# Hyperresolution for Guarded Formulae

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**Abstract.** Recently we have been investigating the use of hyperresolution as a decision procedure and model builder for guarded formulae. In general hyperresolution is not a decision procedure for the entire guarded fragment. However we show that there are natural fragments which can be decided by hyperresolution [9]. As hyperresolution is closely related to various tableaux methods the work is also relevant for tableaux methods. We compare our approach to hypertableaux, and mention the relationship to other clause classes solvable by hyperresolution.

The guarded fragment of first-order logic was introduced in Andréka, van Benthem and Neméti [1, 2]. It extends the modal fragment which corresponds to basic modal logic (via the relational translation) and is an important decidable class which contains many extended modal logics and description logics. Among the most notable properties of the guarded fragment in addition to decidability are Craig interpolation, bisimulation invariance, Beth definability, finite model property, and preservation under submodels.

Several extensions of the guarded fragment, like the loosely guarded fragment [7, 12], guarded fixpoint logic [11], or monadic $GF^2$ with transitive guards [8] have been shown decidable. The various decision procedures exploit the finite model property, use ordered resolution, alternating automata, or embeddings into monadic second-order logic. This is an interesting contrast to the literature on decidable modal logics and description logics, where tableaux-based decision procedures are predominant for testing satisfiability (see for example [5, 10]).

In [17] Lutz, Sattler and Tobies investigate whether tableaux-based decision procedures exist for subclasses of the guarded fragment. They introduce the fragment $GF1^-$ which is obtained from the first guarded fragment GF1 [2] by restricting the way the variables may occur in guards, and show that the fragment is decidable by semantic tableaux [17].

In [9] we continue their line of investigation. However, we make use of the close correspondence between tableaux-based decision procedure for modal logics and hyperresolution combined with eager splitting on an encoding of modal formulae in clausal logic, as described in [4, 15], and in [13, 14] for description logics. By using a structure preserving transformation of guarded formulae into clausal form we are able to recast the method in a first-order setting using hyperresolution, combined with positive factoring and eager splitting.

The method of proving termination of hyperresolution combined with positive factoring and eager splitting for the relational translation of extended multi-modal logics used in [4, 15] does not generalise to $GF1^-$. We investigate a different argument which takes into consideration the form of the derived clauses. The obtained decision procedure is practical. Standard resolution provers can be used without adaptation.

Furthermore, following the approach we show decidability of a larger class of formulae. We describe such generalisations of GF1$^-$ in [9].

We also investigate how our method relates to other inference methods such as hypertableaux [3], and how the work fits into the bigger picture of hyperresolution as a decision procedure [6, 16].

Currently we are looking into defining an abstract atom complexity measure $\mu$ in analogy to Leitsch [16] which would generalise the specific complexity measures and orderings used in the termination proofs presented in this paper and in [4, 13–15]. We are also attempting to define a larger solvable class which would accommodate more formulae outside the guarded fragment. Further it would be of interest to extend the approach to the entire guarded fragment, by using proper blocking conditions in the context of resolution.

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