Ranking-dependent Decision Support Methods for Weakly-structured Domains

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Abstract

The paper presents an overview of decision support methods based on pair-wise comparisons of alternatives, that take ordinal relations between them into consideration. The accent is made on the original method of expert pair-wise comparisons, based on a certain order of alternative estimation. This order is defined based on a preliminary rough ranking of compared alternatives. The described method allows us to improve the credibility of expert session results, obtain more consistent expert data, and, if necessary, reduce the labor intensity of expert estimation through reduction of required number of pair-wise comparisons.

Keywords

decision-making support, expert estimation, pair-wise comparison matrix, alternative ranking, weakly-structured subject domain.

1. Introduction

Weakly-structured subject domains often call for decisions, that require us to select the best alternative from a set, or rate the whole set of alternatives from a given set according to some "fitness" criterion. At the same time, every weakly-structured subject domain is characterized by high levels of uncertainty, which does not allow us to formally describe the domain and alternative decision variants, or obtain qualitative information about them. Consequently, expert estimates of alternatives, criteria, factors, and decision variants (options) often turn out to be the only credible source of such information. Particularly, we should mention the estimates presented in the form of pair-wise comparisons. It is worth noting that in actual weakly-structured domains, when it comes to decision-making, people often resort to primitive ad hoc-type procedures. At the same time, usage of more formal and universal expert estimation procedures would allow us to improve the credibility of expert session results.

Pair-wise comparisons (PS) define the relations between unknown relative weights of compared alternatives. These alternative weights are calculated based on a PC matrix (PCM). For this purpose, one of the many available methods (eigenvector [1], LLSM [2], GM [3], combinatorial [4] or other) can be used.

In order to compare n alternatives among themselves, one needs to perform $n(n-1)/2$ PC. So, the number of PC grows with dimensionality with the speed of $O(n^2)$. That is why, it makes sense to develop methods that would allow us to reduce the number of required PC without losing or distorting expert data.

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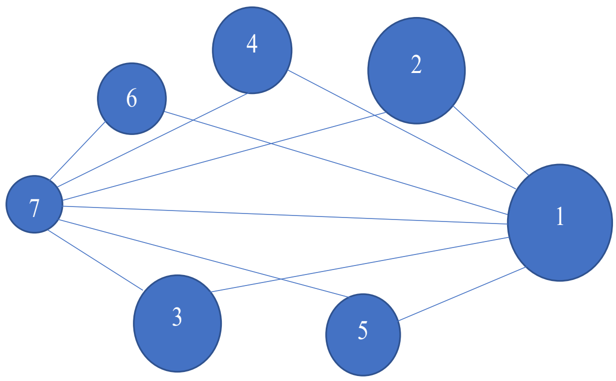
2. Overview of available methods that take the information on alternative ranking into account

In order to be able to reduce the number of PC, the expert session organizer might need some additional information on the alternatives and preference structures. For example, he might follow some certain preference structures, corresponding to minimum-diameter regular graphs, as described in [5,6]. Also, he might use the information on ordinal relation between compared alternatives.

In fact, a set of PC methods envisions usage of information on alternative ranking. Some ordinal factorial analysis methods rely on alternative rankings only [7,8]. Many of ranking-dependent approaches are based on experiments in cognitive psychology, such as the ones described in [9]. These experiments indicated that people estimate objects more precisely if these objects are presented to them in the order of decrease of the estimation criterion (i.e., from largest to smallest, from best to worst etc).

Historically, the first method using this approach was, probably, TOPSIS [10]. In this method, "ideal best" and "ideal worst" alternatives are generated based on multi-criteria expert estimates of alternatives. After that, alternatives are rated according to their distance from these ideal objects. We should also mention approaches used by Harker [11] and Wedley [12], which rely on preliminary rough ranking of alternatives. During the last decade several mono-criterial PC methods emerged, which relied on information on the ordinal relation between the alternatives. For instance, best\worst method [13,14] envisions comparing all alternatives with the best and the worst one in the set (instead of complete enumeration of pairs). TOP2 (best\second best) method [15] envisions comparing all alternatives from the set only with the first two alternatives in the ranking. Best\worst and TOP2 methods are illustrated by Fig. 1.



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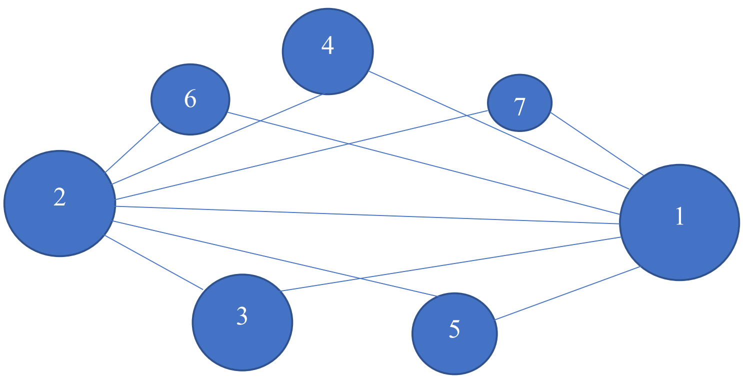

b

Fig. 1 Best\worst (a) and TOP2 (b) methods for the case of $n = 7$ alternatives.

3. Comparisons of the most distant alternatives: outline of the method

Another method, using the information on preliminary alternative ranking, has been suggested by the authors. It has been outlined in several conference papers, for instance, in [16]. Recently we have obtained some more data based on experimental results. In the following sections we will share the most recent findings related to the method.

The approach envisions comparing the alternatives in the order of decreasing distances between them in the ranking. So, PC are performed in queues.

Let the alternatives be numbered according to their preliminary ranking: $a_1 > a_2 > \cdots > a_n$, where a_i is an alternative with number and rank i, $i = \overline{1, n}$, while n is the general number of alternatives. Then, the sequence of expert PC (which ensures the greatest adequacy of expert session results to the expert's inner judgments) is as follows:

queue 1: (a_1, a_n) (ranks differ by $(n-1)$);
queue 2: (a_1, a_{n-1}) or (a_2, a_n) (ranks differ by $(n-2)$);
queue 3: (a_1, a_{n-2}) or (a_2, a_{n-1}) or (a_3, a_n) (ranks differ by $(n-3)$);
…
queue $(n-1)$: (a_1, a_2) or (a_2, a_3) or … or (a_{n-1}, a_n) (ranks differ by 1).

Once PC input is finished, alternative weights can be calculated using eigenvector or any other chosen method.

4. Outcomes of experimental research of the method

Experimental research of the method, outlined in [16], indicated that the relative weights of alternatives, calculated using eigenvector based on PCM, corresponding to the suggested PC sequence, were more adequate to experts' judgments about subject domains they considered themselves competent in (in comparison to weights, obtained based on other PC sequences).

Subject domains and problems chosen by respondents for decomposition were rather diverse. Some of the common subject domain examples, chosen by experts, are as follows:

- Development of a data analysis system development for a smart home
- Smartphone selection (based on design, functionality, aesthetical features)
- Hotel selection
- Analysis of competitors
- Developing a communication platform for a university department
- Starting a smart surveillance camera development project
- Choosing a bank for getting a deposit
- Car selection
- Choosing a cinema to watch a premiering movie
- Choosing a city to live in (based on location, living standards etc)
- Choosing a platform for app development
- Tour selection
- Choosing a foreign university for a master program
- POS system development
- Selecting a laptop for personal use
- Starting up a new band
- Development of a role-play (action, shooter) game
- Choosing a Smart TV
- Promoting an IT startup
- Choosing a music streaming service

- Choosing a mobile app for photo editing

So, as we can see, in comparison to previous experiments, which focused on one and the same subject domain or criterion (such as [12]), our experiment has been a more universal one.

Our recent research shows that even the criteria PCM, constructed according to the suggested PC sequence, are, generally, more consistent than PCM, constructed according to other sequences.

Several hundreds of experiment instances were conducted. Out of these, 83 meaningful instances, devoid of rank reversals, have been selected.

Table 1 shows the filtered (screened) results of the experiment. The 1st column indicates the number of experiment instance. The 9th column indicates how the expert him(her)self has ranked the PC sequences (or orders) A (most distant alternatives compared first), B (random order), and C (closest alternatives compared first). The 10th (last) column indicates the ranking of sequences A, B, and C, according to the value of consistency ratio CR of PCM, obtained based on respective sequences.

Table 1. Aggregated experiment results

#	Number of alternatives	Round # 1		Round # 2		Round # 3		Expert ranking	Ranked by CR value
		order	CR (%)	order	CR (%)	order	CR (%)		
1	7	B	4	C	5,5	A	3,5	ABC	ABC
2	5	A	4,8	C	2,7	B	4,3	CBA	CBA
3	6	C	5,1	A	5,9	B	6,6	BAC	CAB
4	5	C	11,9	B	11	A	1,2	ABC	ABC
5	6	B	9,6	A	4,1	C	6,4	CBA	ABC
6	6	B	5,2	A	3,4	C	5,4	CAB	ABC
7	5	B	24,8	C	31,8	A	16,8	ABC	ABC
8	7	C	7,1	B	8,2	A	8	ABC	CAB
9	5	C	8,9	A	6,3	B	11,7	CAB	ACB
10	5	A	7,3	C	8,7	B	3,9	CAB	BAC
11	5	C	9,4	B	9,9	A	10,5	ACB	CBA
12	5	A	5,1	B	2,3	C	4,8	BCA	BCA
13	6	A	4,2	B	4,1	C	7,7	CAB	BAC
14	6	C	10,4	A	7,5	B	7,3	ABC	BAC
15	5	C	15,3	B	20,3	A	9	ACB	ACB
16	6	C	30,7	A	25,7	B	22,6	BAC	ACB
17	6	A	7,9	C	18,2	B	22,1	ACB	ACB
18	5	C	12,4	B	26,9	A	10,4	CBA	ACB
19	5	C	6	A	5,1	B	3,8	BAC	BAC
20	5	A	3,5	B	6,1	C	3,6	CBA	ACB
21	5	B	9,6	C	9,8	A	13,3	ABC	BCA
22	6	B	23,6	C	11,8	A	32,5	BCA	CBA
23	5	B	12,7	A	1,5	C	1	BCA	CAB
24	5	B	3,8	C	1,1	A	3	ACB	CAB
25	5	C	2,7	A	1,1	B	2,7	ABC	ABC (CB)
26	5	B	10,6	A	3,5	C	8,4	ACB	ACB
27	5	C	6,2	A	2,6	B	11,6	CAB	ACB
28	5	A	7,2	C	8,5	B	2,1	BCA	BAC
29	6	B	7,3	C	5,9	A	6,3	CAB	CAB
30	6	A	4,3	C	2,4	B	2,3	ACB	BCA
31	6	B	3,23	A	4,8	C	3,21	ABC	CBA
32	6	B	15,2	A	21,6	C	31,4	ABC	BAC
33	5	C	15,5	B	19	A	32,4	CAB	CBA
34	6	B	23,9	A	20,5	C	21,9	ACB	ACB
35	6	C	10,4	A	8,4	B	5,3	CAB	BAC
36	6	C	8,9	A	8,2	B	4,9	ACB	BAC
37	6	C	10,9	A	8,2	B	8	ACB	BAC
38	6	C	13,7	B	20,1	A	14,2	CAB	CAB
39	5	C	15,46	A	15,49	B	12,3	ABC	BCA
40	5	C	13,9	B	15	A	17,6	ACB	CBA

Continue with the Table 1.

#	Number of alternatives	Round # 1		Round # 2		Round # 3		Expert ranking	Ranked by CR value
		order	CR (%)	order	CR (%)	order	CR (%)		
41	5	A	22,5	C	15,7	B	29	ABC	CAB
42	5	A	26,1	C	25,4	B	27,2	ABC	CAB
43	5	C	30,9	A	28,3	B	37,9	ABC	ACB
44	6	A	10,6	B	4,9	C	8,4	ABC	BCA
45	6	B	14	C	11,4	A	9,2	CAB	ACB
46	5	C	11,7	B	10,9	A	9,4	ABC	ABC
47	5	A	5,3	C	5,2	B	2,4	BCA	BCA
48	5	B	4,6	C	12,7	A	5,3	CAB	BAC
49	5	A	1,3	C	5,5	B	7,7	ACB	ACB
50	6	A	5,5	B	2,5	C	6,1	BCA	BAC
51	5	B	3,4	C	3,3	A	3,8	ABC	CBA
52	7	B	10,1	C	8,2	A	12,8	ACB	CBA
53	5	A	4	C	13,6	B	7	CAB	ABC
54	6	B	8,6	A	10	C	8,2	ABC	CBA
55	6	C	17,2	B	13,5	A	6,5	ACB	ABC
56	5	B	9,5	A	6,4	C	4,2	BAC	CAB
57	7	B	13,3	A	7,2	C	8,2	ABC	ACB
58	6	C	10,5	B	1,7	A	2,2	CAB	BAC
59	7	B	5,1	A	2,3	C	3,3	CAB	ACB
60	5	B	3,1	C	3,2	A	2,3	ABC	ABC
61	6	B	9,1	A	11,2	C	16,1	ABC	BAC
62	5	C	10,4	B	15,5	A	7,1	CAB	ACB
63	5	A	13,7	C	12	B	15,1	ACB	CAB
64	7	B	11,1	C	15,7	A	12,1	ABC	BAC
65	6	C	11,8	B	6	A	8,6	ABC	BAC
66	5	C	14,3	A	4,3	B	2,4	ABC	BAC
67	7	B	6,6	A	8,6	C	12,5	CBA	BAC
68	5	C	7,7	B	5,5	A	7,9	ABC	BCA
69	7	A	1,5	B	23,2	C	27,1	BCA	ABC
70	5	A	4,1	B	28,9	C	28,1	BCA	ACB
71	6	C	19,5	B	25,3	A	25,8	ABC	CBA
72	5	B	18,8	A	16,8	C	21,2	BAC	ABC
73	6	A	9,8	B	7,9	C	4,6	CAB	CBA
74	6	C	14,8	A	19,7	B	18,2	CBA	CBA
75	5	A	22,9	B	12,4	C	13,1	ABC	BCA
76	7	B	10,7	C	16,7	A	14,5	ABC	BAC
77	5	B	15,7	A	14,7	C	20,7	BCA	ABC
78	6	C	19,9	B	20,9	A	27,6	ABC	CBA
79	5	B	18,1	C	22,8	A	29,7	ABC	BCA
80	6	A	17,3	C	23,7	B	20,6	ABC	ABC
81	5	C	8,2	A	22,7	B	8,3	CBA	CBA
82	7	B	7,08	C	7,05	A	16,1	ABC	CBA
83	6	C	10,5	B	11,6	A	15,9	BAC	CBA

Table 2 shows the summary of experimental research of the method. As we can see from the table, in most cases experts ranked the suggested PC sequence as the first (i.e., they considered alternative weights obtained based on this sequence, more adequate to their "inner picture" of chosen subject domains). At the same time, average values of (in)consistency ratio (*CR*) [1] of PCMs, constructed according to the suggested PC sequence A, were lower than those of matrices, constructed according to other PC sequences. Also, in terms of *CR*, in most cases (30 out of 83), the suggested PC sequence yielded the best results among the three PC sequences, used in the experiment.

Many *CR* values in Table 1 exceed the heuristic threshold of 10%, suggested by Saaty [1]. However, while conducting the experiment, the main purpose of the authors was to obtain results, devoid of rank reversals. For instance, a respondent initially provides some ranking of the objects (or criteria) and then provides PC set, based on which relative object weights are calculated. Within our experiment, only incidents, in which rankings of these relative weights coincided with initial rankings, have been considered acceptable. Consistency of experiment results was not an initial priority and consistency studies have been conducted only recently, however, their results still speak in favor of the suggested approach.

Table 2. Results of experimental study of the method: a summary

Type of PC sequence	Number of experts who assigned the respective rank to the given PC sequence			Average value of consistency ratio CR, %	Share (%) and number of precedents (in brackets), in which the results of PC were more consistent
	1	2	3		
A (most distant objects first)	45	22	16	10,73	36,14 (30)
B (random PC order)	15	38	30	11,61	32,53 (27)
C (closest objects first)	23	23	37	11,88	31,33 (26)

5. Comparing the method with other ranking-based PC methods

One of obvious directions of future research would be comparison of the suggested PC method ("most distant objects first") with other similar ranking-based methods (such as TOP2 and best\worst). We have a simulation-type experiment in mind. Its phases are as follows:
1) generate a random weight vector;
2) reconstruct an ideally consistent PCM (ICPCM) based on the generated vector;
3) fluctuate the ICPCM with some noise matrix;
4) calculate the weight vector from the fluctuated PCM, using different methods (PC of the most distant objects first, TOP2, best\worst);
5) calculate and compare weight calculation errors across the three methods.

At the same time, such an experiment is problematic to conduct before we clarify some issues.

First, both TOP2 and best\worst are incomplete PC methods. They do not require experts to fill all PCM cell. Each of these two methods requires the respondent to perform just $2n - 3$ comparisons (see Fig. 1) and fill just $2n - 3$ cells of the respective PCM:

$$\begin{pmatrix} 1 & a_{12} & \cdots & \cdots & a_{1n} \\ & 1 & \cdots & & \\ & & 1 & \cdots & \cdots \\ & & & 1 & a_{n-1,n} \\ & & & & 1 \end{pmatrix} \begin{pmatrix} 1 & a_{12} & \cdots & \cdots & a_{1n} \\ & 1 & a_{23} & & a_{2n} \\ & & 1 & \cdots & \cdots \\ & & & 1 & \dots \\ & & & & 1 \end{pmatrix}$$

At the same time the suggested method, where the most distant objects are compared first, allows us to build a complete PCM (following the suggested sequence of comparisons Fig. 2), or, if necessary, to reduce the number of comparisons to minimum ($(n - 1)$ PC) [16].

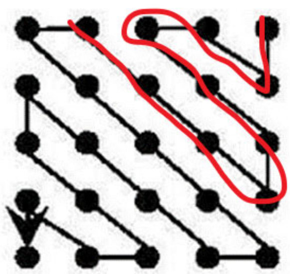

Fig. 2 PCM filling pattern for the suggested PC sequence

Fig. 3 illustrates a spanning tree, obtained according to the suggested PC sequence.

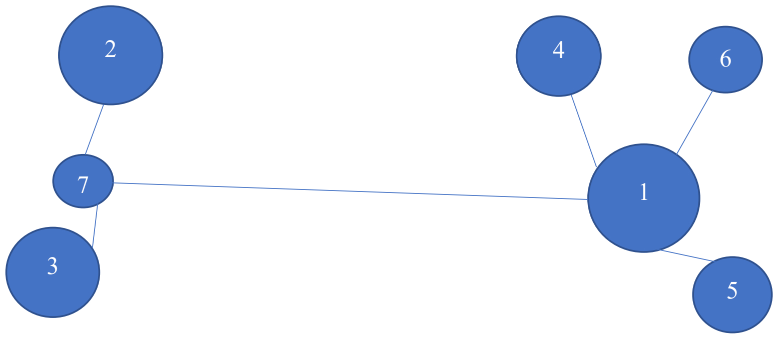

Fig. 3 Spanning tree, obtained for 7 alternatives based on their ranking and suggested PC order

So, the question "how many of the above-mentioned queues and PC from these queues will we need to perform within the experiment?" remains open.

Second, we need to determine, how to complete an incomplete PCM (as in order to apply the eigenvector method for weight calculation, we will need a complete matrix) based on available comparisons.

Third, it is unclear, which methods of weight calculation, besides eigenvector method, should we should apply (LLSM, combinatorial method, GM, other [1-4]).

6. Application of the method to enumeration of spanning trees

Much of recent research (including [3,4]) has been dedicated to reduction of computational complexity of the combinatorial method of spanning tree enumeration. In [4] it has been shown that the trees of smaller diameter accumulate smaller expert errors, than trees with larger diameter. So, it makes sense to consider these trees in the first place during aggregation.

As alternative ranking provides additional information on relations between alternatives, this information can be used for modification of the combinatorial method. Each spanning tree is a graph, consisting of $(n-1)$ edges. So, in order to sort the spanning trees from more informative to less informative ones, we can assign a rating (or utility) to each edge, based on the number of PC queue this edge belongs to:

$$u(T) = \sum_{i=1}^{n-1} \sum_{j=1}^{n-1} d_{ij}, \tag{1}$$

where:

$$d_{ij} = \begin{cases} j-1, & if \quad E_i \in Q_j, \\ 0, & if \quad E_i \notin Q_j; \end{cases} \tag{2}$$

So, spanning trees can be ranked and sorted according to their ratings (or utilities).

The suggested ratings provide an alternative order of spanning tree enumeration. Spanning trees, containing more information, should be enumerated in the first place during aggregation. While the approach outlined in [4] is based solely on the diameter of spanning trees and does not depend on specific object numbers, the approach suggested here relies on alternative ranks and, thus, is not invariant in terms of alternative numbers. At the same time, it is interesting to note that the trees containing just comparisons of neighboring alternatives are the "least informative" ones in terms of both graph diameter and comparison sequence.

To illustrate the sorting of spanning trees according to their ratings, let us consider the examples of 4 and 5 alternatives. Ranking of the spanning trees for $n = 4$ alternatives is shown in Table 3. Sorting of these trees is shown on Fig. 4. Ranking of the spanning trees for $n = 5$ alternatives is shown in Table 4. Sorting of these trees is shown on Fig. 5.

Table 3. Ranking of the spanning trees for $n = 4$

Tree rank	Utility value	Number of spanning trees
1	2	1
2	3	4
3	4	6
4	5	4
5	6	1

Table 4. Ranking of the spanning trees for $n = 5$

Tree rank	Utility value	Number of spanning trees
1	4	2
2	5	6
3	6	15
4	7	24
5	8	29
6	9	26
7	10	16
8	11	6
9	12	1

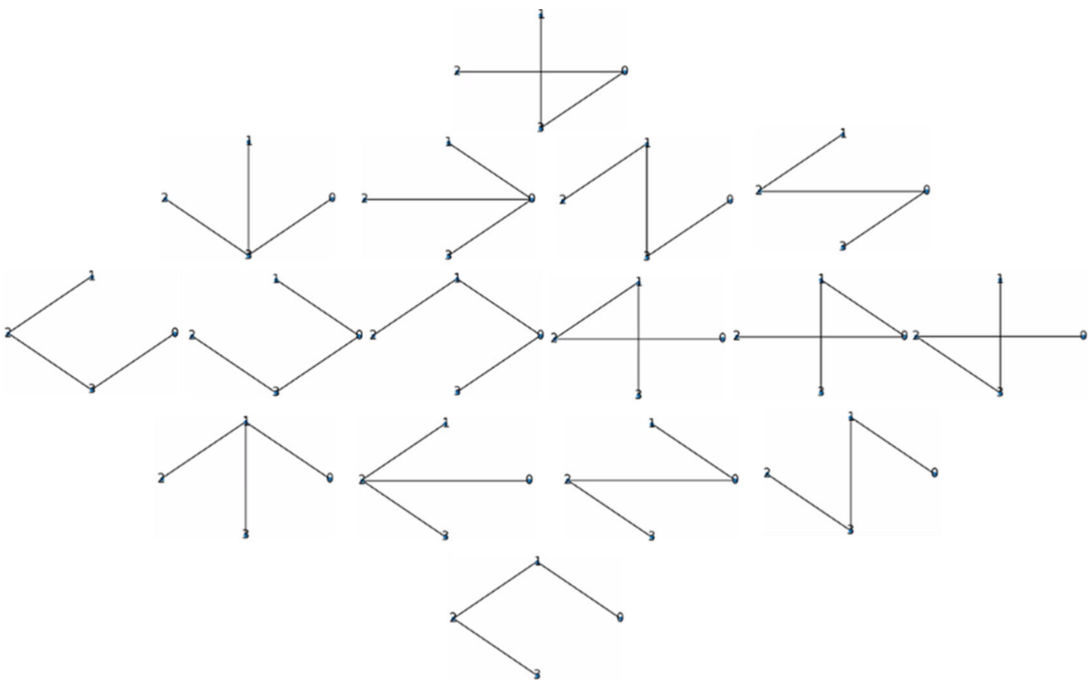

Fig. 4 Sorting of spanning trees by rating (utility) for 4 alternatives

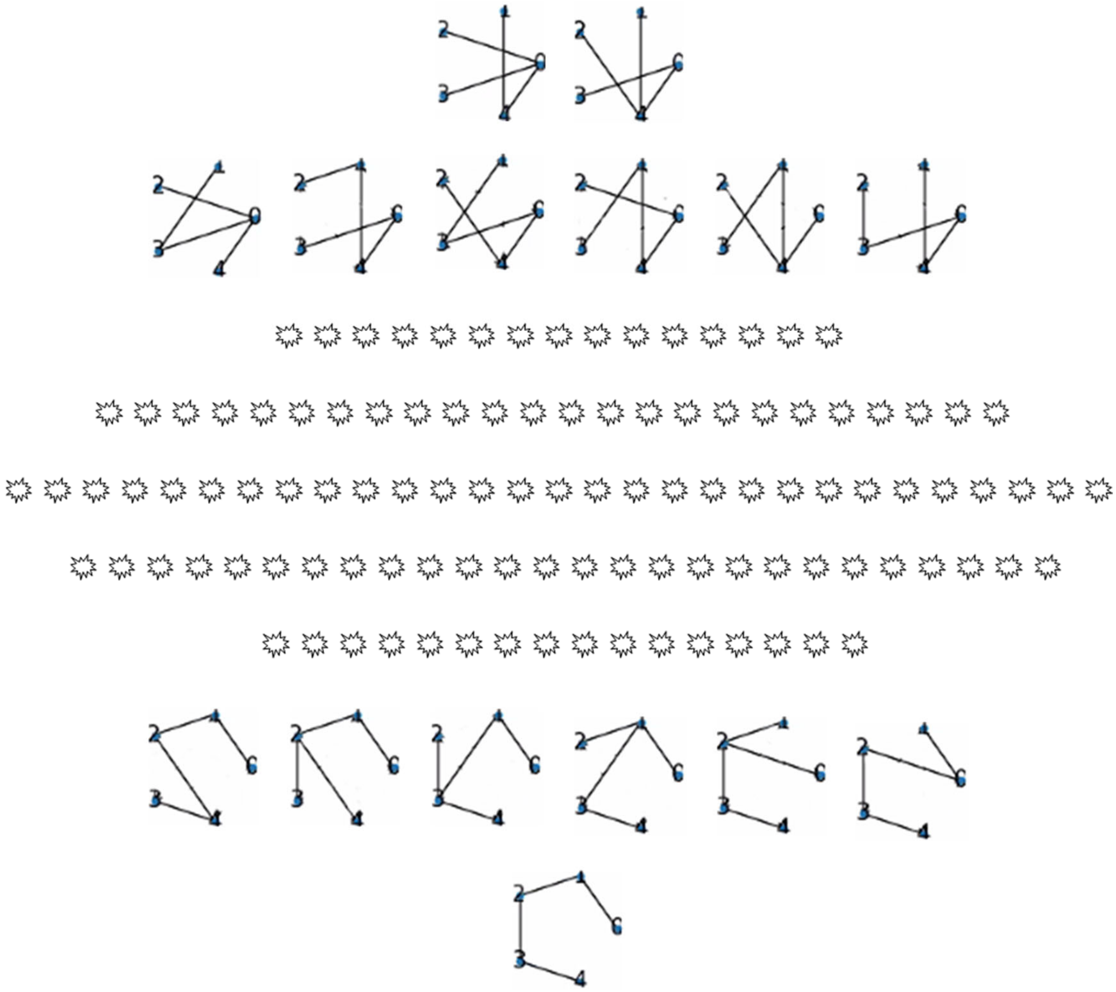

Fig. 5. Sorting of spanning trees by rating (utility) for 5 alternatives

7. Conclusions

In the paper we have presented an outline of expert estimation methods, based on preliminary ranking of alternatives. It has been shown that consideration of ordinal relation between alternatives allows us to reduce the number of PC without significant losses and distortions of expert information. We have suggested an original PC method, based on comparisons of the most distant objects in the ranking. Experimental results show that the method allows us to achieve better credibility and consistency of expert data. Moreover, if necessary, it allows us to reduce the number of required PC to the minimum. Additionally, it allows us to improve the computational complexity of combinatorial method of weight calculation. Further research will be focused on comparison of the method with other ranking-dependent PC methods, such as best\worst and TOP2.

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