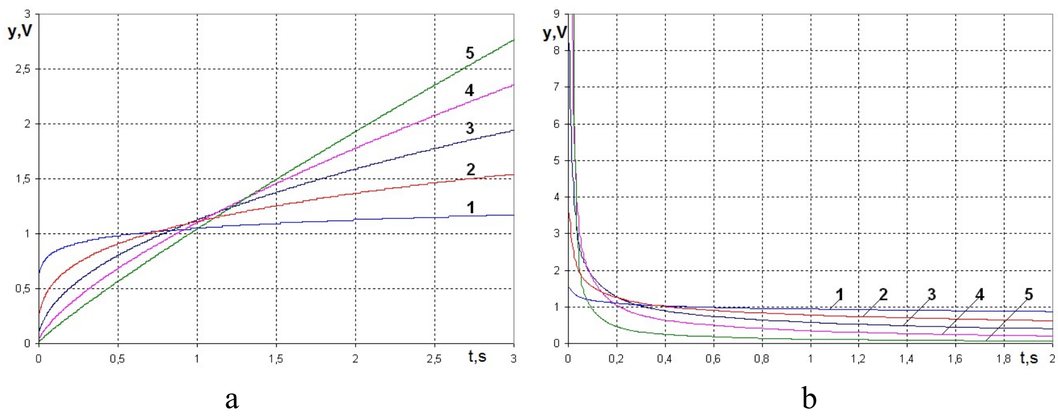


Figure 3: Transition functions of integral (a) and differential (b) fractional order parts with changes α within 0.1÷1.0 for the order of approximation N=2

Based on a pre-designed and debugged program in the MATLAB environment [14] implemented fractional order PI$^{\lambda}$D$^{\mu}$ controller using the programming language C and boards Arduino Mega 2560 and Arduino DUE that can work with a computer.

Each of these methods of implementing PI$^{\lambda}$D$^{\mu}$ controller has disadvantages and advantages. There are advantages to using the MATLAB software environment on a computer with an Arduino Mega 2560 motherboard connected, because in this case the programming language is at a higher level and provides easy and convenient programming and debugging of the controller. Disadvantages of this approach include the effect of computer load on the speed of calculations, which sometimes leads to a twofold increase in calculation time. In addition, the exchange rate between the computer and the board is limited to 115,500 baud, which also significantly affects the signal delay at the output of the controller. It should be noted that this version of the implementation of the fractional controller is appropriate for the use of computer control of the frequency converter.

The use of Arduino Mega 2560 and Arduino DUE boards autonomous using the proposed method of calculating the instantaneous value of the output voltage of the controller has shown its effectiveness. It consists in the fact that it is possible to provide a sampling period of calculations at the level of 0.0025s, ie a significant increase in the speed of obtaining the signal of the controller. In addition, there is the possibility of long-term operation of fractional controllers in stand-alone mode compared to the option when using a computer. The speed of information exchange between the computer and the Arduino DUE board in the mode when the computer needs to control it in this case increases significantly and is 250,000 baud.

Appropriate software has been developed that implements the digital PI$^{\lambda}$D$^{\mu}$ fractional order controller. As an example, consider a TF of fractional order controller (the question of synthesis of PI$^{\lambda}$D$^{\mu}$ controller is not considered in this paper), namely:

$$W_C(s) = 3 + \frac{1}{1.0s^{0.5}} + 1.0s^{0.5}. \tag{9}$$

Experimental studies of fractional order controllers were performed, in particular, according to expression (9). To build them, we used [15] the frequency converter board MFC1000/10 induction electric drive. Figure 4a shows the transition process of the of integer order PI$^{\lambda}$D$^{\mu}$ controller, and on Figure 4b – fractional order ($\lambda = 0.5$, $\mu = 0.5$).

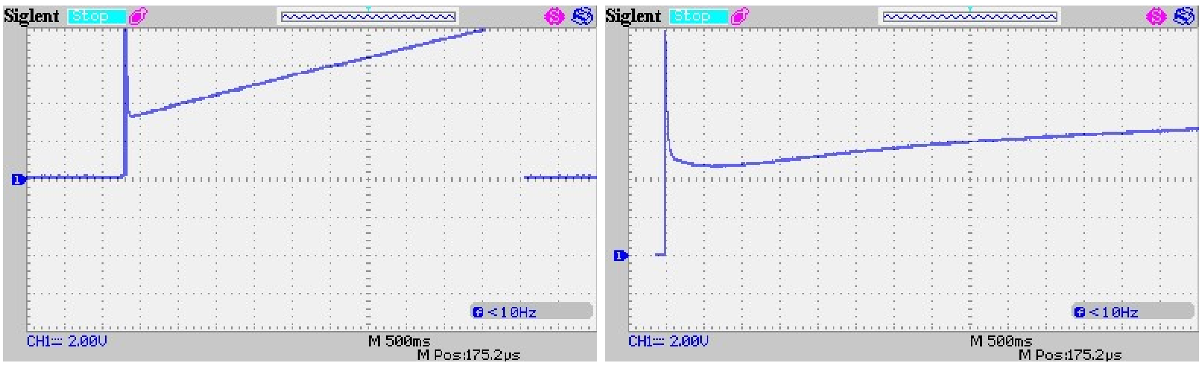


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Figure 4: Transition process of the integer order PI$^{\lambda}$D$^{\mu}$ controller ($\lambda = 1$, $\mu = 1$), implemented using the converter board MFC1000/10 (a) and transition process of the fractional order PI$^{\lambda}$D$^{\mu}$ controller ($\lambda = 0.5$, $\mu = 0.5$) implemented using the converter board MFC1000/10 (b)

Of course, of considerable interest are the possibility of implementing fractional controllers in the ACS. For this purpose, a PI$^{\lambda}$D$^{\mu}$ controller was used as a part of the system "frequency converter - induction motor" (FC-IM).

Figure 5a shows the transition speed, which corresponds to TF (9).

The green graph corresponds to the signal at the output of the PI$^{\lambda}$D$^{\mu}$ controller, the red graph corresponds to the signal at the output of the speed sensor, and the blue colour indicates the set speed. All curves have the appropriate scaling.

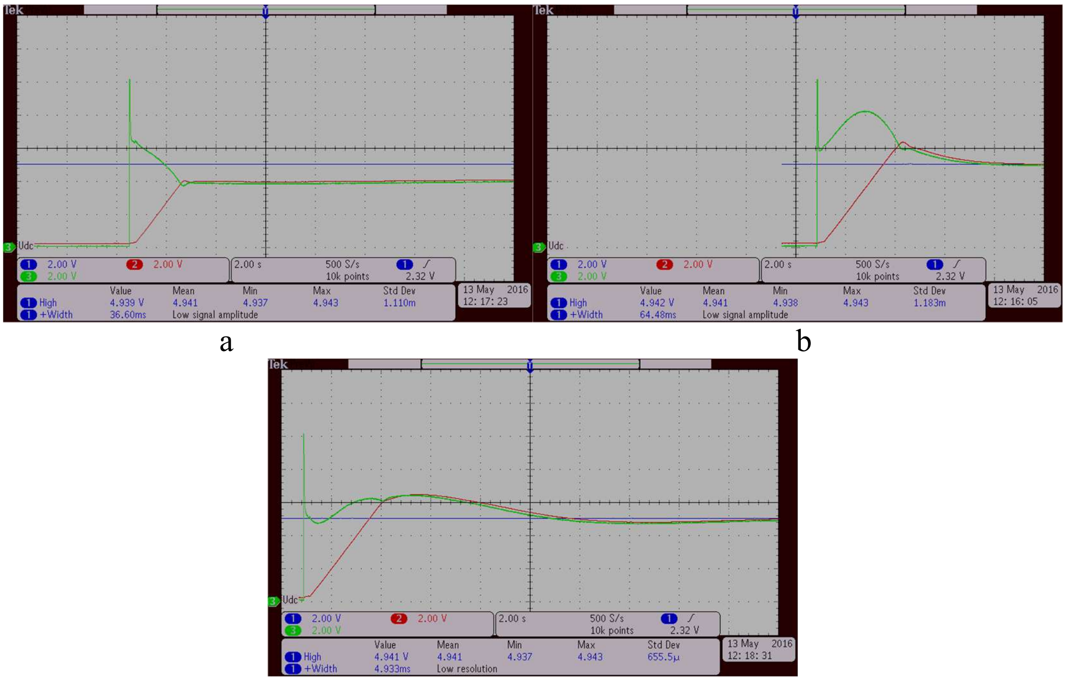
The oscillogram clearly shows the effect of fractional I$^{\lambda}$ – component on the speed of the FC - IM system.

It is of interest to implement a fractional order controller, if the result of its synthesis is a fractional and integer component of the TF controller. It turned out that it is possible to implement such a setting. Figure 5b shows the oscillogram of the dynamic processes of the ACS with such a controller.

Obviously, in such a system it is possible to provide dynamic processes based on the results of the corresponding synthesis of ACS.

In the above studies, $PI^{\lambda}D^{\mu}$ controllers were considered, in which the fractional order is in the range from 0 to 1. It may be necessary to implement fractional order controllers, where this condition is not met. Therefore, experimental studies were conducted for this case.

In Figure 5c shows the transition process of the speed, which corresponds to the TF of the controller with I^{λ} – component when $\lambda = 1.5$ and D^{μ} – component when $\mu = 0.5$. The oscillogram demonstrates the possibility of operation of the developed $PI^{\lambda}D^{\mu}$ controller if $\lambda > 1$.



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c

Figure 5: Transition process of speed in the system FC-IM with $PI^{\lambda}D^{\mu}$ controller $W_C(s) = 3 + 3s^{-0.5} + 1.0s^{0.5}$ (k_p = 3, k_i = 3, k_d = 1, λ = -0,5, μ = 0,5) (a), $W_C(s) = 3 + 5s^{-1.0} + 1.0s^{0.5}$ (k_p = 3, k_i = 5, k_d = 1, λ = -1, μ = 0,5) (b), controller $W_C(s) = 3 + 3s^{-1.5} + 1.0s^{0.5}$ (k_p = 3, $k_i = 1/T_i$ = 3, T_d = 1s, λ = -1,5, μ = 0,5) (c)

The speed of such ACS decreases with a simultaneous increase in the amount of overshooting. These parameters of dynamic processes can change due to the implementation of other criteria for the synthesis of the system.

4. Conclusions

Computer research of algorithm and programs realization of differentiating and integrating fractional order parts, as components of $PI^{\lambda}D^{\mu}$ controllers has shown efficiency of application of approximation of fractional order TF.

The application of the decomposition theorem of rational fractions allowed to construct structural schemes from parallel connected aperiodic units for the realization of approximated arbitrary order TFs.

It is experimentally proved that the implementation of fractional order controllers based on approximated TFs can work in real time as a part of highly dynamic ACS. Tests of the FC MFC 710 option with the $PI^{\lambda}D^{\mu}$ - fractional order controller in the speed control system using the Twerd experimental stand have confirmed its efficiency in terms of expanding the regulatory capabilities of such ACS.

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