

# Development of a Method for Deciding on the Distribution of Efforts to Destroy the Air Enemy Between Anti-Aircraft Missile Forces and Fighter Aircraft

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## Abstract

The publication shows the possibility of using the mathematical apparatus of the multicriteria method of analysis of hierarchies in conjunction with the theory of fuzzy sets, at the stage of assessing the significance of selected parameters that affect the distribution of forces and means in accordance with the conditions of the task, during the task. Binary operations of symmetric difference and drastic sum are used to determine the set of alternatives for the joint use of fighter aircraft and anti-aircraft missile forces, taking into account the projected loss estimate. The conducted mathematical apparatus makes it possible to form a set of rules on the basis of which it is possible to make a decision on the distribution of efforts to destroy the air enemy. Using the method of analysis of hierarchies at one of the stages uses fuzzy logic to form a measure of evaluation of the parameters being evaluated, but the end result has an exact value that requires recalculation when changing at least one of the parameters. Instead, using set theory, namely the operation of symmetric difference and drastic sum, it is possible to determine the set of values that forms alternative decision-making depending on the result obtained. That is, the target that will be in a certain range will be immediately distributed relative to the fighter aircraft and anti-aircraft missile forces, without recalculating the parameters and their coefficients relative to each other.

## Keywords

Method of analysis of hierarchies, fuzzy sets, evaluation criterion, symmetric difference, drastic sum.

## 1. Introduction

The experience of wars and armed conflicts of the twentieth and twenty-first centuries shows a significant increase in the use of air strikes [4]. The capabilities of air strikes are evolving along with scientific and technological progress, which allows them to solve not only tactical but also strategic tasks. Instead, the use of air defense is not only a defense system against air strikes but also a deterrent, the impact of which is assessed in the first place. The increase in the number of tasks before the means of air attack correlates with the

number of tasks performed by air defense, namely [1, 2]:

- destruction of enemy air attack means;
- gaining and maintaining an advantage in the air;
- destruction of unmanned aerial vehicles, which are used in the conditions of hybrid warfare, and represent an element of uncertainty for a set of means of automation or automated control system and require a clear definition (shock or reconnaissance);
- cover of important state and military facilities, their transformation into the national defense system of Ukraine;

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increasing the level of combat capability of the Armed Forces of Ukraine and other components of the defense forces with the achievement and maintenance of certain capabilities for fire defeat of the enemy.

The effectiveness of joint combat use of troops (forces) in repelling air strikes is achieved by realizing their combat capabilities under the condition of coordinated interaction and centralized management, a single automated control system using the decision support system (DSS). It is also necessary to take into account the scale, nature of the actions of the means of air attack, combat composition, combat capabilities, weather and geographical conditions.

The decision on the distribution of efforts in the destruction of air between the anti-aircraft missile forces and fighter aircraft is made by the decision-maker on the basis of proposed alternative solutions to DSS. However, the decision-maker will use the received recommendations only if he trusts this system, and his trust can be built on the degree of validity of the recommendations.

Therefore, at present, the question of choosing a mathematical decision-making apparatus remains relevant, with the possibility of explaining the recommendations in DSS, which provides proposals for the distribution of efforts in the destruction of air strikes between anti-aircraft missiles and fighter aircraft.

## 2. The main part

The distribution of efforts is carried out in order to achieve the maximum effect in the destruction of air attack means by joint efforts, as well as to ensure the safety of their aircraft. Given the fact that anti-aircraft missile forces can be used for ground (surface) purposes, it follows that the scope is common.

Depending on the combat capabilities and the nature of the tasks to be solved, fighter aircraft and anti-aircraft missile forces share efforts:

- in space (in directions, boundaries, areas, strips, sectors, zones, altitudes);
- by time;
- method of performing the task [3].

The distribution of goals according to the forces that must be allocated for their destruction is carried out by the decision-maker from a particular point of control. To make such decisions requires not only a preliminary assessment of the conditions of use of troops

(forces) but also an assessment of the relative losses that can be achieved on the one hand and on the other, resource indicators and time parameters from analysis of the situation to completion of the combat mission.

Determine the sequence of calculations using the method of analysis of hierarchies (MAH) [5, 6, 7], which includes the steps:

1. Selection of troops (forces) of fighter aircraft and anti-aircraft missile forces in accordance with the existing ones;

2. Determination of sets of parameters that characterize certain troops (forces) (tactical and technical capabilities; time of readiness to perform the task; means of destruction used; training of personnel; preparation for re-use; others);

3. The choice of quality indicators that are compared (quality indicators in turn can be both local and global, which directly affects the priority of the parameter);

4. Calculation of the generalized criterion (global assessment for each fighter or anti-aircraft missile system, fighter aircraft and anti-aircraft missile forces in general relative to others).

Since MAH is based on estimates of the degree of influence of lower hierarchy factors on the criteria and indicators of higher levels of the hierarchy, we assume that the more factors and indicators will be taken into account, the more accurate the final result will be. On the other hand, the assessment of the significance of some parameters may be insignificant in comparison with other estimates of parameters, which implies the effect of insignificance of the selected parameters, which will give a negligible advantage to the parameters that should not be considered at all.

To select the optimal grouping of troops (forces) that will be tasked, we use the algorithm of expert assessment of the impact of the characteristics of complex technical systems on quality indicators, proposed by T. Saaty using a nine-point scale for comparing alternatives Tab. 1. However, this measure of comparison is not a dogma and, if necessary, it is possible to use your own.

If several experts take part in the assessment, the assessment is agreed by consensus, or each expert builds his own table, and the estimates are presented as geometric averages.

It is important to emphasize the importance of choosing the right priorities, because in the case of a gross mistake, the priority may not be important, but important will be ignored, which

will lead to the accumulation of error or even error and making the wrong decision. Therefore, for a more accurate assessment, you need to specify a more accurate set of input data, assess their

reliability, as well as take into account possible errors.

**Table 1**  
Comparison of alternatives

Relative importance	Definition	Explanation
1	Equal importance of indicators	Equal contribution of two indicators to the assessment
3	A slight advantage of one over the other	Experience and research give a slight advantage of one indicator over another
5	Great advantage	Experience and research give a great advantage of one indicator over another
7	A significant advantage	The advantage is so strong that it becomes significant
9	Full advantage	The obvious advantage is most fully confirmed
2,4,6,8	Intermediate values	Used as an alternative

After calculations, we obtain a matrix of priorities in accordance with certain conditions (known). This will allow you to make a decision on the distribution of effort on the basis of the average value, or using special data analysis software [8] to determine at what level of the hierarchy the defined indicators have the greatest impact.

For example, using a color gradation of the appropriate range. However, changing the conditions of the task, taking into account the risks or making a decision in conditions of complete uncertainty requires a recalculation of the priorities of the parameters on the basis of new input data.

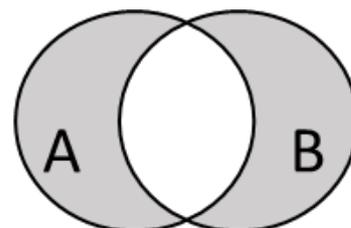
Although the set of comparative alternatives is used at the stage of expert evaluation in MAH, it affects only the initial level of evaluation of coefficients, which does not allow to determine the set of alternatives of the final result.

Using instead of the specified exact value, a fuzzy unlimited multivalued estimate, we can increase the set of values, all values of which satisfy the task and are within acceptable limits. However, when considering the distribution of efforts in one area, the security of their troops (forces) remains essential, so the set of targets set for fighter aircraft should not intersect with multiple targets that will apply to anti-aircraft missile forces,

unless they operate jointly in one area but separated by sometimes.

Therefore, it would be rational to apply a symmetrical difference (drastic product  $A \Delta B$ ) [9, 10, 11] of two sets where A is the set of targets that will be attributed to the targets of fighter aircraft and B is the set of targets that will be attributed to the targets of anti-aircraft missile forces.

Also  $(A \Delta B)$  based on the definition  $(A \cup B) \setminus (A \cap B)$  can be described as Fig.1., that is, a set that includes all elements of both sets that are not common to the two sets or  $(A \setminus B) \cup (B \setminus A)$ , a set that includes all elements of the first set that are not included in the second set, as well as elements of the second set that are not included in the first set.



**Figure 1:** Image of the symmetrical difference of sets A and B using Euler circles

Using a system of algebraic equations can be represented as follows:

$$m_{AAB}(x) = \begin{cases} m_B(x), & \text{if } m_A(x) = 1 \text{ for } \forall x \in X \\ m_A(x), & \text{if } m_B(x) = 1 \text{ for } \forall x \in X, \\ 0 & \text{in other cases} \end{cases} \quad (1)$$

where  $X$  is the set of real numbers,  $m_A(x)$  is the set of targets of fighter aircraft,  $m_B(x)$  is the set of targets of anti-aircraft missile forces.

From the point of view of the use of fighter aircraft and anti-aircraft missile forces, it can be described as used jointly in one area but divided into ranges, in the purpose for which fighter aircraft are used, anti-aircraft missile forces are not considered and vice versa.

On the other hand, it is necessary to consider the whole set of purposes for which fighter aircraft and anti-aircraft missile forces can be used based on their capabilities, given the limit of joint use to determine this set, we use a drastic sum ( $A \nabla B$ ) which can be written as:

$$m_{A \nabla B}(x) = \begin{cases} m_B(x), & \text{if } m_A(x) = 0 \text{ for } \forall x \in X \\ m_A(x), & \text{if } m_B(x) = 0 \text{ for } \forall x \in X, \\ 1 & \text{in other cases} \end{cases} \quad (2)$$

In the case of the use of a symmetric sum, a set of purposes became known which can be considered for joint use, and in the case of the use of a drastic sum using the previously described  $\max(\delta_{A \nabla B})$ , we define the boundaries of joint use, which can be described as:

$$m_{A \nabla B}(x) = \begin{cases} \max(m_B(x)), & \text{if } \min(m_A(x)) = 0 \text{ for } \forall x \in X \\ \max(m_A(x)), & \text{if } \min(m_B(x)) = 0 \text{ for } \forall x \in X, \\ 1 & \text{in other cases} \end{cases} \quad (3)$$

then the symmetric difference can be written as:

$$m_{A \Delta B}(x) = \begin{cases} \min(m_B(x)), & \text{if } \max(m_A(x)) = 1 \text{ for } \forall x \in X \\ \min(m_A(x)), & \text{if } \max(m_B(x)) = 1 \text{ for } \forall x \in X, \\ 0 & \text{in other cases} \end{cases} \quad (4)$$

Using a symmetrical difference and a drastic sum to determine the boundaries of alternatives to the decision to use fighter aircraft and anti-aircraft missile forces, there are other cases in which joint use will take place.

Let us denote the joint application by  $\delta$ , since it is necessary to consider all possible cases, then the system of equations of the drastic sum takes the form:

$$m_{A \nabla B}(x) = \begin{cases} m_B(x), & \text{if } m_A(x) = 0 \text{ for } \forall x \in X \\ m_A(x), & \text{if } m_B(x) = 0 \text{ for } \forall x \in X, \\ \delta_{A \nabla B} & \end{cases} \quad (5)$$

Symmetrical difference, respectively.

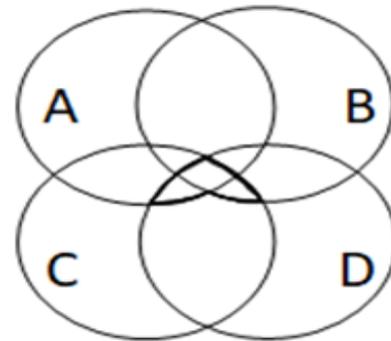
Given the safety of joint use, we define the set  $C$  - the estimated estimate of the loss of fighter aircraft and  $D$  - the estimated estimate of the loss of anti-aircraft missile forces in joint use. Based on the above use, we must take into account that the projected loss estimate will be relative to the set of common use, so similarly use the symmetric difference and the drastic sum to determine  $k_{C \nabla D}(x)$  and  $k_{C \Delta D}(x)$  where  $k_C(x)$  and  $k_D(x)$  - the set of projected losses.

To determine the set of decision-making alternatives, taking into account the projected estimate of losses when used together, consider  $\max(\delta_{A \nabla B})$ ,  $\min(\delta_{A \Delta B})$  and  $\max(\delta_{C \nabla D})$ ,  $\min(\delta_{C \Delta D})$ , in the figure it will look like a limit, and its division is described by determining the maximum of the maximum allowable value and the minimum of the minimum allowable. We get:

$$\max(\delta_{A \nabla B}) = \max(\max(m_B(x)) \cup \max(\max(m_A(x))) \quad (6)$$

$$\min(\delta_{A \Delta B}) = \min(\min(m_B(x)) \cup \min(\min(m_A(x))) \quad (7)$$

Accordingly, for  $\max(\delta_{C \nabla D})$ ,  $\min(\delta_{C \Delta D})$ . Using Euler's circles, the set of decisions takes the form of Fig.2



**Figure 2:** A variant of many decision-making alternatives

To describe the knowledge for decision-making based on the given set of values, we use the production model [12, 13]. In our case we can form the following rules:

1. If  $\max(\delta_{A \nabla B}) \cup \max(\delta_{C \nabla D})$  then use fighter aircraft;
2. If  $\min(\delta_{A \Delta B}) \cup \min(\delta_{C \Delta D})$  then use anti-aircraft missile forces;
3. If  $\max(\delta_{A \nabla B}) \cup \min(\delta_{A \Delta B})$  then joint use.

Otherwise, it is possible to form a set of hierarchically dependent rules depending on the level of the production model, or the level of hierarchy of the method of analysis of hierarchies.

### 3. Conclusions

The use of air strikes is a challenge, the response to which is the joint use of fighter aircraft and anti-aircraft missile forces, both in one air defense system and individually in certain areas. Their joint application encourages the development of methods and models that will allow the use of available forces and means with minimal costs and maximum planned effect.

Using the method of analysis of hierarchies at one of the stages uses fuzzy logic to form a measure of evaluation of the parameters being evaluated, but the end result has an exact value that requires recalculation when changing at least one of the parameters. Instead, using set theory, namely the operation of symmetric difference and drastic sum, it is possible to determine the set of values that forms alternative decision-making depending on the result obtained. That is, the target that will be in a certain range will be immediately distributed in relation to fighter aircraft or anti-aircraft missile forces, without recalculating the parameters and their coefficients relative to each other.

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