Taking into Account a Priori Uncertainty in the Model of Maintenance of Objects with Time Redundanc

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Abstract

The article presents the results of a theoretical study of maintenance of restored objects of continuous use, which provides for temporary redundancy − the permissible time for performing current repairs and maintenance. The formulation of the problem is formulated and a formalized description of the process of servicing these objects is given, a distinctive feature of which is the presence of incomplete information about the distribution functions of the recovery time of an object after a failure and the time for performing maintenance (only two initial moments of these random variables are known).

A general approach to solving the formulated problem is proposed, based on the sequential solution of a number of particular interrelated problems: obtaining formulas for indicators of the quality of maintenance with complete initial information, which contain functionals depending on the type of distribution functions, information about which is limited; obtaining two-sided estimates of these functionals; synthesis of particular results using the proposed method for calculating the boundary values of the optimal frequency and bilateral estimates of service quality indicators − the coefficient of technical utilization and average unit costs. A numerical example is considered to illustrate the practical application of the results obtained.

Keywords [1]

Maintenance, prior uncertainty, temporary redundancy.

1. Introduction

Ensuring high reliability of the functioning of modern complex technical systems includes a wide range of problems, which are addressed by the efforts of many specialists of various profiles in the design, manufacture, testing and operation of such systems. Among these problems, an important place is occupied by the organization of effective maintenance, the main task of which is to maintain the required level of reliability of complex systems during operation.

However, the analysis of a number of publications of domestic and foreign specialists indicates a decrease in attention to the issues of researching the reliability and maintenance of complex technical systems, in particular telecommunications equipment of communication networks [1]. The results known in this subject area are devoted to the study of mainly traditional methods of maintenance and are obtained, as a rule, without a complex consideration of a set of factors that significantly affect the efficiency of maintenance. These factors include the usage of various types of redundancy, which is a necessary condition for ensuring the reliability of the functioning of almost any technical system. In particular, the usage of temporary redundancy imposes certain conditions on the organization and

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conduct of such operational activities as maintenance, routine repairs and others, therefore, this factor must be taken into account when organizing the operation of technical facilities, since a time reserve objectively exists in many real systems or can be provided by carrying out organizational and technical measures. In well-known publications, such factors are not always taken into account that characterize the modes of usage of objects for their intended purpose, as well as the parameters of the flows of failures and equipment recovery. All this leads to results that are difficult to use in practice.

The subject of the study are technical facilities with time redundancy (systems "object time"), which provide for periodic maintenance [2, 3]. In contrast to the work [4 − 7], the article considers continuous use systems, examples of which are telecommunication systems of communication networks, automated control systems (ACS) for various purposes and other systems. In addition, the study of these systems is carried out in conditions of a priori uncertainty, which means incomplete source information about the laws of distribution of determinants of random variables, when the exact type of these distribution functions is not specified, and only some numerical characteristics of random variables are known. The purpose of the article is to substantiate a general approach to the study of maintenance of the mentioned above class of technical objects under conditions of limited initial information about the distribution function of the object recovery time and the distribution function of the maintenance execution time (only two initial moments of these random variables are known), which allows one to obtain design relations for two-way estimates (lower and upper bounds) of the optimal frequency of maintenance and the corresponding extreme values of quality indicators, values of service quality indicators: the maximum value of the coefficient of technical utilization of the facility and the minimum value of the average unit costs.

2. Statement of the problem

We will assume that the object is represented by one structural element, the time to failure of which t_0 is distributed according to an arbitrary law $F(t) = P\{t_0 < t\}$ with the ending mathematical expectation $\bar{t}_0$, and failures are manifested at the time of their occurrence. The system provides for the implementation of two types of restoration work: periodic maintenance, which is based on the implementation of planned preventive maintenance over time $T = \text{const}$ (frequency of maintenance) and emergency (current) repairs after the failure of the object. Maintenance duration is a random variable t_m with an arbitrary distribution function $F_m(t)$ and the ending mathematical expectation. If the maintenance is performed within the allowable time $t_{a1} = \text{const}$, which determines the time reserve used in the system, it refers to the useful time of operation of the system, otherwise (at $t_m > t_{a1}$) – to downtime. hen a failure occurs, the object begins to recover, the duration of which is a random variable t_R with an arbitrary distribution function $F_R(t)$ with the ending mathematical expectation $\bar{t}_R$. If the object is restored for a time not exceeding the allowable value $t_a = \text{const}$ ($t_a \le t_R$), then it refers to useful time, otherwise (at $t_R > t_a$) – to system downtime. Note that in the particular case of magnitude t_a and t_{a1} can have the same values [8]. Let the exact type of the function of distribution of recovery time $F_R(t)$ and maintenance time $F_m(t)$ of the object is unknown, and only two of their initial moments are known:

$$\beta_1 = \bar{t}_R = \int_0^\infty t dF_R(t), \quad \beta_2 = \int_0^\infty t^2 dF_R(t), \quad \beta_1^2 < \beta_2, \tag{1}$$

$$s_1 = \bar{t}_m = \int_0^\infty t dF_m(t), \quad s_2 = \int_0^\infty t^2 dF_m(t), \quad s_1^2 < s_2. \tag{2}$$

Let's also assume that when you restore an object, the average cost per unit time is c_R, and when performing maintenance – c_m. After completing any kind of restoration work, the system (object + time reserve) is completely restored, the moment of the next maintenance is rescheduled, and the whole process is repeated. In this case, we will assume that during the period of operation of the object under consideration, the planned types of repairs are not carried out.

To assess the quality of maintenance of the object, we use two indicators: a comprehensive indicator of the reliability of the system "object time", which is considered – the coefficient of technical use $K_{tu}(T)$ and cost indicator – average costs per unit time of the system in working order – average unit costs $\overline{C}(T)$ [1, 10]. For the above conditions of the system, it is necessary to obtain analytical ratios to determine bilateral estimates (limit values) of the optimal frequency of maintenance and the corresponding extreme values of service quality indicators: the maximum value for the coefficient of technical use of the object and the minimum for average unit costs.

2.1. A general approach to solving a problem

The analysis showed that to obtain in the general case the exact values of maintenance quality indicators in terms of the above problem is not possible, and the task is to find two-way estimates (exact upper and lower limits) of these indicators, when the unknown distribution functions $F_R(t)$ and $F_m(t)$ belong to some fixed class of distributions K [11].

To solve this problem, it is advisable to use a method that takes into account the distinctive feature of the calculated ratios for quality indicators of maintenance of objects with time redundancy, obtained with complete source information. This feature is that these formulas include functionals of a special type, the values of which depend on the type of distribution functions of the original random variables, a priori information about which is limited.

In accordance with the formulation of our problem in the formulas for quality indicators of maintenance (coefficient of technical usage $K_{tu}(T)$ and average unit costs $\overline{C}(T)$) should include functionals that characterize the average time of restoration work (repair and maintenance), which are completed before the backup (allowable time) t_a and t_{a1} respectively. This time refers to the useful time of using the system with time redundancy and helps to increase the efficiency of its operation. The above functionals for known (or given) distribution functions $F_R(t)$ and $F_m(t)$ look like:

$$I(F_R) = \int_0^{t_a} [1 - F_R(t)] td = \int_0^{\infty} g(t) dF_R(t), \tag{3}$$

where
$$g(t) = \begin{cases} t & \text{at } 0 \le t < t_a, \\ t_a & \text{at } t \ge t_a; \end{cases}$$

$$I(F_m) = \int_0^{t_{a1}} [1 - F_m(t)] td = \int_0^{\infty} h(t) dF_m(t), \tag{4}$$

where
$$h(t) = \begin{cases} t & \text{at } 0 \le t < t_{a1}, \\ t_{a1} & \text{at } t \ge t_{a1}; \end{cases}$$

If we obtain analytical relations for the exact lower and upper limits (for bilateral estimates) of these functionals and present these values in the formulas for maintenance quality indicators obtained with complete initial information, it is possible to construct bilateral estimates of quality indicators in a priori uncertainty.

This approach is the basis for solving the formulated problem. It includes the sequential solution of the following partial problems:

– building a model of maintenance of objects with complete source information;

– obtaining bilateral assessments of functionals $I(F_R)$ and $I(F_m)$ that characterize the quality of maintenance;

– obtaining the limit values of the optimal frequency of maintenance and the corresponding extreme values of quality indicators.

Synthesis of the results of solving these problems allows to obtain general solutions to the problem.

2.2. Construction of a service model with complete source information

We will keep the initial conditions of our problem, but we will assume that the functions of distribution of recovery time $F_R(t)$ and maintenance duration $F_m(t)$ are known or given, and we will build a model of maintenance of objects of continuous use.

Under the maintenance model we will understand the mathematical (in our case analytical) model that establishes the relationship between the quality of maintenance of the object and the characteristics of its reliability, maintenance parameters and the process of operation of the object. Construction of such model will allow to receive settlement ratios for indicators of quality of maintenance at the full initial information [12]. It is easy to see that the process of functioning of the object of continuous use with time redundancy can be described using the model of the regenerating random process $x(t)$, the graph of states and transitions of which is shown in Fig. 1.

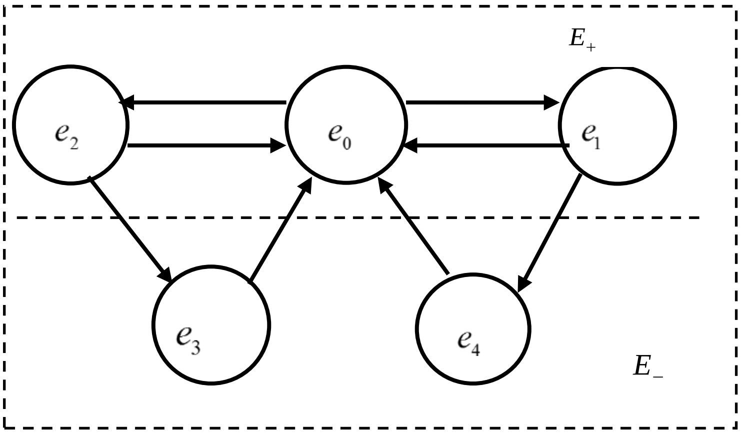

Figure 1: Graph of states and transitions of a random process $x(t)$, which describes the functioning of the system of continuous use

According to fig. 1, the process $x(t)$ at any time may be in one of the following states:

e_0 – the condition in which the facility is operational;

e_1 – the condition in which the object is inoperable, but its restoration is carried out over time $t_R \le t_a$;

e_2 – the condition in which the facility is maintained for time $t_m \le t_{a1}$;

e_3 – the condition in which the facility is serviced for a time exceeding the allowable value t_{a1};

e_4 – the condition in which the object restores its operability for a time exceeding the allowable value t_a.

In fig. 1 through E_+ and E_- marked areas (subsets) of working and inoperable states of the system, respectively. Note that in accordance with the accepted initial conditions of the states e_1 and e_2, in which restoration works (repair and service) for time less than admissible are carried out on object, we carry to area E_+.

It is important to note that for this service strategy at the time of completion of restoration work (moments of time t_k $(k = 0,1,2...)$ transition to the state e_0) further course of the process $x(t)$ does not depend on the past, as in these moments the system (object + time reserve) is completely updated, and the next service is planned. These moments t_k are moments of regeneration of the process $x(t)$, and a sequence of time intervals $\tilde{X}_k = t_k - t_{k-1}$ $(k = 1,2...)$ between the regeneration points forms a recurrent recovery process. The values $\tilde{X}_k$ is a sequence of positive, equally distributed, mutually independent random variables. Whereas the process $x(t)$ has regeneration points, then random variables $X_k^{(i)}$, showing how much time the system has spent in the state e_i by k period between the moments of regeneration, are mutually independent and equally distributed random variables.

With prolonged usage, the proportion of time K_i, which the system conducts in the state e_i, equal to the ratio of the average regeneration time $\mathrm{M}X^{(i)}$, conducted in the state e_i for the period between the moments of regeneration, to the average duration $\mathrm{M}\tilde{X}$ of this period.

Therefore, for the system under consideration, the operation of which is described by a random process $x(t)$ (Fig. 1), the coefficient of technical usage can be represented as follows:

$$K_{tu} = \frac{\sum_{i \in E_+} \mathrm{M}X^{(i)}}{\mathrm{M}\tilde{X}}, \tag{5}$$

where $E_+ = \{e_0, e_1, e_2\}$ is a subset of the operating states of the system.

Average time $\mathrm{M}X^{(0)}$ of the system is in a state e_0 is defined as the mathematical expectation of the minimum of two quantities: operating time before the failure of the object t_0 and time before scheduled maintenance (periodicity τ), namely

$$\mathrm{M}X^{(0)} = \mathrm{M}[\min(t_0, \mathrm{T})] = \int_0^T [1 - F(t)]dt. \tag{6}$$

Average time $\mathrm{M}X^{(1)}$ of the system is in a state e_1 is equal to the product of the probability of fulfillment of the condition $t_0 < t$ (i.e. the probability that the object will fail before the start of scheduled maintenance) on the mathematical expectation of the minimum of two values: the duration of recovery t_R and allowable time t_a, namely

$$\mathrm{M}X^{(1)} = P\{t_0 \le T\}\mathrm{M}[\min(t_R, t_a)] = F(t)\int_0^{t_a} [1 - \bar{F}_R(t)]dt. \tag{7}$$

Average time $\mathrm{M}X^{(2)}$ of the system is in a state e_2 is equal to the product of the probability that the object will not refuse until the start of scheduled maintenance on the mathematical expectation of the minimum of two values: the duration of maintenance t_m and allowable time τ_{a1}, namely

$$\mathrm{M}X^{(2)} = P\{t_0 > T\}\mathrm{M}[\min(t_m, t_{a1})] = [1 - F(t)]\int_0^{t_a} [1 - \bar{F}_m(t)]dt. \tag{8}$$

Time interval $\tilde{X}$ between adjacent system upgrade points (between process $x(t)$ regeneration points) $x(t)$) consists of two intervals: the interval from the end of the previous update to the beginning of the restoration work, which is equal to $\min(t_0, \tau)$, and a recovery interval representing some random variable ξ. According to the formula of complete mathematical expectation we obtain:

$$\begin{aligned} \mathrm{M}\tilde{X} &= \mathrm{M}[\min(t_0, T)] + \mathrm{M}\xi = \mathrm{M}[\min(\bar{t}_0, T)] + \bar{t}_R P\{t_0 \le T\} + \bar{t}_m P\{t_0 > T\} = \\ &= \int_0^T [1 - F(t)]dt + \bar{t}_R F(T) + \bar{t}_m [1 - F(T)]. \end{aligned} \tag{9}$$

Substituting formulas (6) – (9) in (5), can be obtained:

$$K_{tu} = \frac{\int_0^T [1 - F(t)]dt + F(T)\int_0^T [1 - F_R(t)]dt + [1 - F(T)]\int_0^{t_{a1}} [1 - F_m(t)]dt}{\int_0^T [1 - F_R(t)]dt + \bar{t}_R F(T) + \bar{t}_m [1 - F(t)]}. \tag{10}$$

Consider further the cost indicator – the average cost of restoration work per unit time of the system in working order $\overline{C}(T)$.

The average residence time of the system in the field of working conditions for the period between the points of regeneration of the process $x(t)$ (Fig. 1) is determined by the formula:

$$\mathrm{M}X^{(0)}+\mathrm{M}X^{(1)}+\mathrm{M}X^{(2)}=\int_0^T[1-F(t)]dt+F(T)\int_0^{t_a}[1-F_R(t)]dt+[1-F(T)]\int_0^{t_{a1}}[1-F_m(t)]dt.$$

Average costs for the same period are determined by the formula of full mathematical expectation:

$$c_R\bar{t}_R P\{t_0\le T\}+c_m\bar{t}_m P\{t_0>T\}=c_R\bar{t}_R F(T)+c_m\bar{t}_m[1-F(T)].$$

Therefore, the expression for average unit costs $\overline{C}(T)$ will look like:

$$\overline{C}(T)=\frac{c_R\bar{t}_R F(T)+c_m\bar{t}_m[1-F(T)]}{\int_0^T[1-F(t)]dt+F(T)\int_0^{t_a}[1-F(t)]dt+[1-F(T)]\int_0^{t_{a1}}[1-F_m(t)]dt}. \tag{11}$$

Thus, the calculated relations (formulas (10) and (11)) for indicators of quality of maintenance (objective functions) at the complete initial information which include linear functionals $I(F_R)$ and $I(F_m)$ (formulas (3) and (4)) are received.

We obtain formulas for determining the optimal values of the frequency of maintenance T^* and T_1^*, that provide extreme values of quality indicators $K_{tu}(T^*)$ and $\overline{C}(T_1^*)$.

Differentiating expression (10) by T and equating the derivative to zero, we obtain the following equation to find the optimal value of the frequency of maintenance T^*:

$$\frac{\bar{t}_m-I(F_m)}{[\bar{t}_R-I(F_R)]-[\bar{t}_m-I(F_m)]}=-F(t)+\lambda(T)\left\{\int_0^T[1-F(t)]dt+\frac{\bar{t}_R I(F_m)-\bar{t}_m I(F_R)}{[\bar{t}_R-I(F_R)]-[\bar{t}_m-I(F_m)]}\right\}, \tag{12}$$

where $\lambda(T)=\dfrac{F'(T)}{1-F(T)}$ – the failure rate of the object.

Equation (12) is a necessary condition for the existence of the optimal value of the frequency of maintenance, at which the coefficient of technical use of the system takes the maximum value. We obtain formulas for checking the fulfillment of a sufficient extremum condition of the objective function (10). Assume that $\bar{t}_m<\bar{t}_R$ and $\lambda'(t)>0$ and denote by $V(T)$ the right part of the equation (12). It is easy to see that when $T=0$ we receive $V(0)=0$ and fair inequality:

$$\frac{\bar{t}_m-I(F_m)}{[\bar{t}_R-I(F_R)]-[\bar{t}_m-I(F_m)]}\ge V(0).$$

Therefore, for $T=0$ the derivative of function (10) is positive.

Let now $T\to\infty$. If at the same time

$$\frac{\bar{t}_m-I(F_m)}{[\bar{t}_R-I(F_R)]-[\bar{t}_m-I(F_m)]}<\lim_{T\to\infty}V(T), \tag{13}$$

then equation (12) will have at least one root, and the objective function (10) is the absolute maximum at $T\in[0,\infty)$.

Considering that $\lim_{T\to\infty}\lambda(T)=\lambda(\infty)$ exists, from expression (12) we obtain:

$$\lim_{T\to\infty}V(T)=-1+\lambda(\infty)\left[\bar{t}_0+\frac{\bar{t}_R I(F_m)-\bar{t}_m I(F_R)}{[\bar{t}_R-I(F_R)]-[\bar{t}_m-I(F_m)]}\right], \tag{14}$$

where $\bar{t}_0$ – the average operating time of the object to failure.

Then inequality (13) takes the form:

$$\frac{\bar{t}_m - I(F_m)}{[\bar{t}_R - I(F_R)] - [\bar{t}_m - I(F_m)]} < -1 + \lambda(\infty)\left[\bar{t}_0 + \frac{\bar{t}_R I(F_m) - \bar{t}_m I(F_R)}{[\bar{t}_R - I(F_R)] - [\bar{t}_m - I(F_m)]}\right]. \tag{15}$$

Inequality (15) is a sufficient condition for the existence of the absolute maximum of function (10) for $T \in [0,\infty)$. From this formula it is seen that when $\lambda(\infty) = \infty$ inequality (15) holds for any values of other parameters. If $\lambda(\infty) < \infty$, it is necessary to perform a calculation according to formula (15) for a given initial data and when the condition under test, to draw conclusions about the feasibility of maintenance after a finite time. Note that the calculation of values T^* using formula (12) it is advisable to perform the graphical method. Let T^* – the point at which the absolute maximum of the objective function is reached. Then to determine this maximum, you can use the formulas that are obtained provided that the value of T^* satisfies equation (12):

$$\max_T K_{tu}(T) = K_{tu}(T^*) = \begin{cases} \dfrac{\bar{t}_m + I(F_R)}{\bar{t}_m + \bar{t}_R}, & T^* = \infty, \\ \dfrac{1 + \lambda(T^*)[I(F_R) - I(F_m)]}{1 + \lambda(T^*)(\bar{t}_R - \bar{t}_m)}, & T^* < \infty. \end{cases} \tag{16}$$

From formula (16) it is seen that if $T^* = \infty$, then it is impractical to perform maintenance, and we get a formula for the system readiness factor.

Next, we investigate the objective function (11) (cost quality of maintenance) at a minimum of T. Differentiating expression (11) by T and equating the derivative to zero, can be obtained the equation to determine the optimal values of the frequency of maintenance T_1^*:

$$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_m \bar{t}_m} = -F(T) + \lambda(T) = \left[\int_0^T [1 - F(T)]dt + \frac{c_R \bar{t}_R I(F_m) - c_m \bar{t}_m I(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m}\right]. \tag{17}$$

In its structure, equation (17) coincides with equation (12), so, repeating the previous reasoning, we arrive at the following inequality:

$$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_m \bar{t}_m} < 1 + \lambda(\infty)\left[\bar{t}_0 + \frac{c_R \bar{t}_R I(F_m) - c_m \bar{t}_m I(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m}\right], \tag{18}$$

which is a sufficient condition for the existence of the optimal frequency of maintenance T_1^* in terms of average unit costs (formula (11)).

If T_1^* – the point at which the absolute minimum of function (11) is reached, then given that when $T_1^* \neq \infty$ equation (17) holds, we obtain

$$\min_T \bar{C}(T) = \bar{C}(T_1^*) = \begin{cases} \dfrac{c_R \bar{t}_R}{\bar{t}_0 + I(F_R)}, & T_1^* = \infty, \\ \dfrac{\lambda(T_1^*)(c_R \bar{t}_R - c_m \bar{t}_m)}{1 + \lambda(T_1^*)[I(F_R) - I(F_m)]}, & T_1^* < \infty. \end{cases} \tag{19}$$

Next, to solve this problem it is necessary to find bilateral evaluations of the functionals $I(F_R)$ and $I(F_m)$, characterizing the quality of service.

2.3. Bilateral evaluations of functionals $I(F_R)$ and $I(F_m)$

At the preliminary stage of the solution for the case of complete initial information the calculated relations for determination of optimum values of periodicity of maintenance T^* and T_1^* are received (formulas (12) and (17)), to verify the fulfillment of sufficient conditions for the existence of the extremum of maintenance quality indicators (formulas (15) and (18)) and to calculate the extreme values of maintenance quality indicators (formulas (16) and (19)).

These formulas include linear functionals $I(F_R)$ and $I(F_m)$ (expressions (3) and (4)), the value of which depends on the type of function of the recovery time of the object $F_R(t)$ and duration of maintenance $F_m(t)$, the exact form of which is unknown, and only two initial points are known β_1, β_2 and s_1, s_2 (formulas (1) and (2)). Let the distribution function $F_R(t)$ belongs to the set of distribution functions K_2, satisfying the restriction (1), and the distribution function $F_m(t)$ – set of distribution functions L_2, satisfying constraint (2). For the above conditions, it is necessary to obtain accurate lower and upper estimates of the functional $I(F_R)$:

$$I_*(F_R) = \inf_{F_R \in K_2} I(F_R), \qquad I^*(F_R) = \sup_{F_R \in K_2} I(F_R);$$

provided that

$$\int_0^\infty dF_R(t) = 1, \quad \int_0^\infty [1 - I(F_R)]dt = \beta_1, \quad \int_0^\infty t^2 dF_R(t) = \beta_2, \ 0 < \beta_1^2 < \beta_2 < \infty;$$

and functional $I(F_m)$:

$$I_*(F_m) = \inf_{F_m \in L_2} I(F_m), \quad I^*(F_m) = \sup_{F_m \in L_2} I(F_m);$$

provided that

$$\int_0^\infty dF_m(t) = 1, \quad \int_0^\infty [1 - I(F_m)]dt = s_1, \quad \int_0^\infty t^2 dF_m(t) = s_2, \ 0 < s_1^2 < s_2 < \infty.$$

Record $F_R \in K_2$ (or $F_m \in L_2$) means that the distribution function is unknown $F_R(t)$ (or $F_m(t)$) belongs to the set of distribution functions $K_2(\beta_1, \beta_2)$ (or $L_2(s_1, s_2)$) positive random variables with fixed initial moments β_1, β_2 (or s_1, s_2). A general approach to the analytical solution of such problems was developed by L.S. Stoykova [13]. The main theorems underlying this approach, which determine the necessary and sufficient conditions for the existence of an extremum (exact upper and lower bounds) of some functionals, are given in the paper [14]. Here we obtain two-sided estimates of the linear functional, which for our case have the form:

$$I_*(F_R) = \inf_{F_R \in K_2} I(F_R) = \begin{cases} t_a\left(\beta_1^2/\beta_2\right) & \text{at } t_a < (\beta_2/2\beta_1), \\ 0{,}5\left(\beta_1 + t_a - \sqrt{t_a^2 - 2t_a\beta_1 + \beta_2}\right) & \text{at } t_a \ge (\beta_2/2\beta_1), \end{cases} \tag{20}$$

$$I^*(F_R) = \sup_{F_R \in K_2} I(F_R) = \begin{cases} t_a & \text{at } t_a < \beta_1, \\ \beta_1 & \text{at } t_a \ge \beta_1, \end{cases} \tag{21}$$

$$I_*(F_m) = \inf_{F_m \in L_2} I(F_m) = \begin{cases} t_{a1}\dfrac{s_1^2}{s_2} & \text{at } t_{a1} < \dfrac{s_2}{2s_1}, \\ 0{,}5\left(s_1 + t_{a1} - \sqrt{t_{a1}^2 - 2t_{a1}s_1 + s_2}\right) & \text{at } t_{a1} \ge \dfrac{s_2}{2s_1}, \end{cases} \tag{22}$$

$$I^*(F_m) = \sup_{F_m \in L_2} I(F_m) = \begin{cases} t_{a1} & \text{at } t_{a1} < s_1, \\ s_1 & \text{at } t_{a1} \ge s_1. \end{cases} \tag{23}$$

3. Method of calculating the limit values of the optimal periodicity and bilateral assessments of service quality indicators

We now turn to obtain the calculated relations that determine the overall solution of the problem. To do this, we use the results of solving two parts of the problems obtained in the previous stages of the study. If substitute the limit values $I_*(F_R)$, $I^*(F_R)$ and $I_*(F_m)$, $I^*(F_m)$ functionals $I(F_R)$ and $I(F_m)$ (expressions (20) – (23)) in formulas (12), (15 – 19) it is easy to obtain expressions to verify the conditions of existence of the extremum of the lower and upper estimates of quality indicators (Table 1), the equation for determining bilateral estimates of optimal periodicity maintenance (Table 2) and the calculated ratios for two-way assessments of extreme values of quality indicators (Table 3) [15, 16, 17].

Table 1
Formulas for checking the fulfillment of sufficient conditions for the existence of an extremum of quality indicators

Time reserve	Quality indicator	Formulas for checking the sufficient condition for the existence of the extremum of the lower estimate of the indicator	Formulas for checking the sufficient condition for the existence of the extremum of the upper estimate of the indicator
$t_a > 0$ $t_{a1} > 0$	$K_{tu}(T)$	$\frac{\bar{t}_m - I_*(F_m)}{[\bar{t}_R - I_*(F_R)] - [\bar{t}_m - I_*(F_m)]} < -1 + \lambda(\infty)\left\{\bar{t}_0 + \frac{\bar{t}_R I_*(F_m) - \bar{t}_m I_*(F_R)}{[\bar{t}_R - I_*(F_R)] - [\bar{t}_m - I_*(F_m)]}\right\}$	$\frac{\bar{t}_m - I^*(F_m)}{[\bar{t}_R - I^*(F_R)] - [\bar{t}_m - I^*(F_m)]} < -1 + \lambda(\infty)\left\{\bar{t}_0 + \frac{\bar{t}_R I^*(F_m) - \bar{t}_m I^*(F_R)}{[\bar{t}_R - I^*(F_R)] - [\bar{t}_m - I^*(F_m)]}\right\}$
	$\bar{C}(T)$	$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_m \bar{t}_m} < -1 + \lambda(\infty)\left[\bar{t}_0 + \frac{c_R \bar{t}_R I^*(F_m) - c_m \bar{t}_m I^*(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m}\right]$	$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_R \bar{t}_R} < -1 + \lambda(\infty)\left[\bar{t}_0 + \frac{c_R \bar{t}_R I_*(F_m) - c_m \bar{t}_m I_*(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m}\right]$

Note: $\bar{t}_R = \beta_1 = \int_0^\infty x dF_R(x)$; $\bar{t}_m = s_1 = \int_0^\infty x dF_m(x)$; $\bar{t}_0 = \int_0^\infty x dF(x)$; $I_*(F_R)$ – formula (20); $I^*(F_R)$ – formula (21); $I_*(F_m)$ – formula (22); $I^*(F_m)$ – formula (23); $\lambda(t) = \frac{F'(t)}{1 - F(t)}$.

Note: $\bar{t}_R = \beta_1 = \int_0^\infty x dF_R(x)$; $\bar{t}_m = s_1 = \int_0^\infty x dF_m(x)$; $\bar{t}_0 = \int_0^\infty x dF(x)$; $I_*(F_R)$ – formula (20); $I^*(F_R)$ – formula (21); $I_*(F_m)$ – formula (22); $I^*(F_m)$ – formula (23); $\lambda(t) = \frac{F'(t)}{1 - \bar{F}(t)}$.

Note: $I_*(F_R)$ – formula (20); $I^*(F_R)$ – formula (21); $I_*(F_m)$ – formula (22); $I^*(F_m)$ – formula (23); $\bar{t}_R = \beta_1 = \int_0^\infty x dF_R(x)$; $\bar{t}_m = s_1 = \int_0^\infty x dF_m(x)$; $\lambda(t) = \frac{F'(t)}{\bar{F}(t)}$; $\bar{t}_0 = \int_0^\infty x dF(x)$

Thus, the following method can be used to calculate the limit values of the optimal periodicity and two-way evaluations of service quality indicators.

1. Initial data for calculation:

– distribution function $F(t)$ (or $\bar{F}(t) = 1 - F(t)$) operating time t_0 object to failure with a mathematical expectation $\bar{t}_0$;

– distribution density $f(t) = F'(t)$ of random variable t_0;

– the failure rate of the object $\lambda(t) = \frac{F'(t)}{\bar{F}(t)} = \frac{f(t)}{\bar{F}(t)}$;

– the first $\beta_1 = \bar{t}_R$ and the second β_2 initial moments of recovery time $F_R(t)$;

– the first $s_1 = \bar{t}_m$ and the second s_2 initial moments of maintenance duration;

– the amount of time reserve used in recovery (t_a) and when performing maintenance (t_{a1});

– average costs per unit of time for recovery (c_R) and when performing maintenance (c_m).

Table 2
Equation for determining bilateral estimates (lower and upper limits) of the optimal frequency of maintenance

Time reserve	Quality indicator	Equation to determine the lower limits of the optimal frequency of maintenance $\underline{T}^*$ and $\underline{T}_1^*$	Equation to determine the upper limits of the optimal frequency of maintenance $\overline{T}^*$ and $\overline{T}_1^*$
$t_a > 0$ $t_{a1} > 0$	$K_{tu}(T)$	$\frac{\bar{t}_m - I_*(F_m)}{[\bar{t}_R - I_*(F_R)] - [\bar{t}_m - I_*(F_m)]} = -F(T) + \lambda(T) \times \left\{ \int_0^T \overline{F}(t)dt + \frac{\bar{t}_R I_*(F_m) - \bar{t}_m I_*(F_R)}{[\bar{t}_R - I_*(F_R)] - [\bar{t}_m - I_*(F_m)]} \right\}$	$\frac{\bar{t}_m - I^*(F_m)}{[\bar{t}_R - I^*(F_R)] - [\bar{t}_m - I^*(F_m)]} = -F(T) + \lambda(T) \times \left\{ \int_0^T \overline{F}(t)dt + \frac{\bar{t}_R I^*(F_m) - \bar{t}_m I^*(F_R)}{[\bar{t}_R - I^*(F_R)] - [\bar{t}_m - I^*(F_m)]} \right\}$
	$\overline{C}(T)$	$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_m \bar{t}_m} = -F(T) + \lambda(T) \times \left[\int_0^T \overline{F}(t)dt + \frac{c_R \bar{t}_R I^*(F_m) - c_m \bar{t}_m I^*(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m} \right]$	$\frac{c_m \bar{t}_m}{c_R \bar{t}_R - c_m \bar{t}_m} = -F(T) + \lambda(T) \times \left[\int_0^T \overline{F}(t)dt + \frac{c_R \bar{t}_R I_*(F_m) - c_m \bar{t}_m I_*(F_R)}{c_R \bar{t}_R - c_m \bar{t}_m} \right]$

Table 3
Bilateral assessments of extreme values of maintenance quality indicators

Time reserve	Quality indicator	The lower limit of the extreme value of quality indicators $\min K_{tu}(\underline{T}^*)$, $\min \overline{C}(\underline{T}_1^*)$	The upper limit of the extreme value of quality indicators $\max K_{tu}(\overline{T}^*)$, $\max \overline{C}(\overline{T}_1^*)$
$t_a > 0$ $t_{a1} > 0$	$K_{tu}(T^*)$	$\begin{cases} \frac{\bar{t}_0 + I_*(F_R)}{\bar{t}_0 + \bar{t}_R} & T^* = \infty \\ \frac{1 + \lambda(\underline{T}^*)[I_*(F_R) - I_*(F_m)]}{1 + \lambda(\underline{T}^*)(\bar{t}_R - \bar{t}_m)} & \underline{T}^* < \infty \end{cases}$	$\begin{cases} \frac{\bar{t}_0 + I^*(F_R)}{\bar{t}_0 + \bar{t}_R} & T^* = \infty \\ \frac{1 + \lambda(\overline{T}^*)[I^*(F_R) - I^*(F_m)]}{1 + \lambda(\overline{T}^*)(\bar{t}_R - \bar{t}_m)} & \overline{T}^* < \infty \end{cases}$
	$\overline{C}(T_1^*)$	$\begin{cases} \frac{c_R \bar{t}_R}{\bar{t}_0 + I^*(F_R)} & T_1^* = \infty \\ \frac{\lambda(\underline{T}_1^*)(c_R \bar{t}_R - c_m \bar{t}_m)}{1 + \lambda(\underline{T}_1^*)[I^*(F_R) - I^*(F_m)]} & \underline{T}_1^* < \infty \end{cases}$	$\begin{cases} \frac{c_R \bar{t}_R}{\bar{t}_0 + I_*(F_R)} & T_1^* = \infty \\ \frac{\lambda(\overline{T}_1^*)(c_R \bar{t}_R - c_m \bar{t}_m)}{1 + \lambda(\overline{T}_1^*)[I_*(F_R) - I_*(F_m)]} & \overline{T}_1^* < \infty \end{cases}$

2. Restrictions and assumptions:

– after the completion of any of the restoration work, the system (object + time reserve) is completely updated;

– it is necessary to comply with the following conditions:

$[\bar{t}_R - I_*(F_R)] > [\bar{t}_m - I_*(F_m)]$; $[\bar{t}_R - I^*(F_R)] > [\bar{t}_m - I^*(F_m)]$; $c_R \bar{t}_R > c_m \bar{t}_m$; $t_a < \frac{\beta_2}{2\beta_1}$; $t_a < \beta_1$; $t_{a1} < \frac{s_2}{2s_1}$; $t_{a1} < s_1$

3. Calculation of limit values of quality indicators.

3.1. Calculation of limit values $I_*(F_R)$, $I^*(F_R)$ and $I_*(F_m)$, $I^*(F_m)$ functionals $I(F_R)$ and $I(F_m)$ (formulas (20) – (23)).

3.2. Checking the fulfillment of sufficient conditions for the existence of the extremum of quality indicators using the inequalities listed in table. 1.

Execution of inequalities indicates the existence of finite values of the optimal periodicity of maintenance, providing extremes of quality indicators. Otherwise it is necessary to draw a conclusion that at the set values of initial data carrying out service is inexpedient $\left(T^* = \infty\right)$.

3.3. Definition of bilateral assessments (lower $\underline{T}^*$, $\underline{T}_1^*$ and upper $\overline{T}^*$, $\overline{T}_1^*$ boundaries) optimal frequency of maintenance using the equations given in table. 2.

3.4. Calculation of bilateral estimates (lower $\min K_{tu}\left(\overline{T}^*\right)$, $\min \overline{C}\left(\underline{T}_1^*\right)$ and upper $\max K_{tu}\left(\underline{T}^*\right)$, $\max \overline{C}\left(\overline{T}_1^*\right)$ boundaries) of extreme values of quality indicators using the formulas of table. 3.

Numerical example. Consider a system of continuous use with time redundancy, which has the following characteristics:

– the operating time of the object before failure is distributed according to Erlang's law of the and order

$$F(t) = 1 - e^{-\lambda t}(1 + \lambda t)$$

with the parameter $\lambda = 0{,}02$ 1/h and mathematical expectation $\bar{t}_0 = 2/\lambda = 100$ h;

– the failure rate of the object

$$\lambda(t) = \frac{\lambda^2 t}{1 + \lambda t} = \frac{4 \cdot 10^{-4} t}{1 + 0{,}02\,t} \text{ 1/h};$$

– the first and second initial moments of the recovery time $\beta_1 = \bar{t}_R = 2{,}0$ h, $\beta_2 = 8$ h^2 and maintenance time $s_1 = \bar{t}_m = 0{,}5$ h, $s_1 = 0{,}5$ h^2;

– allowable recovery time $t_a = 1{,}0$ h and maintenance $t_{a1} = 0{,}2$ h;

– average costs per unit of time for recovery $c_R = 60$ c.y./h and when performing maintenance $c_m = 40$ c.y./h.

For these initial data we will define limit values of optimum periodicity of carrying out maintenance and corresponding bilateral estimations of indicators of quality – a factor of technical use and average specific expenses. Solution. Consider first the solution for the coefficient of technical use of the system.

1. We calculate bilateral estimates $I_*(F_R)$, $I^*(F_R)$ and $I_*(F_m)$, $I^*(F_m)$ functionals $I(F_R)$ and $I(F_m)$ (formulas (20) – (23)).

Whereas $t_a = 1{,}0$ h less than value $\frac{\beta_2}{2\beta_1} = \frac{8}{4} = 2$ h, so

$$I_*(F_R) = t_a \frac{\beta_1^2}{\beta_2} = \frac{1 \cdot 4}{8} = 0{,}5 \text{ h}; \quad (24)$$

value $I^*(F_R)$ can be found at $t_a < \beta_1$:

$$I^*(F_R) = t_a = 1{,}0 \text{ h}. \quad (25)$$

It can be defined similarly

$$I_*(F_m) = t_{a1} \frac{s_1^2}{s_2} = 0{,}2 \frac{0{,}25}{0{,}5} = 0{,}1 \text{ h}, \quad (26)$$

$$I^*(F_m) = t_{a1} = 0{,}2 \text{ h}. \quad (27)$$

Using formulas (24) – (27), we make sure that the conditions specified in the restrictions and assumptions are met.

2. Whereas $\lim\limits_{t \to \infty} \lambda(t) = 0{,}02$ – the final value, then check the fulfillment of a sufficient condition for the existence of the optimal frequency of maintenance, using the formulas of Table 1 for the indicator $K_{tu}(T)$. Calculate the left and right parts of the formula, the lower score: the left part, which is equal to 0.364, less than the right part (0.999). Similar calculations are performed for the upper assessment of the indicator $K_{tu}(T)$ (Table 1): the left part is 0.428, and the right – 0.997. Therefore, sufficient conditions for the existence of the optimal frequency of maintenance are met for the lower and upper estimates of the indicator $K_{tu}(T)$.

3. Using the formulas of Table. 2 for the indicator, write the equation to determine the limit values ($\underline{T}^*$ and $\overline{T}^*$) of optimal frequency of maintenance. Denote by $\underline{V}(T) = f_1(T)$ and $\overline{V}(T) = f_2(T)$ right

parts of these equations.

Then

$$\underline{V}(T)=-F(T)+\lambda(T)\left\{\int_0^T \overline{F}(t)dt+\frac{\bar{t}_R I_*(F_m)-\bar{t}_m I_*(F_R)}{\left[\bar{t}_R-I_*(F_R)\right]-\left[\bar{t}_m-I_*(F_m)\right]}\right\}=-1+e^{-0,02T}(1+0,02T)+$$
$$+\frac{4\cdot 10^{-4}T}{1+0,02T}\left[100\left(1-e^{-0,02T}\right)-Te^{-0,02T}+\frac{2\cdot 0,1-0,5\cdot 0,5}{(2-0,5)-(0,5-0,1)}\right] \quad (28)$$

(recall that the left side of the equation is 0.364).

$$\overline{V}(T)=-F(T)+\lambda(T)\left\{\int_0^T \overline{F}(t)dt+\frac{\bar{t}_R I^*(F_m)-\bar{t}_m I^*(F_R)}{\left[\bar{t}_R-I^*(F_R)\right]-\left[\bar{t}_m-I^*(F_m)\right]}\right\}=-1+e^{-0,02T}(1+0,02T)+\frac{4\cdot 10^{-4}T}{1+0,02T}\times$$
$$\times\left[100\left(1-e^{-0,02T}\right)-Te^{-0,02T}+\frac{2\cdot 0,2-0,5\cdot 1}{(2-1)-(0,5-0,2)}\right] \quad (29)$$

(the left part of the equation is 0.428).

4. Solving these equations graphically (Fig. 2), we obtain $\overline{T}^*=122$ h and $\underline{T}^*=96$ h.

Substitute the found limit values of the optimal frequency of maintenance in the formulas of table. 3 and get the lower and upper estimates of the coefficient of technical use $\min K_{tu}\left(\overline{T}^*\right)=0{,}9858$; $\max K_{tu}\left(\underline{T}^*\right)=0{,}991$. When the maintenance is not performed $(T=\infty)$ and the time reserve in the system is not provided $(t_a=t_{a1}=0)$, the coefficient of technical usage is converted into a coefficient of readiness K_h:

$$K_{tu}=K_h=\frac{\bar{t}_0}{\bar{t}_0+\bar{t}_R}=\frac{100}{100+2}=0{,}98.$$

We move on to the definition of bilateral estimates of the cost indicator of $\overline{C}(T)$. Using the formulas of table. 1, calculate the value of the right-hand sides of the inequalities of this indicator and obtain a value close to unity. The left part of these inequalities is equal to 0.2. As we can see, the sufficient condition to be checked is fulfilled, so there are finite limits of the optimal periodicity of service, which provide extreme values of bilateral estimates of average unit costs [15, 16].

Using the formulas of table. 2 for the indicator $\overline{C}(T)$, calculate the right-hand sides of the equations to determine the limit values ($\underline{T}_1^*$ i $\overline{T}_1^*$) of optimal frequency of service:

$$\underline{V}_1(T)=-1+e^{-0,02T}(1+0,02T)+\frac{4\cdot 10^{-4}T}{1+0,02T}\left[100\left(1-e^{-0,02T}\right)-Te^{-0,02T}+\frac{60\cdot 2\cdot 0,2-40\cdot 0,5\cdot 1}{60\cdot 2-40\cdot 0,5}\right];$$

$$\overline{V}_1(T)=-1+e^{-0,02T}(1+0,02T)+\frac{4\cdot 10^{-4}T}{1+0,02T}\left[100\left(1-e^{-0,02T}\right)-Te^{-0,02T}+\frac{60\cdot 2\cdot 0,2-40\cdot 0,5\cdot 0,5}{60\cdot 2-40\cdot 0,5}\right];$$

while the left parts of these equations are 0.2.

It is easy to make sure that for the received source data $\underline{V}_1(T)\approx\overline{V}_1(T)$. Therefore, we construct one dependence curve $V_1(T)=f(T)$ (Fig. 3) and determine the value $T_1^*\approx 50$ ч. Substituting this value of the optimal frequency of service in the formulas of table. 3 for the indicator $\overline{C}\left(T_1^*\right)$, we get the bottom ($\min \overline{C}\left(T_1^*\right)$) and the top ($\max \overline{C}\left(T_1^*\right)$)estimates of average unit costs:

$$\min \overline{C}\left(T_1^*\right)=0{,}992 \text{ c.y./h}; \quad \max \overline{C}\left(T_1^*\right)=0{,}996 \text{ c.y./h.}$$

For comparison, we determine the average unit costs for the case when maintenance is not performed $(T=\infty)$ and there is no time reserve $(t_a=t_{a1}=0)$. In this case

$$\overline{C}\left(T_1^*=\infty\right)=\frac{c_R\bar{t}_R}{\bar{t}_0}=\frac{60\cdot 2}{100}=1{,}2 \text{ c.y./h}.$$

4. Summary

The given theoretical research of process of maintenance of objects with time reservation allows to draw the following conclusions:

1. The proposed approach to take into account the a priori uncertainty (incompleteness of the source information) when building a model of maintenance of objects of continuous use allowed to obtain relatively easy analytical formulas for quality indicators of maintenance, convenient for practical use. These relationships establish a relationship between the quality indicators and reliability characteristics of the object, the values of periodicity and duration and the parameters of time redundancy.

2. Taking into account in the developed model the parameters of time redundancy (allowable time of recovery and maintenance) the ability of objects to function normally under the influence of various destabilizing factors (failures, equipment failures, etc.). It is the presence of time reserve, which is a system parameter, that can in many cases explain why objects perform their functions more successfully than is the result of equipment failure

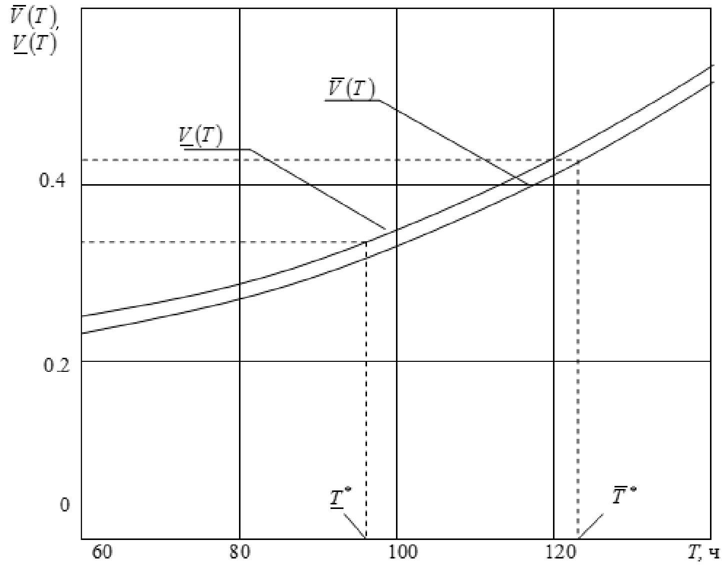

Figure 2: Graph of functions $\overline{V}(T)$ and $\underline{V}(T)$ to determine the limit values of the optimal frequency of maintenance

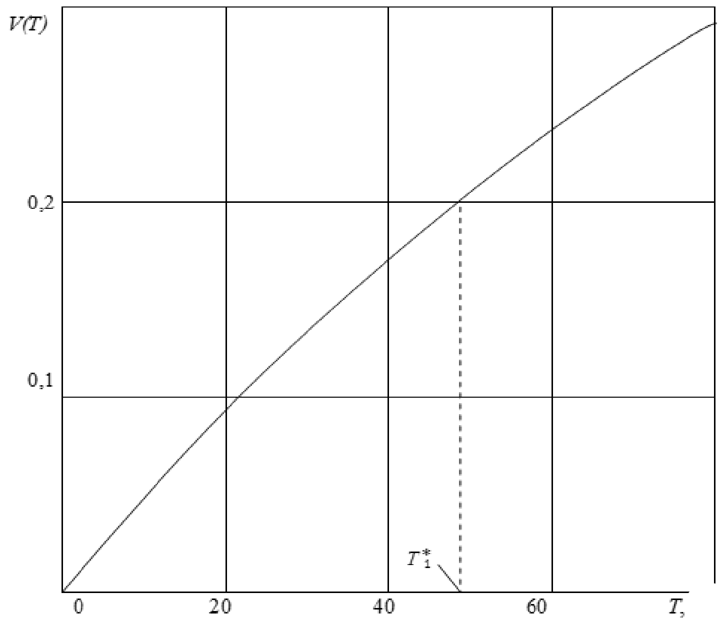

Figure 3: Graph of the function $V(T)$ to determine the value of the optimal frequency of maintenance T_1^*

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