A Model of Information Influences on the Base of Rectangular Stochastic Matrices in Chains of Reasoning with Possible Contradictions

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Abstract

A way of implementing the model "state-probability of choice", which describes probabilities of making a choice between certain alternatives, into chains of uncertain reasoning has been suggested. That means associating a special matrix, called rectangular stochastic matrix, with each node of the reasoning chain, whereas a node corresponds either to a given fact or to a rule of inference, maybe uncertain. A binary case, when a resolution has to be either rejected or accepted, has been regarded. A situation of a dynamic equilibrium, which means that none of these alternatives has advantage over the other, and possible ways of how agents of influence can break such a situation, have been demonstrated. Some ways of resolving conflicts between contradictory sources of evidence have been suggested.

Keywords [1]

Dynamic equilibrium of alternatives, chain of reasoning, contradictory evidence, rectangular stochastic matrices, agents of influence

1. Introduction

Uncertain reasoning takes a significant place in decision making. An agent, who has to make a choice between two or more given alternatives, often is not certain enough of facts affecting their decisions. Moreover, an agent's knowledge may be inconsistent, incomplete and contradictory. And there may be agents of influence, which are trying to affect what other agents believe in or how they behave. Belief networks typically based on Bayesian reasoning (Bayes networks [1]) or maybe on the Dempster-Shafer theory [2] are commonly used for modelling processes related to reasoning. It appears promising to integrate such approaches with knowledge graphs, which are referred to as a new trend in artificial intelligence [3, 4].

Studies of information influence comprise modeling information dissemination and exploring how information impacts affect levels of trust and belief of an agent. Issues relating to social modeling, spread of information and news, especially of fake news, establishing trust networks [5, 6, 7] are of great interest now. There is a lot of approaches to modeling individual and collective behavior of agents in multi-agent systems, a review of these approaches and models can be found in [8]. It appears helpful and important to enforce behavioral aspects of these and related models.

The model "state-probability of choice" based on the concept of rectangular stochastic matrices [9] was suggested in [10]. It involves into consideration distributions of probabilities regarded as certain states and a Markov chain describing changes of those probabilities in terms of transitions between the states. Within the model, an agent of influence can try to change transition probabilities, and thereby probabilities of choice and, speaking more generally, agents' behavior. That can be carried out by providing influences which can affect transition probabilities of the Markov chain.

In this paper we are developing an approach aimed at introducing the model "state-probability of choice" into a chain of reasoning, which might be a part of a more complicated belief network, by

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associating main elements of this model with nodes of the network. Insofar as an agent's knowledge may be uncertain or contradictory, we can talk about probabilities that an agent is going to accept or reject a specific resolution. Some experiments illustrating possible information influences will be presented.

2. Methods and tools

The model "state-probability of choice" can be shortly described as follows [10].

Let *n* be a number of alternatives to be chosen by an agent, and *m* be a number of states, each state represents a certain distribution of probabilities among the alternatives.

The model comprises two main components:

- the matrix $Z = (z_{ij}, i = \overline{1,m}; j = \overline{1,n})$ called the "state-probability of choice" matrix, where z_{ij} is a probability that when being in the *i*-th state, the agent will choose the *j*-th alternative;
- the stochastic matrix of transition probabilities $\Pi = (\pi_{ij}, i = \overline{1,n})$, where π_{ij} is a probability that when being in the *i*-th state, the agent will move on to the *j*-th one.

The sum of any row in the matrix Z equals *1*. By analogy with square stochastic matrices, such matrices can be referred to as rectangular stochastic matrices []. In [4] some properties of rectangular stochastic matrices have been established. A matrix Z can be chosen rather arbitrarily, but this arbitrariness can be significantly mitigated on the base of dividing the states into some groups having more or less clear meaningful interpretation. This will be explained below in the paper.

The matrix Π determines a Markov chain, which describes how probabilities of choice may change. It is well known that under some conditions a vector of stationary probabilities $p = (p_1, \dots, p_n)$, which is the main left eigenvector of Π, exists. Moreover, the corresponding eigenvalue equals *1*, and this vector satisfies the equation

$$p\Pi = \Pi \tag{1}$$

Therefore, in some situations within the model "state-probability of choice" we may postulate *p* instead of Π. Then the overall probability that an agent will choose the *j*-th alternative equals [4]

$$v = pZ \tag{2}$$

The vector $v = (v_1, \dots, v_n)$ contains such probabilities for all alternatives. A very important problem within the model is a problem of reaching a dynamic equilibrium, which is a situation when

$$\forall i\; v_i = \frac{1}{n} \tag{3}$$

Dynamic equilibrium is of especial importance in the case if *n=2* and decisions are being made collectively by majority of votes. Then it means that none of the two alternatives has advantages over the other. If the number of agents is large enough, a situation of the dynamic equilibrium is practically the only situation when the parity of alternatives holds, so they are rotated and win by turn, which was showcased in [10]. For the case n=2, the relation (3) takes the view

$$v = (0.5, 0.5) \tag{4}$$

Finding of dynamic equilibrium is related to the concept of balanced matrices [10]. A rectangular stochastic matrix is said to be balanced if sums of all its columns equal *1*. An important theorem, which states that if *n=2*, Z and p in (2) are a balanced matrix and a symmetric vector respectively, then the dynamic equilibrium holds, has been proved in [10]. If an agent of influence wants to promote an alternative which is losing at the moment, they may need first of all to reach the nearest point of the dynamic equilibrium and then to move it away from this point in the desired direction. Some illustrations of such a situation and of moving away from it it were presented in [10, 11]. Now we are going to discuss how to use the model "state-probability of choice" in chains of inference and reasoning.

3. A simple rule of inference

Let's consider a simple rule

$$A \Rightarrow B \tag{5}$$

We are interested in the probability that the resolution *B* will be accepted or rejected. So, we have two alternatives: ACCEPT and REJECT, and therefore *n=2*. In the simplest case, we may take

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

So, in this case the matrix *Z* is binary, that is its elements equal either *0* or *1*. This matrix is obviously a balanced rectangular stochastic one. For the rule $A \Rightarrow B$ we have to construct another matrix "state-probability of choice" *R*, which connects states of *A* and *B*. We introduce two states for it: supporting *A* called *FOR_A* and opposing *A* called *AGAINST_A*. Let's introduce two random variables: $\xi^{A\Rightarrow B}$ which can take values r_1 = *FOR_A* or r_2 = *AGAINST_A* and ξ^B, whose possible values correspond to states z_1 and z_2 represented by the rows of *Z*. Then we can consider *R* having elements

$$r_{ij} = P(\xi^B = z_j | \xi^{A\Rightarrow B} = r_j) \tag{7}$$

For instance, we may take

$$R = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix} \tag{8}$$

This matrix was chosen to be the balanced rectangular stochastic matrix. It is important that such matrices should be centrocymmetric [12, 13].

Given *A* as a known fact, we have to postulate either stationary probabilities p or transitive ones Π and then obtain p from Π. Anyway, it is easy to see that

$$v = pRZ \tag{9}$$

which ensues from the total probability rule.

It is important to mention that the matrix product *RZ* itself is a balanced stochastic matrix, so we can talk about some chain of balanced stochastic matrices alongside the whole chain of inference or reasoning. For this simple case, we will postulate p directly. In addition to this, an analysis carried out in [4, 5] allows us to choose p so that the dynamic equilibrium will hold. Insofar as p ought to be symmetric, there is no other possibility to reach that than to put $p = (0.5, 0.5)$. It gives

$$v = (0.5, 0.5) \tag{10}$$

So, dynamic equilibrium holds. If an influencer manages to change *p*, they may break the dynamic equilibrium. For example, taking *p=(0.55, 0.45)* gives *v=(0.53, 0.47)*. The analysis performed in [10] shows that if a decision is being made by majority of votes and the number of agents is large enough, the probability that *B* will be accepted is close to *1*. This simplest illustrative example gives the same results as the total probability rule does, that is only another way of calculations. Now let's consider a more comprehensive and more flexible example involving larger systems of states and transition probabilities. The following example also aims to demonstrate some behavioral aspects.

4. A more comprehensive example

The inference rule is the same as regarded above: $A \Rightarrow B$. Now we are going to introduce a matrix "state-probability of choice", which represents more states reflecting possible distributions of probabilities of accepting or rejecting *B*, likewise it was suggested in [10].

The matrix *Z* will represent some of such distributions. It should be balanced. Some techniques of building such matrices were suggested in [10]. Let *Z* be as follows:

$$Z = \begin{pmatrix} 1 & 0 \\ 0.9 & 0.1 \\ 0.75 & 0.25 \\ 0.6 & 0.4 \\ 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.25 & 0.75 \\ 0.1 & 0.9 \\ 0 & 1 \end{pmatrix} \tag{11}$$

As it was mentioned above, these basic states can be chosen rather arbitrarily. This arbitrariness can be reduced by explicit distinguishing groups of states which are directly related to the states represented by *Z*. But the similar effect can be reached if we build a matrix *R* which corresponds to the rule of inference in the way illustrated above.

Now we'll consider three groups of states:

- convinced proponents of accepting *B*;
- those who hesitate;
- convinced proponents of rejecting *B*.

Let's take *R* as follows:

$$R = \begin{pmatrix} 0.9 & 0.08 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.2 & 0.4 & 0.2 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.02 & 0.08 & 0.9 \end{pmatrix} \quad (12)$$

Whereas it appears problematic to postulate transition probabilities between the states of *Z*, possible transitions between the new three states appear to be much clearer.

Based on results of [10. 11], we are able to choose a matrix of transition probabilities Π so that the vector of stationary probabilities p would be symmetric, and thereby dynamic equilibrium would hold. In details, the following statement was proven.

Let a $(m \times m)$ -matrix *A* satisfy any of the following relations:

$$\forall i, j: a_{ij} = a_{i,m-j+1} \quad (13)$$

(i.e. the *i*-th and the *j*-th columns of *A* are equal to each other) or

$$\forall i, j: a_{ij} = a_{m-i+1,m-j+1} \quad (14)$$

(i.e. A is a centrosymmetric matrix).

Then the main eigenvector *x* of the matrix *A* is symmetric, i.e.

$$\forall j: x_j = x_{m-j+1} \quad (15)$$

Let's take

$$\Pi = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.5 & 0.4 \end{pmatrix} \quad (16)$$

The matrix Π is centrosymmetric, that means that it satisfies (14). Then the stationary probabilities of being in the groups are as follows:

$$p = (0.25, 0.5, 0.25) \quad (17)$$

This vector is symmetric, indeed. It gives the following vector of final probabilities of rejecting and accepting *B*:

$$v = pRZ = (0.5, 0.5) \quad (18)$$

So, dynamic equilibrium holds. An obvious way for an influencer to break the dynamic equilibrium may be related to affecting the transition probabilities. Changes of transition probabilities may occur when receiving additional information or on the base of reinforcement learning [14]. Some ways of applying reinforcement learning to establishing trust networks within the model called the Integrated Trust Establishment Model (ITE) were proposed in [15]. Really, if an agent gets a positive experience related to *B*, or if they find out a good fact about it, the probability of their transition to a group with a better attitude to *B* may increase. And vice versa, negative experience or information may push an agent to a group with a worse attitude.

Let Π be slightly changed and take the following view:

$$\Pi = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.25 & 0.6 & 0.15 \\ 0.1 & 0.5 & 0.4 \end{pmatrix} \quad (19)$$

Now

$$v \approx (0.5406, 0.4594) \quad (20)$$

and the dynamic equilibrium has been broken. B will be permanently accepted, if a number of agents is large enough and decisions are being made by majority of votes.

5. Contradictory rules of inference

Let there be two rules of inference with the same corollary:

$$\begin{gathered} A_1 \Rightarrow B, \\ A_2 \Rightarrow B \end{gathered} \tag{21}$$

Here we are coming back to postulating stationary probabilities explicitly. Let R and Z be the same as in the Section 4. Both A_1 and A_2 may be associated with different vectors p_1 and p_2.

Let's take

$$\begin{aligned} p_1 &= (0.8, \quad 0.1, \quad 0.1) \\ p_2 &= (0.1, \quad 0.3, \quad 0.6) \end{aligned} \tag{22}$$

Then we get two respective vectors, corresponding to different probabilities of accepting or rejecting B:

$$\begin{aligned} v_1 &\approx (0.8409, \quad 0.1591) \\ v_2 &\approx (0.2565, \quad 0.7435) \end{aligned} \tag{23}$$

This means that when reasoning on the base of A_1, B should be accepted, but on the base of A_2 it should be rejected. There are many approaches to how to combine such contradictory pieces of evidence. We are developing the one related to importance of the evidence.

6. Weighting evidence

An agent may regard some pieces of evidence more important than some other ones. Then we may consider a convex combination

$$p = \sum_i \alpha_i \, p_i \tag{24}$$

where

$$0 \le \alpha_i \le 1,$$

$$\sum_i \alpha_i = 1$$

The weight coefficients are interpreted as degrees of importance assigned to the known facts. The larger is α_i, the more important is the evidence. So, for achieving their goals, an influencer can try to affect not only transition probabilities but degrees of importance α_i as well.

Let's look at how to find weight coefficients ensuring a situation of dynamic equilibrium. Relation (9) gives the clear clue to this. Since

$$\forall i \; v_i = p_i RZ, \tag{25}$$

substituting (24) into (9) gives

$$v = pRZ = \left(\sum_i \alpha_i \, p_i RZ \right) = \sum_i \alpha_i v_i \tag{26}$$

Therefore, for ensuring a dynamic equilibrium we have to choose such α_i , for which the relation

$$\sum_i \alpha_i v_i = (0.5, \ldots, 0.5) \tag{27}$$

holds.

For two vectors, the relation (27) takes a view

$$\alpha v_1 + (1 - \alpha) v_2 = 0.5, \tag{28}$$

where c_1 and c_2 are the first elements of $v_1$and v_2 respectively.

After some simple transformations we get

$$\alpha = \frac{0.5 - v_2}{v_1 - v_2} \tag{29}$$

assuming $v_1 > 0.5 > v_2$.

On this base, let's find a situation of dynamic equilibrium for the example from the Section 5. We have there

$$c_1 = 0.8409; \;\; c_2 = 0.2565, \tag{30}$$

$$\alpha \approx 0.417,$$

and

$$p = \alpha p_1 + (1 - \alpha)p_2 = (0.3917, \qquad 0.2167, \qquad 0.3917)$$

The vector p is symmetric. Then

$$v = pRZ = (0.5, 0.5) \tag{31}$$

which means the dynamic equilibrium.

7. Another way of getting weights

For this simple case, there is another way of getting weight coefficients.

For finding α, which would make a symmetric convex combination, instead of solving the equation (28) we might try to apply a similar technique directly to p_1 and p_2.

Let's re-write elements of p_1 and p_2 in the following form:

$$\begin{aligned} p_1 &= (a, x, b) \\ p_2 &= (c, y, d) \end{aligned} \tag{32}$$

with given a, x, b, c, y, d.

A convex combination

$$p = \alpha p_1 + (1 - \alpha)p_2 \tag{33}$$

will be symmetric if the relation

$$\alpha a + (1 - \alpha)c = \alpha b + (1 - \alpha)d \tag{34}$$

holds. The middle elements x and y don't matter within this context.

From (34) we can get the solution

$$\alpha = \frac{d - c}{a - c - b + d} \tag{35}$$

This gives the same results as those in the Section 6. However, that is the very simple case when there are only two vectors representing two different facts, and each of them has only three elements representing three different levels either of certainty of given facts or maybe of attitude to the resolution under consideration. In more complicated cases, for instance when considering more groups of states and agents or if there are many different and maybe contradictory sources of evidence, a situation probably will be more intricate. But on the other hand, possible ways of getting situations of a dynamic equilibrium may become more flexible and multi-faceted.

8. Conclusions and discussion

This paper reports how the model "state-probability of choice", which was described and explored in [10], can be implemented in chains of uncertain inference and reasoning, which may be a part of a more complicated belief network. It has been suggested that each node of such a chain of reasoning, either a given fact or a rule of inference, shall be associated with a special rectangular stochastic matrix. So, it was shown in the paper that a sequence of reasoning can be represented as a chain of matrix products alongside the chain of reasoning.

In this paper the regarded states corresponding to the rules of reasoning were related mainly to various levels of support for accepting or rejecting the final resolution under consideration. But systems of states may be very different. It is possible to consider different levels of confidence about the rules or something else.

We regarded two basic cases: first of them is a simple action of reasoning on the basis of the modus ponens rule, and the other is a situation of two inference rules with the same corollary. Various systems having different amounts of states were considered. Even though these cases are very simple, they should be considered as building blocks for constructing AND-OR-graphs, belief networks based either on Bayesian networks or on the Dempster-Shafer theory, production systems [1, 2] etc. As regards a knowledge graph, stationary probabilities and intermediate matrices might be associated with its ground extensional and intentional components respectively, and the reasoning process within the model "state-probability of choice" is carried out by means of multiplying vectors and matrices. Approaches to implementing the model "state-probability of choice" on the basis of merging the regarded blocks into such composite structures should be developed.

We have considered a case of two contradictory rules in order to contribute a wide range of studies related to logical systems which may be incomplete or inconsistent [16, 17]. Some ways of resolving conflicts based on introducing weight coefficients, which reflect degrees of certainty about the evidence or maybe trust to them, were suggested in the paper. There is an importing question about how to assign these coefficients. Some approaches to that, based first of all on estimating experts' competence and that appear mostly promising to be combined with the model "state-probability of choice", were suggested in [18, 19].

It appears very reasonable to consider what may be a common point of paraconsistent and uncertain reasoning. Really, if an inconsistent system of assertions allows us to infer both a statement and its negation, we are able to talk about a plausibility of this statement, and therefore about a probability of accepting it, even though the available evidence and the process of reasoning itself may not be of probabilistic nature. Such considerations appear to be of great importance for social modelling in terms of multi-agent systems, in which resolutions are being made collectively, first of all by majority of votes. For studying such systems, behavioral aspects and factors relating to affecting and changing opinions of agents appear to be of a first-rank significance. The model "state-probability of choice" just aims to point out such behavioral aspects in a more or less clear and articulate way.

We consider possible wishes of influencers who are trying to increase or decrease levels of support for certain decisions and resolutions. We have explicitly introduced the states representing possible agents' attitude to certain resolutions, and within the suggested approach those agents of influence can affect transition probabilities between these states. Another way of influence is to affect degrees of certainty about the evidence and levels of trust to the sources of evidence.

Within this context, finding situations of a dynamic equilibrium appears to be a quite important issue. Dynamic equilibrium means that none of the alternatives has advantages over the others. In the particular case regarded in the paper, dynamic equilibrium means that there is the equal probability that a considered resolution would be accepted or rejected. So, if the resolution is going to fail, and an influencer wants to maintain and push it forward, they may try first of all to reach the nearest point of the dynamic equilibrium and then to move away from it in a desired direction [11]. In the paper, some ways of finding a dynamic equilibrium for the case of two contradictory facts with the same corollary based on selecting relevant weights of these facts have been suggested.

Even though the model figures out some parameters that agents of influence can affect, namely the transition probabilities between states and the levels of certainty and trust, they typically are not able to affect these parameters directly. Instead, some information influences should be delivered, and there is a special issue how to explore possible effects of such information influences. It appears promising to apply methods of the game theory, the algorithmic game theory, the theory of mechanism design [20, 21] as well. After all, it's worth mentioning that a multi-node network approach, which is being considered in this paper, seems to be rather fruitful for developing distributed multi-agent architectures for various applications such as [22, 23, 24, 25] etc.

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