Computer Model of Tuned Oscillator

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Abstract

Processes in tuned oscillators are studied. It is stressed that those processes are complex and analytical approach to understanding them is time-consuming and not flexible enough. Computer simulation is put forward as a method alternative to the analytical one. An approach is proposed to the development of a computer simulation model of a tuned oscillator and a computer simulation model based on this approach is designed. The computer model development procedure in the first stage involves drawing a system of differential equations for the analyzed circuit, in the second – transformation of the differential equations into the ones of difference and in the final stage – presenting the equation of difference in Matlab programming environment. Designed computer model of the tuned oscillator has a number of varying parameters that increase its potential to analyze different modes of operation and regimes of the circuit. In this paper, dependencies of the currents and voltages in the input and output circuits of the oscillator upon the feedback factor are analyzed and the threshold value of this feedback factor is ascertained. Obtained results clearly indicate validity of the tuned oscillator model and its applicability and effectiveness for studying processes in such circuits. Designed model can be used for performing promising research or assessing efficiency of already designed oscillators in the field.

Keywords [1]

Electronic communications, tuned oscillators, differential equations, equations of difference, computer simulation, computer models, Matlab

1. Introduction

Computer simulation is permeating more and more into different branches of technologies as a powerful tool of design and performance evaluation [1, 2]. This process is primarily driven by the rapid developments in the computer science and technologies and the obvious advantages that computer simulation brings with it. Among those obvious benefits are the cost of the computer simulation, which is usually much lower than creation of the natural models, and the precision and flexibility it delivers to the processes of design and assessment of the proposed solutions and approaches in different areas of engineering. As a result, computer simulation models have been successfully used in such diverse fields of engineering as adaptive antenna arrays, long transmission lines [3], pulse-forming networks and class-C power amplifies [4] with very good results. One of the branches of electronic engineering that so far has insufficiently deployed computer simulation models is that of generators or oscillators of sinusoidal waves [5-7]. To cover this gap, this paper proposes an approach to modeling and the designed operational computer simulation model in Matlab environment, which allows one to study in-depth the underlying processes in oscillators and assess their performance characteristics for a wide range of those oscillators' parameters. The computer simulation model design procedure starts with differential equations that represent processes in the

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different circuits of the oscillator, then transforms those differential equations into the equations of difference and, in the end, presents these equations of difference in the Matlab code [8-10]. The model has a wide range of changeable parameters, what creates unique opportunities for studying oscillators in different modes of operation and regimes.

Before proceeding to design of the computer simulation model and performing different experiments with it, the authors think it is relevant to describe the construction as well as operation of the chosen particular sinusoidal oscillator.

2. General Description of the Oscillator

2.1. Diagram of the Oscillator

Tuned oscillators play an important role in electronic communications for creating stable sinusoidal waveforms with the required frequency [11-13]. Let us consider fundamental operation and design of such a type of oscillator, as well as its operation principle. The tuned power oscillators are electronic circuits without a driver, in which oscillations are maintained by the feedback signal coming from the sinusoidal output voltage [14-16]. They are used in many applications [17, 18] and are similar in their design to the power amplifiers of different classes [19, 20].

It is known from the literature that tuned power oscillators might experience problems with design and frequency instability [6, 7]. Therefore, it is necessary to have an effective and useful tool to test such oscillators' usages and to understand their functioning mechanisms. Not least, such a tool would be important to perform design strategies and check performances of the developed device. Such tool is proposed to be a computer simulation model that is in the focus of this paper.

Information flow and the controlling inputs in the oscillator are shown in figure 1.

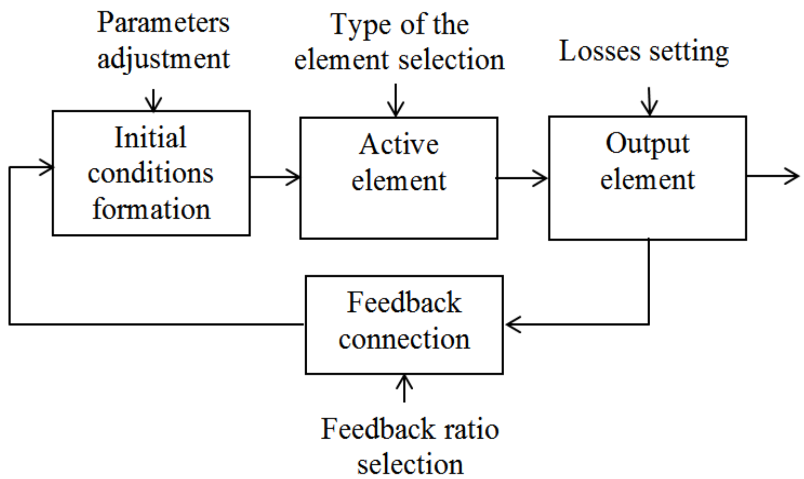

Figure 1: Information flow and the controlling inputs in the oscillator

For the sake of better understanding of the tuned oscillators operation, it is useful to compare them with tuned or resonance amplifiers [6]. The latter usually include DC voltage source, amplifying active element (transistor), the output element and some auxiliary elements. Auxiliary element includes voltage divider to bias the electronic device (set its operational point), thermo-stability circuits and some filtering condensers and inductances [6, 7]. The crucial distinction between tuned oscillator and the tuned amplifier is that the oscillator has a feedback connection while the amplifier includes an external driver to create the sine form at the output [6].

It is with this application of the feedback connection in the role of the driving signal that makes the oscillator work in the self-oscillation mode. In this type of oscillators, the signal coming over feedback route is a sinusoidal type because the output voltage is considered to have a sinusoidal waveform as well. Among other roles, the feedback element should adjust phase shift between output voltage and the voltage at the base of the transistor (active element), as well as required amplitude of the base oscillation. These two requirements are known as the balance of phases and the balance of amplitudes [6].

In the diagram shown in figure 1, capacitance C and inductance L can represent output element (the tuned output circuit or active element resonance load); feedback coil L_l that transfers some required energy from the output of the oscillator to its input – feedback connection; and resistor divider R_1 and R_2 that sets the bias (operation mode of the active element) – initial conditions formation part [6]. All the elements are subject to the controlling and disturbing inputs or influences.

2.2. Principle of operation and oscillation emergence and establishment

There are two possible ways that the sine waveform in the oscillators emerges. One of them is an electronic noise in the active element (transistor) and the other is power voltage step-function that emerges when the power is turned on [6]. The noise is thought to be wideband, so it has the necessary frequencies that excite the tuned circuit while the step function due to its steep front is supposed to create a broad range of harmonics that include the one that resonates with the tuned circuit as well. Both of these hypotheses are difficult to study analytically while the computer simulation model can easily prove or disprove them. Any of the mentioned two sources can secure the emergence of initial oscillations on the resonant frequency of the collector tuned circuit. At the same time, establishment of the oscillations happens due to the positive feedback connection presence as shown in figure 1. As the amplitude of the oscillations increases, the limitations of the collector current-base voltage characteristic of the transistor (active element) grow. This increase manifests itself in exhibiting nonlinear behavior, and, as a result, amplitude of the oscillations stops growing; hence, stationary regime gets established.

Strictly speaking from the mathematical point of view, for the emergence and establishment of the oscillations, amplitude balance condition and phase balance condition need to be present [6, 9-11]. The former means that the oscillator loop amplification should be bigger than unity at first and reach unity from above in the steady-state or stationary operation regime, while the latter implies that the sum of all oscillator loop phase shifts must be equal to zero or $2\pi n$, where n= 1, 2 . . . ..

In any case, studying tuned oscillators analytically is a difficult process [6] and, as we will see below, computer simulation can solve almost all the problems related to the oscillators.

3. Synthesis of the oscillator computer simulation model

3.1. Development of the differential equations system that describes the oscillator

There are three main units connected in the loop to form the oscillator (figure 1): the properly biased active element (transistor), which is described by the collector current – base voltage characteristic; output element (resonant circuit), behavior of which is determined by a system of differential equations for the tuned circuits with losses; and the feedback connection, which is described by one differential equation for the inductive connection.

Let us first consider collector current – base voltage characteristic [6]. This characteristic is determined by the biasing of the transistor, its type and usually shows some nonlinearity. Typically, it is approximated in the analytical studies by a function or a piecewise function. For the sake of our research, the authors propose to use a sine wave approximation of the following form:

$$i_{col} = \frac{I_{col\max}}{2}\left(1 + \sin\left(\frac{p}{2}\left(\frac{2u_B}{U_{B\max}} - 1\right)\right)\right), \tag{1}$$

where i_{col} is the transistor collector current; $I_{col\max}$ – maximum collector current; u_B – base voltage; and $U_{B\max}$ – maximum base voltage.

The curve determined by (1) is shown in figure 2 for maximum collector current equaling 20 A and maximum base voltage reaching 3 V.

There are many different approximations similar to the one shown in figure 2. They can be easily created and incorporated into the computer simulation model, but for this paper we use the one determined by (1). It is also necessary to stress that although (1) is slightly nonlinear, it does not include any derivatives or integrals and is not a differential equation.

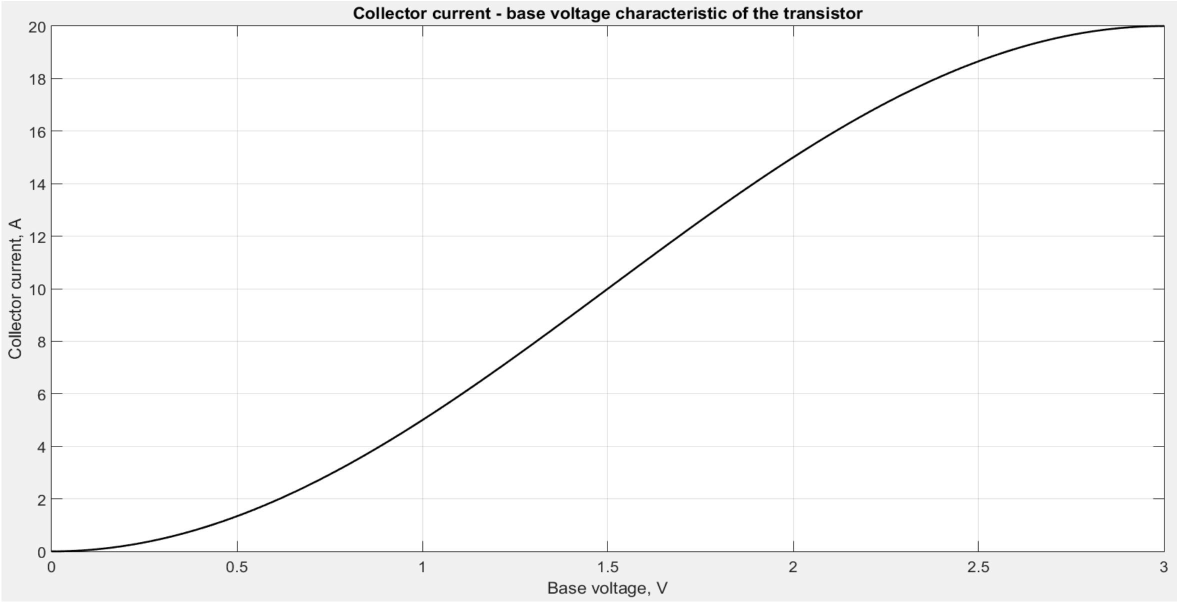

Figure 2: Collector current-base voltage characteristic of the transistor

Now, let us proceed to describing output element (tuned collector circuit) presented in figure 1.

The tuned output circuit of this type is fed by the collector current of the transistor – $i_{col}(t)$ [6]. As real tuned circuits as well have unwanted active impedance, this impedance is accounted for by adding resistor *R*. Now we can create the mathematical model for the output element in figure 1 as one of the parts of the oscillator.

There are two energy-storage elements in this circuit [6, 8]: one of them is inductor *L* and the other is the capacitor *C*. As a result, the mathematical model includes two first-order ordinary differential equations for the capacitor voltage $u_C(t)$ and inductor current $i_L(t)$,

$$C\frac{du_C(t)}{d(t)} = i_C(t), \tag{2}$$

$$L\frac{di_L(t)}{d(t)} = u_L(t). \tag{3}$$

Then, applying Kirchhoff's current law [6, 8], one can easily come to the following equation:

$$i_{col}(t) - i_C(t) - i_L(t) = 0. \tag{4}$$

After that, substituting (2) in (4) and moving this member to the opposite side of the equation, one obtains

$$C\frac{du_C(t)}{d(t)} = i_{col}(t) - i_L(t). \tag{5}$$

In the next stage of the transformations, one is required to find an expression for the voltage over the inductor *L*. Here, it is possible to apply Kirchhoff's voltage law [6, 8]. As a result, the following equation presents itself:

$$-u_C(t) + u_R(t) + u_L(t) = 0. \tag{6}$$

Moving $u_C(t)$ and $u_R(t)$ in (6) to the right-hand side, as well as plugging (3) into this equation instead of $u_L(t)$ and using Ohm's law for active impedance *R*, the following equation can be written:

$$L\frac{di_L(t)}{d(t)} = u_C(t) - i_L(t)R. \tag{7}$$

Equations (5) and (7) are considered to be the mathematical model of the output element (tuned circuit with losses) shown in figure 1. They represent the system of differential equations in terms of $u_C(t)$ and $i_L(t)$ while $i_{col}(t)$ is the source of signal feeding it. And finally, one needs to present the mathematical model for the feedback circuit with account of the bias applied to the base of the transistor. As is obvious from figure 1, both the bias (initial conditions) and the feedback voltage (through feedback element) are applied to the base of the transistor. The bias is created by the resistors R_1 and R_2, while the feedback signal is supplied via the inductive link through the L_l. As a result, the input voltage of the active element (transistor) in figure 1 is described by the following equation:

$$u_B(t) = L_m \frac{di_L(t)}{d(t)} \times K_{FB} + U_{Bias}\,, \qquad (8)$$

where $u_B(t)$ is the base voltage; L_m – mutual inductance of the feedback connection in figure 1; K_{FB} – feedback coefficient (determined by the ratio of the coils in the primary and secondary coils in the feedback connection in figure 1); and U_{Bias} – initial bias on the base of the transistor set by the resistors R_1 and R_2 in the initial conditions formation part in figure 1.

Under inspection, equations (1), (5), (7) and (8) represent the mathematical model of the oscillator shown in figure 1 in terms of the first order differential as well as algebra equations, and they are connected in a loop: in the same way as the sub-circuits of the oscillator diagram in figure 1. For example, equation (1) establishes collector current (i_{col}) as a function of the base voltage (u_B); then the formulas (5) and (7) deduce the inductance current ($i_L(t)$) as depending on the collector current (i_{col}), although in this case through solution of the differential equations; and, finally, (8) describes how the base voltage ($u_B(t)$) is linked to the inductance current $i_L(t)$. As it is obvious, one has gone the full circle in the equations.

In addition, equations (1), (5), (7) and (8) have some important input parameters that can be set up before the model testing and which describe the oscillator important design choices. For example, I_{colmax} and U_{Bmax} in (1) are chosen according to the transistor type selection. U_{Bias} in (8) corresponds to the initial bias and, hence, oscillation mode choice among many. Furthermore, parasitic impedance R in (7) describes the losses occurring in the tuned circuits while K_{FB} in (8) establishes the dependence of the performance on the feedback factor. Most importantly, as was mentioned above, initiation of the oscillations can be simulated by the choice of zero for the first value of the U_{Bias}, what creates the step function causing oscillations to occur. All together I_{colmax} , U_{Bmax}, U_{Bias}, R, and K_{FB} create ample opportunities for researching the oscillator presented in figure 1 in different settings.

3.2. Transition from the system of differential equations to the system of the difference equations

For implementation of the designed mathematical model (1), (5), (7) and (8) in Matlab environment, one must perform transition from the system of differential equations to the system of the difference equations. While for this purpose formula (1) is left as it is, equations (5), (7) and (8) are transformed with the help of the well-known Euler relation, as follows:

$$\frac{dx(t)}{dt} = \frac{x(t+\Delta t) - x(t)}{\Delta t}\,, \qquad (9)$$

where Δt is the step size, a small time increment that we assume fixed.

With account of (9), equations (7) and (8) can be presented in the form of the following pair of the equations of difference:

$$u_C(n+1) = u_C(n) + (i_{col}(n) - i_L(n))\frac{\Delta t}{C}\,, \qquad (10)$$

$$i_L(n+1) = i_L(n) + \frac{\Delta t}{L}u_C(n) - \frac{R\Delta t}{L}i_L(n)\,. \qquad (11)$$

Careful application of the equation (9) to the formula (8) leads one to the difference equation shown below,

$$u_B(n+1) = (i_L(n+1) - i_L(n))\frac{L_m}{\Delta t} \times K_{FB} + U_{Bias}\,. \qquad (12)$$

With account of the discrete time, equation (1) can be presented as

$$i_{col}(n) = \frac{I_{col\max}}{2}(1 + \sin(\frac{p}{2}(\frac{2u_B(n)}{U_{B\max}} - 1))) \; . \quad (13)$$

In their entirety, equations (10), (11), (12) and (13) represent computer simulation model for the oscillator shown in figure 1. In addition, it has many useful parameters that can be varied for different set-ups of some computer experiments. All this makes the computer model especially useful for research in the field.

4. Computer simulation of the tuned oscillator computer model

The computer simulation model developed in this paper and described by (10), (11), (12) and (13) allows one to study the oscillator in different modes of operation for various scenarios, but, for the purpose of this research, the authors chose the one that shows how the magnitude of the sine wave in the output tuned circuit, as well as other related signals, depend upon the feedback factor K_{FB} in (12). In this scenario, all other parameters are kept the same and their values are presented in table 1.

Table 1
Parameters of the oscillator computer simulation model and their values

Parameters	Values and Units
I_{colmax}	20 A
U_{Bmax}	3 B
U_{Bias}	1.5 B
L	0.5 Henry
C	2 F
R	0.2 Ohm
K_{FB}	0, 0.02, 0.04, 0.06, 0.08, 0.1
Δt	13 µS

Computer simulation was carried out for the setting presented above in the Matlab environment. As is evident from the table, feedback factor changed from 0 to 0.1 with the step 0.02.

Figure 3 and figure 4 illustrate collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.

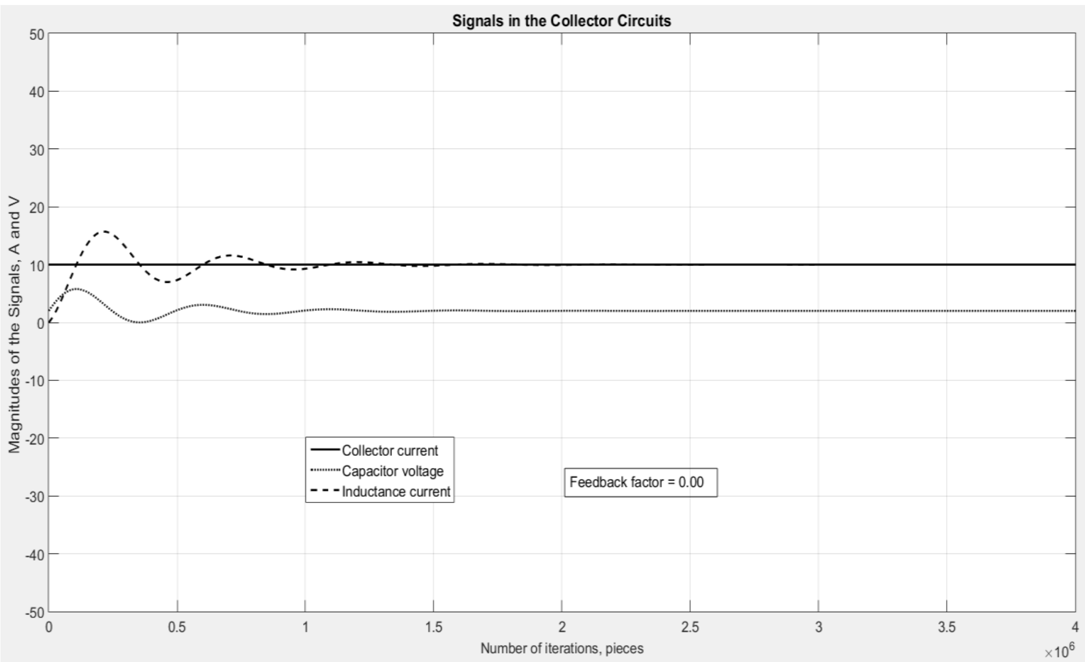

Figure 3: Collector current, capacitor voltage and inductance current for the feedback factor 0

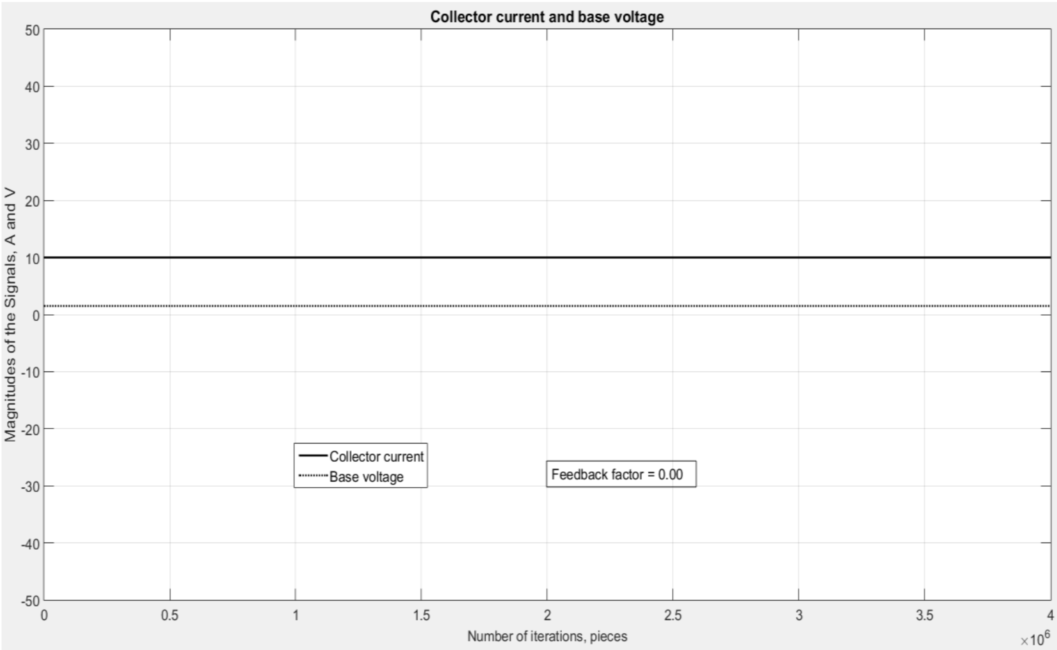

Figure 4: Collector current and base voltage for the feedback factor 0

From these figures, it is clearly seen that initial step-function of the collector current causes some decaying oscillations in the tuned circuit. This decaying process is evidently caused by the presence of the parasitic active impedance, as shown in table 1. One should notice that because feedback factor is zero, the oscillations do not get into the base circuit. It can be easily shown that if this active impedance is zero, the oscillations in the tuned circuit become permanent. Collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.02 are presented in figure 5 and figure 6. By comparing the waveforms in figure 5 and figure 6 with those in figure 3 and figure 4, it is easy to come to the conclusion that non-zero feedback factor facilitates appearance of the oscillations in the base circuit, and this factor helps to support the oscillations in the output tuned circuit; they have bigger magnitude and last longer. Still, it is not enough to overcome the losses brought in by the active impedance and the oscillations eventually subside to zero.

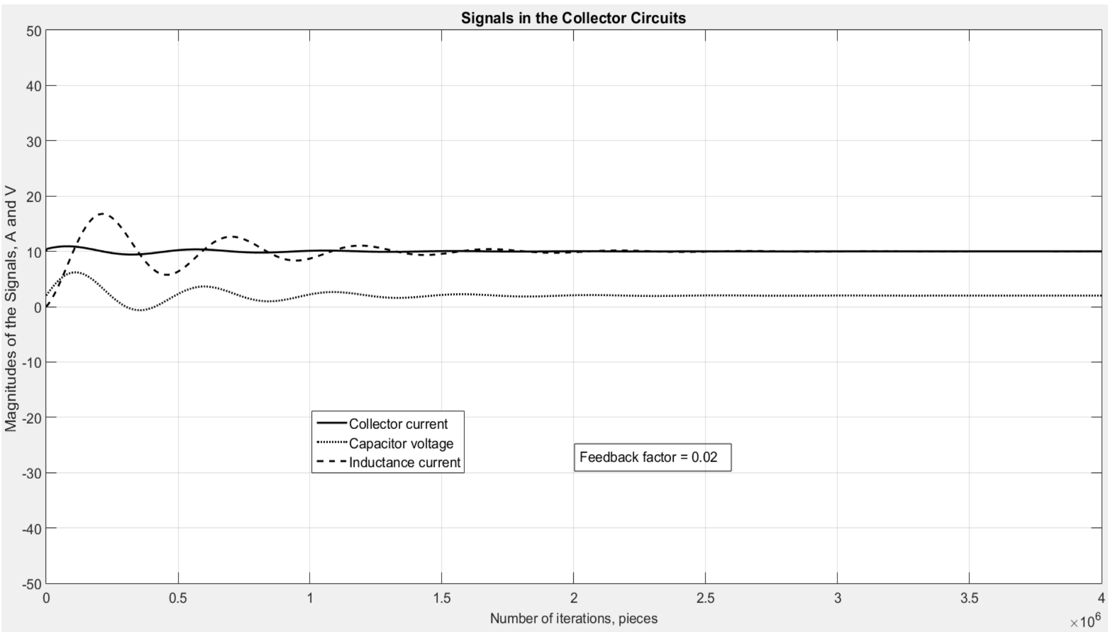

Figure 5: Collector current, capacitor voltage and inductance current for the feedback factor 0.02

Figure 7 and figure 8 demonstrate collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.04.

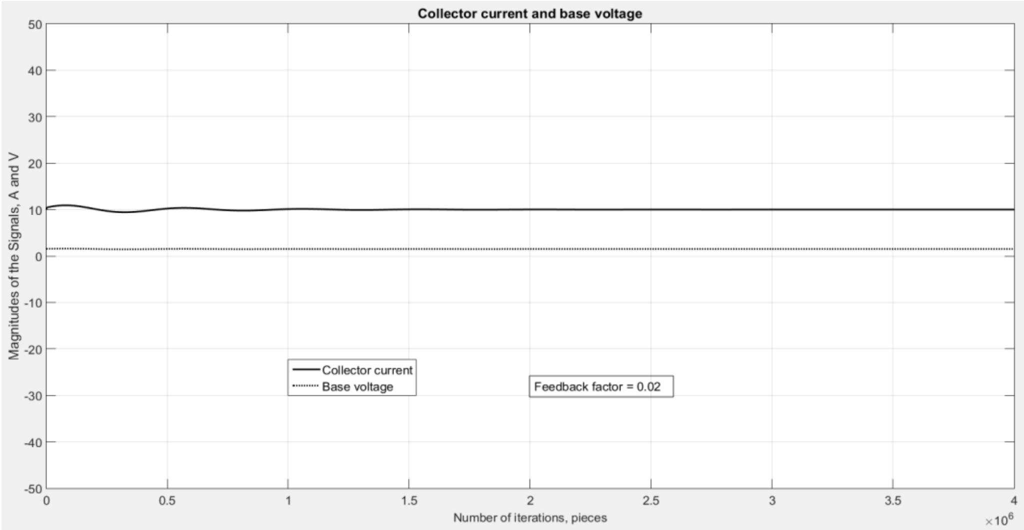

Figure 6: Collector current and base voltage for the feedback factor 0.02

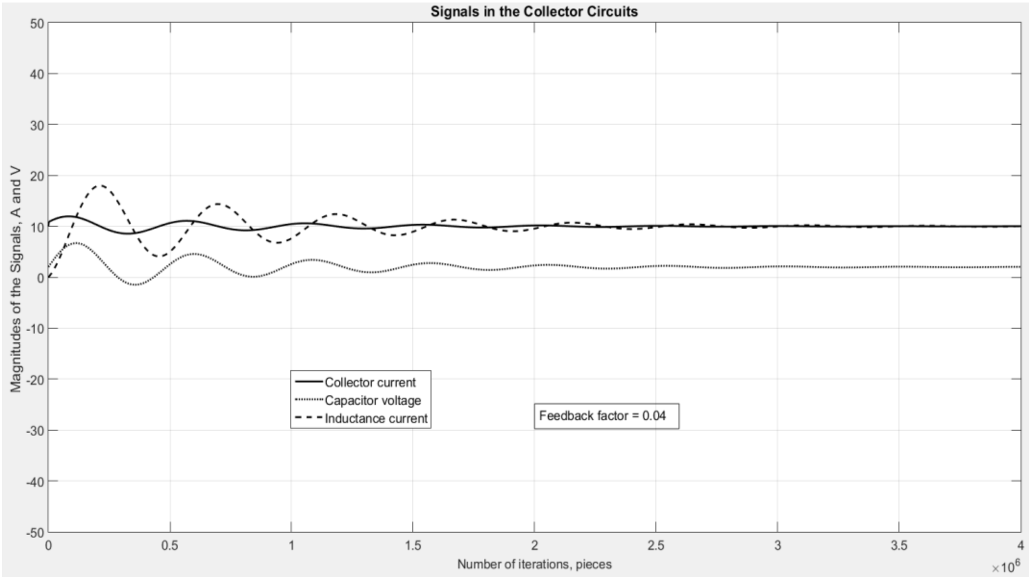

Figure 7: Collector current, capacitor voltage and inductance current for the feedback factor 0.04

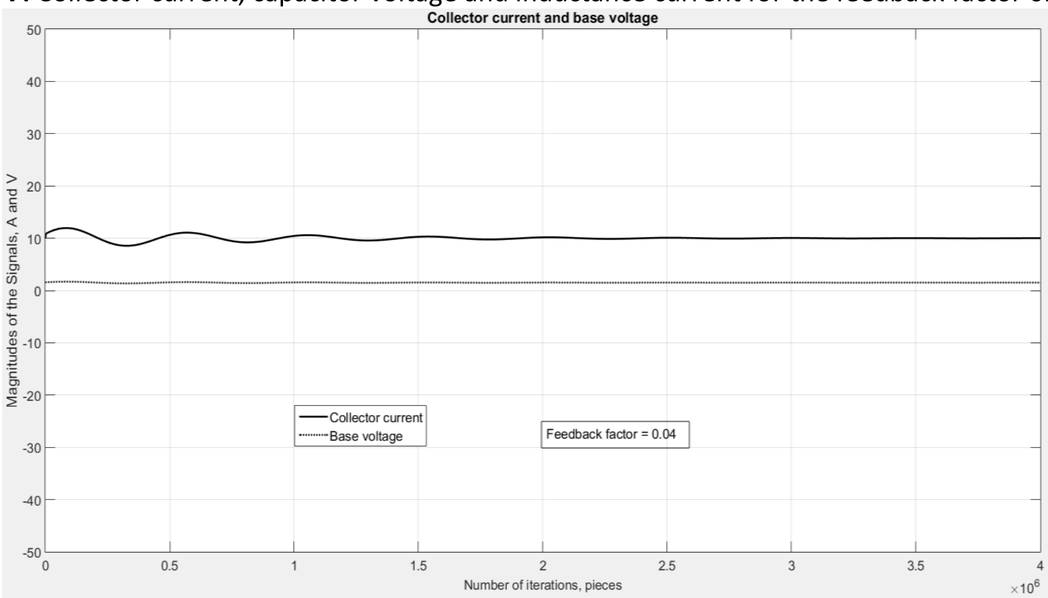

Figure 8: Collector current and base voltage for the feedback factor 0.04

One can see that the trend initiated by the previous feedback value is gathering pace: the oscillations in the collector tuned circuit last longer and their magnitudes are getting bigger than in the previous case. Nevertheless, the feedback factor is yet too small to secure non-decaying oscillations in the oscillator. Collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.06 are shown in figure 9 and figure 10.

As is obvious from the figures, the trend revealed earlier for lower values of the feedback factor continues and the magnitudes and the durations of the oscillations in the tuned output circuit increase even further. But yet again, one can deduce that the power introduced into the output circuit is not just enough to compensate for the losses created by the active impedance in this circuit. From the analytical point of view, one can stress that the balance of amplitude well-known from the literature and mentioned before still does not hold, but it is clear that we are almost near it.

Figure 11 and figure 12 show collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.08. This case is totally different from the previous one. It is clear that the energy introduced into the tuned circuit through the feedback link is higher than the losses suffered in the circuit due to the active impedance, and the oscillations become constant. This fact indicates that the balance of amplitudes is achieved for the feedback approximately equaling 0.07.

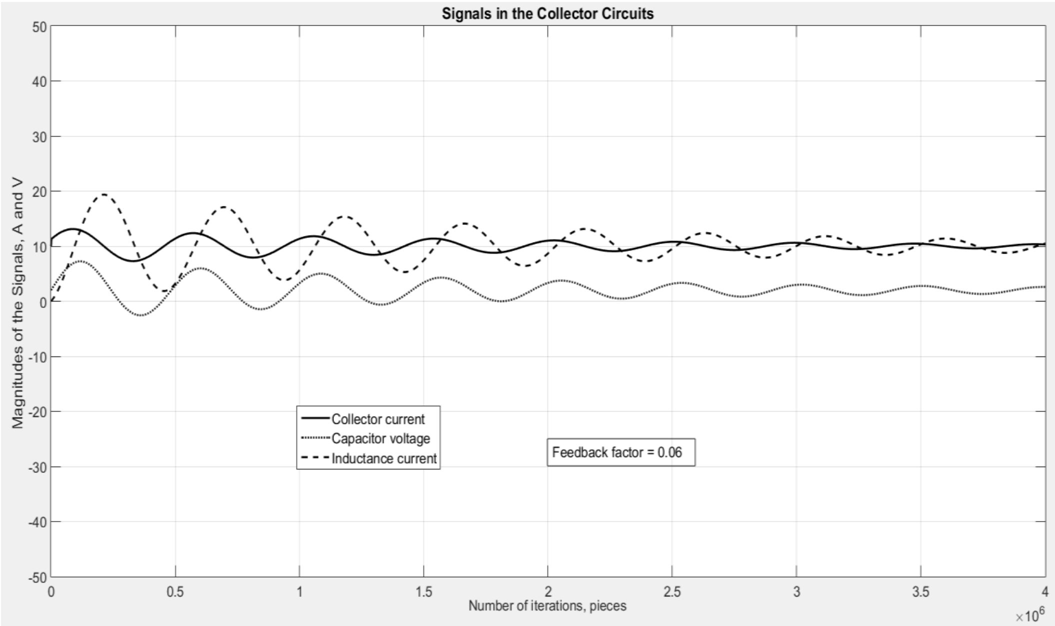

Figure 9: Collector current, capacitor voltage and inductance current for the feedback factor 0.06

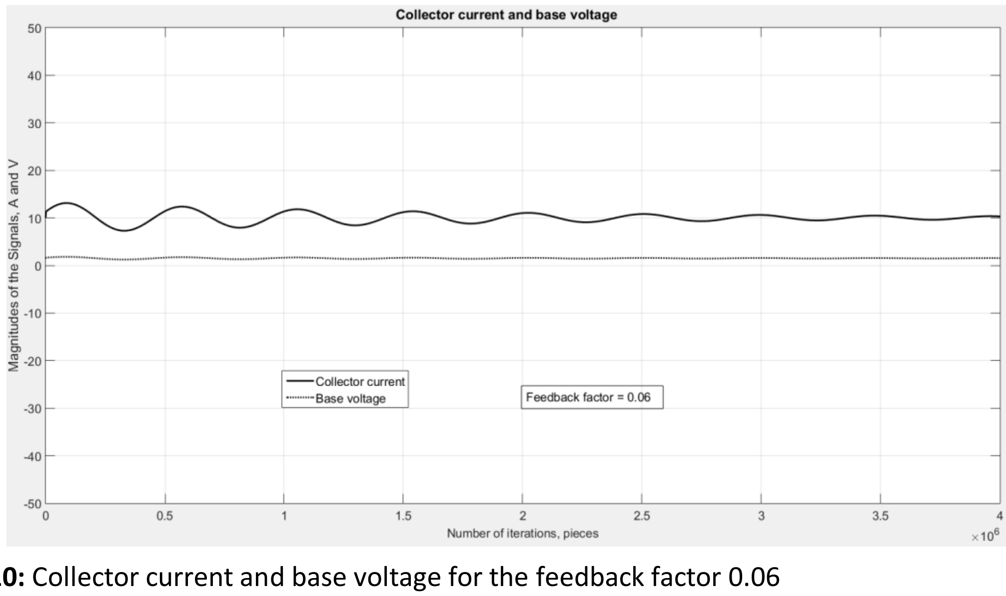

Figure 10: Collector current and base voltage for the feedback factor 0.06

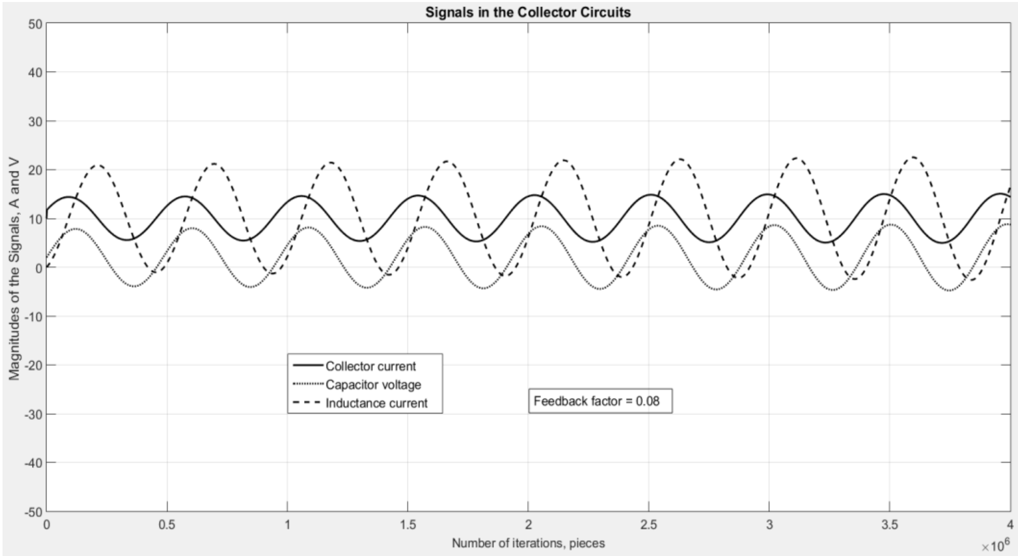

Figure 11: Collector current, capacitor voltage and inductance current for the feedback factor 0.08

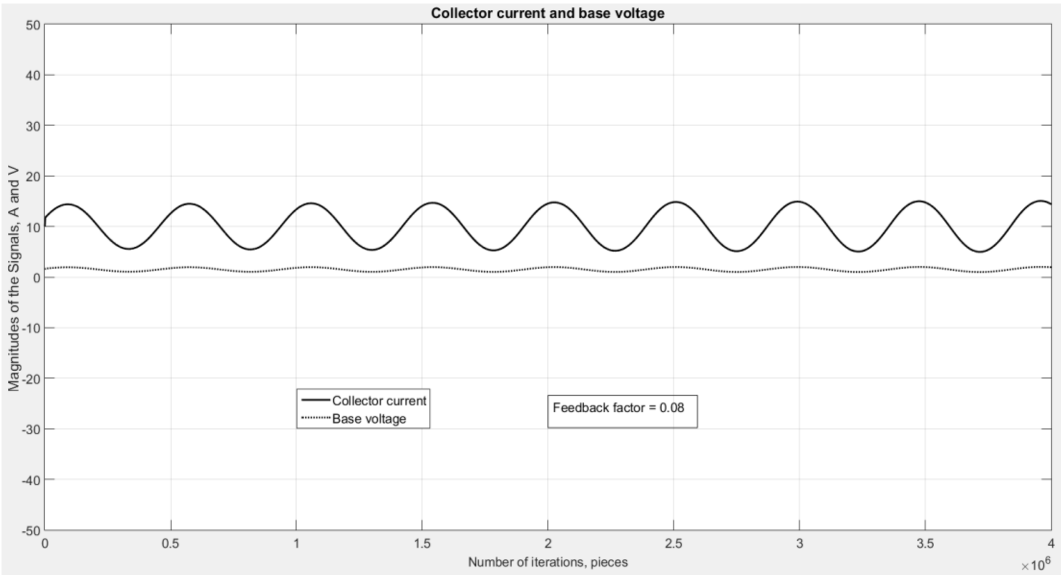

Figure 12: Collector current and base voltage for the feedback factor 0.08

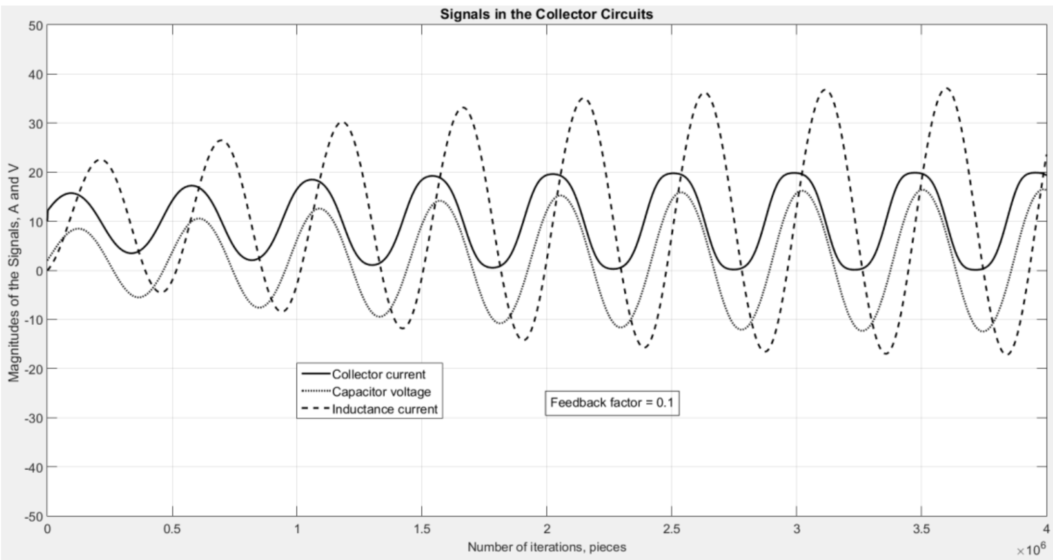

Figure 13: Collector current, capacitor voltage and inductance current for the feedback factor 0.1

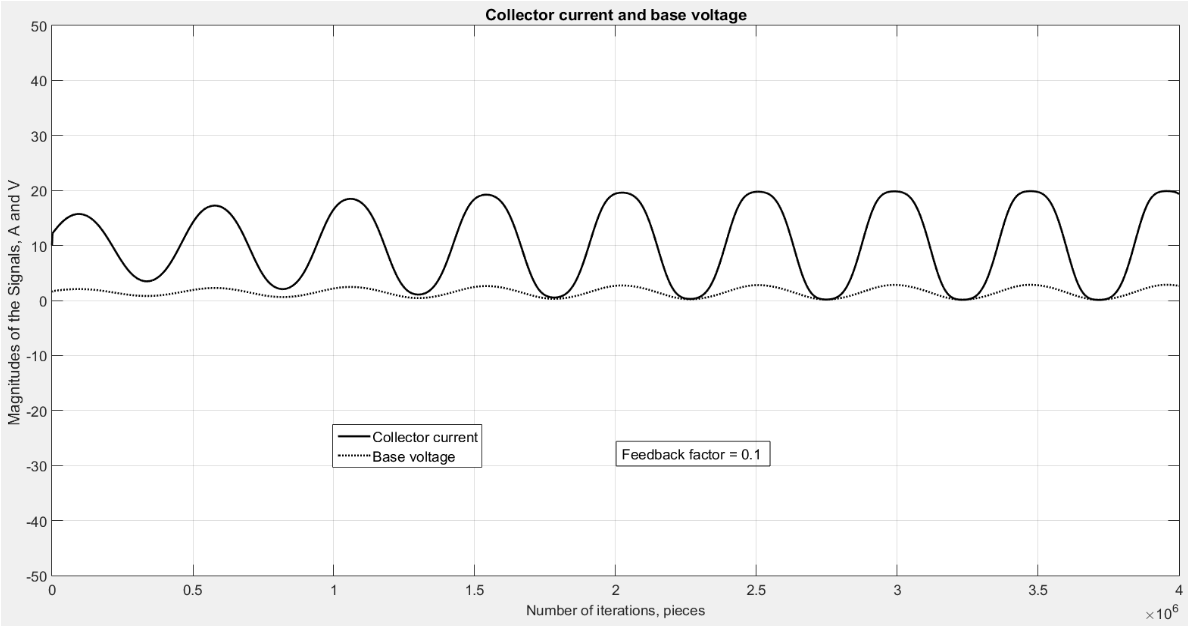

Figure 14: Collector current and base voltage for the feedback factor 0.1

Collector current, capacitor voltage, inductance current and base voltage for the feedback factor equaling 0.1 are shown in figure 13 and figure 14. This feedback ratio clearly leads to higher magnitudes of all the waveforms and higher stationary mode amplitude.

Overall analysis of the computer experiments with the designed computer simulation model shows that the model correctly represents operation of the oscillator shown in figure 1 and due to its simplicity and clarity can be successfully used to study processes in the oscillators presented in figure 1 as well as many others with some minor modifications.

5. Conclusion

In this paper an alternative approach to studying processes in oscillators has been proposed. This approach is based on the computer simulation model implemented in the Matlab environment. The developed model has been tested for the scenario of changing feedback factor in the oscillator. Simulation results proved that the model correctly represents the processes in the oscillator, and they comply with the general understanding of the processes in such devices. Those results also support the conclusions and assumptions made in scientific literature by other authors. This approach is simple, understandable and can be modified to be used for a wide range of tuned oscillators. Designed computer model can be used for doing new promising research in the field as well as improving performance of the existing devices.

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