

# Revisiting pedagogic strategies for supporting students' learning in Mathematical Microworlds.

## Devolving teachers' role to an 'intelligent' facilitator.

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**Abstract.** This paper presents categories of pedagogic strategies for helping students during mathematical explorations in microworlds, that take into account the constructivist theory of learning. We illustrate the strategies using examples from empirical data supported by other research in the field. As precursor to designing intelligent support for exploratory learning environments we discuss ways to operationalise these strategies in order to delegate some of the teacher's responsibilities to what we call an intelligent computer-based facilitator.

**Key words:** microworlds, pedagogic strategies

## 1 Introduction

The Migen project<sup>1</sup> is developing a technical and pedagogical environment to assist students with mathematical generalisations (see [1, 2]). Its core consists of a microworld. Mathematical microworlds belong to a particular genre of exploratory learning environments (ELEs) that allow students to explore not only the structure of accessible objects in the environment, but also construct their own objects and explore the mathematical relationships between and within the objects, as well as the representations that make them accessible [3, 4]. From this perspective they are a generalisation of other ELEs which normally allow the learner only to explore the effects of different variables on a particular model (for a review of other types of ELEs -such as simulation environments- see [5]).

A substantial body of research shows that although students may be able to use the tools available in microworlds (or in other exploratory environments), in order to ensure that students' interaction are effective and meaningful there is a need for significant pedagogic support (see [5],[6, p70-71],[7]) from teachers. To preserve the essence of exploratory learning environment, research suggests that the role of the teacher should be that of a 'competent guide' [8], a 'facilitator' [9] who, apart from structuring activities and promoting the appropriate learning atmosphere, recognises the need for students' autonomy and responsibility, directs their attention accordingly, and can help them organise their environment and plan and monitor their work.

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This role is difficult to achieve in a normal classroom and therefore, teachers often revert to their role as a transmitter of knowledge into a set of ‘empty vessels’ [9]. We envisage that some of the teacher’s responsibilities could be delegated to an intelligent system which could support either the student directly or provide information to teachers, helping them in their role as facilitators. We believe the second option is particularly relevant and timely.

Along with other methods, principled approaches for developing intelligent support can be based on observation of human tutors and students as well as relevant theories of learning [10]. The constructivist theory of learning is particularly relevant since it has been transformed through constructionism to a strategy for learning, particularly applicable in microwords [11]. However, despite the fact that previous research has recognised the need for more explicit research on how students learn when interacting with microworlds and on appropriate types of teacher interventions, relatively few attempts (e.g., [12]) investigate specific strategies and ways to support students’ learning from a constructivist perspective in general. Fewer still (e.g., [7, 9]) are focused specifically on microworlds.

This paper presents pedagogic strategies for helping students during their exploration in microworlds as a precursor to designing intelligent support. In particular, Section 2 provides a brief description of the teacher’s role, shaped by our understanding of our collaborating teachers, and taking into account previous research in the field and the constructivist theory of learning. Section 3 presents an initial framework of pedagogic strategies, drawing examples from empirical data supported by research literature. Section 4, provides suggestions for operationalising our framework at a level of detail that would allow devolving some of the teacher’s responsibilities to an intelligent computer-based facilitator.

## 2 Teacher’s role as a guide and facilitator

In microworlds, understandings are, at least partly, generated during interactions with the system, rather than having to precede them. Although the design of the microworld and the guidance provided by structuring activities should, ideally, enable students to connect their actions and the relationships embedded in the microworld, with the mathematical principles or ideas that a teacher would like them to construct, inevitably they often need explicit support. The teacher’s role as facilitator of this process is indispensable.

A helpful way of understanding this role, in general, is interpreting it as having to be sensitive to the learner’s attention, but also to the way they are attending [13], as well as their preferred strategies for solving a problem [12]. More specifically, in microworlds, a key challenge for teachers is to support exploration that is goal-oriented and which aligns with the teachers’ agenda (see the ‘play paradox’ notion in [6] and other classroom vignettes in [7, 9]). In addition, expert teachers rarely assume the role only of an authority that judges the quality of responses. On the contrary, they create situations where students can *reflect* on their own responses and strategies. Additionally, the teacher promotes *motivation* and supports the *collaboration* between students, not only through the design of the activity but during the activity itself.

### 3 Pedagogic strategies for student support in microworlds

The brief account of teachers' role in Section 2 is the starting point for developing a set of categories of pedagogic strategies and teacher interventions (see Table 1). This is based on our previous work for Logo [9] and dynamic geometry environments (DGEs) [14] and is supported and adapted in the light of empirical data from early prototype microworlds developed for the MiGen project [2].

<ol style="list-style-type: none"> <li>1. Supporting processes of mathematical exploration <ul style="list-style-type: none"> <li>• Supporting students to set and work towards explicit goals.</li> <li>• Directing students' attention.</li> <li>• Helping students organise their working environment.</li> <li>• Provoking cognitive conflict.</li> <li>• Encouraging alternative solutions.</li> </ul> </li> <li>2. Supporting reflection</li> <li>3. Promoting motivation</li> <li>4. Supporting collaboration</li> </ol> <p style="text-align: center;"><b>Table 1.</b> A framework of pedagogic strategies for student support</p>
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These pedagogic strategies, discussed explicitly in the following sections, provide an initial framework for modelling teacher's role. Section 4 revisits them providing details and suggestions for designing a computer-based facilitator.

#### 3.1 Supporting processes of mathematical exploration.

Adhering to their role as facilitators, teachers can support the processes of mathematical exploration by helping students set and monitor their goals, by directing their attention appropriately, by helping them reflect on their actions and the microworld's visual feedback, and by provoking cognitive conflicts that demonstrate the limitation of students' approach. In addition, teachers can help students reflect on their solutions and finally allow them, if not encourage them, to come up with more alternative solutions. These are briefly discussed below.

**Supporting students to set and work towards explicit goals.** The importance of directing students' goals during mathematical exploration was mentioned already. Regardless of who sets activity goals the effective teacher's role is to orient students to work on well-defined investigations [6, 7] (e.g. "Investigate the relationship between these two shapes"). However, a difficulty that students face when solving problems in general is a tendency to lose sight of their overall goal. As emphasised in [13], attention is usually caught up by current actions which are sometimes only intermediate towards a goal. In microworlds, and particularly where interaction is via direct manipulation, some actions may not be directly relevant to the mathematical aspects that are being explored, yet necessary in order to reach a goal. This loss on focus on the goal is often observed in our studies with the MiGen tools and in previous research with DGEs and Logo. For students who are facing difficulties, teachers provide a reminder of their goals trying to re-establish it: "What were you trying to do?", "Do you

remember the question?”. Often simple questions like these, even if they are not answered, can orient students back towards their goal.

Another way of helping students is to provide specific prompts that can guide them towards their goal. Before providing help, effective teachers establish the goal students are trying to achieve and try to adopt their way of thinking rather than the ‘correct’ one. In other words, the teacher needs to maintain a subtle balance between solving problems for students (or providing the way to solve a problem) and, leaving students on their own and unable to proceed if stuck.

**Directing students’ attention.** In order to direct students’ attention, teachers first try to determine of what they are not yet aware, then find ways to prompt them without giving the answers away [13]. For example, if they suspect that a student has not noticed certain facts they may ask a question to direct students’ attention to this fact (e.g. “Did you notice what happens when you resized the circle?”). Questions like this help students to start noticing invariants or other details which are important towards their investigation.

In most cases, empirical data and other research (e.g., [12]) suggest that particularly expert teachers tend not to intervene if students’ attention seems to be directed towards something that teachers believe is useful. If however, students insist on looking or manipulating unnecessary elements of their construction (e.g., dealing with relationships or properties that are not meaningful), teachers eventually intervene by providing hints towards more constructive aspects to be perceived. Finally, sometimes procedural mistakes can be scaffolded, or even ignored [12] in favour of directing students’ attention to more important issues.

**Helping students organise their working environment.** Related to the strategies mentioned above are interventions that teachers make, targeted specifically to helping students organise their working environment, either in order to work effectively towards a specific goal or in order to help them become aware of relationships between objects. For example, they may suggest a specific action (e.g. “Why don’t you make a [certain shape]”), or ask students to change the location of a shape, its properties or delete unnecessary shapes.

The effectiveness of this strategy is supported by the fact that students who cope better with activities, are very good at organising their environment and take specific actions targeted towards their goal. Usually they find ways to place shapes in ways that support their perception and avoid cluttering their interface.

**Provoking cognitive conflicts.** As mentioned in the Introduction, in microworlds students often have to be explicit about the relationships they recognise. For example, in DGEs relationships and shape properties must be made explicit if this shape is constructed and not simply drawn. Teachers employ student-assigned relationships to create a cognitive conflict and help students become aware of the lack of explicit relationships. A typical intervention is providing a counter-example. Another strategy, in DGEs, is “messing-up” [15] which challenges students to generalise a construction by dragging a point to check if its properties (e.g., an intersection point) remain invariant when the variable aspects change. Although the exact technique is usually activity-specific, the strategy is general and has been used effectively in other tasks (e.g., [1]).

**Encouraging alternative solutions.** Teachers who realise the importance of encouraging students' autonomy and responsibility over their learning, allow a margin for different solutions to emerge even if it is not evident from the beginning that an approach will be effective [12]. In microworlds and in activities where there are multiple ways to approach a construction, it is surprising how often students come up with innovative, valid, approaches that were not anticipated in advance. Following a constructivist perspective, it is more desirable, to let students choose their own way. Of course not all of them are elegant or demonstrate perfectly the mathematical ideas that the teacher intended. The teachers' role then is to guide students to reflect on the limitations (or advantages) of their approach, compared to other approaches that are, for example, more efficient, more understandable, etc.

### 3.2 Supporting Reflection

Reflection is important in the process of learning as well as a critical metacognitive skill. When students are working on a task, teachers usually remind them of actions, strategies or even their own previous prompts. This eventually supports students' autonomy to become able to evaluate their own mistakes and progress. To ensure students have not only reached their goals, but also gained knowledge at the end of an activity, studies of human tutors suggest that they often give help both during and after a student's performance [16]. In addition, even in the cases where mistakes are ignored or rectified, it is unlikely that a teacher would proceed without a reflective discussion at the end of the session. In microworlds, and particularly in activities with multiple solutions, an explicit phase of reflection is important. Although students may have started recognising some relationships, internalising them and perceiving the concepts that underpin them in order to use them in subsequent activities requires explicit reflection, and articulation by teacher and students.

### 3.3 Promoting Motivation

Although motivation is usually supported by the overall task-design which should provide an intrinsic motivation and incentive for engagement, expert teachers sense when students are in need of praise or encouragement and provide these by employing several strategies. The right incentive and appropriate praise even for the smallest achievement or effort that students exerts, usually have a positive effect on their attitude towards learning and further progress. Similarly, constant encouragement and support are important.

### 3.4 Supporting Collaboration

Apart from promoting a collaboration culture in classroom through appropriately designed activities, the teacher needs to foster students' collaboration and to facilitate discussion, encourage questioning and, depending on the overall task, help students set challenges to each other. The difficulty for the teacher in a classroom is to monitor all the groups of students and be able to change the group dynamics (e.g., if one student is dominating the discussion). In addition, empirical data reveal another strategy where teachers dynamically allocate competent students who have completed their tasks as 'helpers' for other students.

## 4 Discussion - Suggestions for Intelligent Support

Each pedagogic strategy and the interventions discussed in Section 3 require in depth discussion to be operationalised to the level of detail that would allow its implementation for intelligent support. Although we presented these strategies as ways of helping students directly, we acknowledge that some of them involve a significant amount of uncertain information that an intelligent system cannot always deal with. In what follows we provide brief suggestions on how aspects of the teacher's role might be devolved to an intelligent computer-based facilitator.

### 4.1 On supporting students to set and work towards explicit goals.

Section 3 highlighted the importance of helping students set and prioritise their goals, but also the need to identify what the students' current goals and intentions are, before providing any help. The issue of support for students' goals has been targeted in intelligent simulation environments [5] where students' goals are determined or, at least, inferred, by letting them choose a specific assignment from the environment. Since the goal of this assignment is known to the system, it can offer more contextualised support. Something similar could be achieved in microworlds. For example in [14] students work in a prototype intelligent DGE and select specific tasks and goals. This enables the system to provide support and direct students' attention according to predetermined rules that are described by the activity designer.

In relation to secondary goals (within an activity) we can draw again on an example from simulation environments. For example, the system presented in [17], is designed to allow students to define and keep track of the hypothesis they want to test in an explicit way. Although, ideally, this would require natural language processing capabilities, providing possible goals and hypotheses in the form of multiple choice questions or dropdown menus can provide an effective scaffold for students and is not necessarily restricting, especially if different goals can be chosen. A similar technique has been used in SHERLOCK [18] to help the learner's planning during a diagnostic problem-solving process by choosing their next step from a menu of actions. This provides a window to their intentions but also opportunities for metacognitive scaffolding.

### 4.2 On directing students' attention

**Modelling the structure of attention** Directing students' attention appropriately (or informing teachers about issues related to it) requires inferring students' current goals. In addition, it is important to be aware of the different ways of attending [13] since they determine the kind of help that can be provided to the student. A useful framework for modelling the different ways of attending is provided in [13] and is referred to as, the 'structure of attention'. With this as a starting point and adapting it for microworlds we can distinguish three intertwined but subtly different layers of attention. The first can be referred to as *exploring-manipulating* and involves students spending time in arbitrary object constructions and manipulation, usually at the beginning of the activity,

followed by inspection of properties and more specific construction steps. The role of an intelligent system, when students are still exploring, could be to direct their attention appropriately (e.g., by flagging details they may be missing) or to detect (and inform teachers) whether they are having difficulties and are failing to explore important aspects. The next layer of attention has been referred to as “*getting-a-sense-of*” [13, 6] and involves actions that demonstrate that the student is starting to discern details and recognise important relationships between the concepts involved in a task. In the final layer of attention students are *perceiving general properties or concepts*. It involves students' employing the relationships embedded in the microworld as the basis for their reasoning and is usually manifested across different activities.

**A window on the students' object of attention** Because of the nature of microworlds it is often the case that students create and interact with many objects in their attempts to get a sense of what they are being asked to do. This introduces noise for an intelligent system. However, it is possible to make some inferences about the object of students' attention. For example, during studies on students-tutors interactions in a setup where tutors could observe students' working environments only from a remote location [19], mouse movements, button clicks and other interactions helped tutors infer the object of students' attention. Their inferences can be emulated to help the computer-based facilitator be aware of what students are attending to. Empirical data from students interacting with the exploratory tools developed for MiGen suggest that direct manipulation of objects and inspection of properties can provide substantial information to allow an intelligent system to infer the object of students' attention.

The difficulty the system faces is similar to the one a teacher faces when approaching students who request help in a classroom and does not have a detailed context of their preceding work. A teacher would establish which object students are attending to by asking them directly. It is not too bold to imagine that when the system lacks knowledge about the object of students' attention and before being able to help them, it could require a particular interaction such as highlighting the object they are attending to. In particular, the microworld can be designed in a way that ensures an increased 'bandwidth' (see [20]). For example, instead of displaying all available information at once, the microworld can be designed so as students' interactions are less ambiguous providing evidence for what they are attending to. Examples of such a design for a prototype microworld for generalisation for the MiGen project are presented in [2].

### 4.3 On helping students organise their working environment

In Section 3.1 we mentioned the value of supporting students perceptions by helping them organise their work space. Although the ways to help them achieve this are situation-specific, assuming that the task is known to the system, there are prompts that the system can give or actions it can take to help students directly or through the teacher. Also, for different domains there will be general principles that can be used for supporting students. For example, cognitive psychology principles for the way humans organise perceptual stimuli, were useful in

designing intelligent support for DGEs [14]. Based on these, an intelligent component provides hints for bringing related objects close, for avoiding or trying to make their size really small or large, and in general helps students reorganise the locations of objects in order for their attention to be directed to appropriate places. In addition, we mentioned that competent students avoid cluttering their interface and that teachers employ similar strategies to help students. The computer-based facilitator could inform teachers about students who seem to have difficulties organising their environment.

#### **4.4 On provoking cognitive conflict**

Using strategies adapted from human tutors, it is possible to devolve this aspect of teachers' support to a computer-based facilitator. Apart from providing automatically-generated counter-examples, a system can also take actions that would demonstrate to students limitations of their approach. One example is the aforementioned "messaging-up" strategy that can be easily automated (examples of intelligent support using this strategy are presented in [21]).

#### **4.5 On providing support for multiple and innovative solutions**

Activities in microworlds usually expect students to come up with a construction with explicit relationships and properties and therefore, by observing if these relationships and properties are present, it may be easier than in others contexts (e.g., solving procedural algebraic problems) to support multiple and innovative solutions. This could enable the system to determine students' plans or strategies and help either the teacher or the student directly, by providing notifications of unpredictable or innovative strategies, or by suggesting appropriate next steps employing students' preferred strategies as envisaged in [12]. Some ideas of how this could be achieved using case-based reasoning are presented in [21]. Also, in [22] an approach based on Hidden Markov Models is presented that could be used to allow freedom to learners and cater for innovative solutions.

#### **4.6 On supporting reflection**

An intelligent system could support the teacher's responsibilities (see 3.2) and automatically generate or propose activities that give students the opportunity to reflect on their actions or important parts of an activity. Students' proficiency to perceive important concepts and use them in other situations can be 'assessed' by designing activities that expect knowledge acquired in previous activities to be re-used. The computer-based facilitator can then observe if students are using their previous understandings or, as they often tend to: reinvent the wheel.

#### **4.7 On promoting motivation**

In the field of Artificial Intelligence in Education (AIEd) there have been attempts (and some success) to detect and adapt to aspects of students' affective and motivational characteristics. However, in certain cases it may be difficult and inappropriate for a computer-based tutor to respond or adapt to students' affective characteristics. Assuming that adequate intrinsic motivation is provided



from the activities and the overall environment, intelligent support can be limited (but still very useful) in communicating to the teacher diagnoses of students' motivational states. AIED research suggests that it is possible to detect factors such as confidence and effort [19] as well as off-task behaviour [23]. The latter could be particularly useful in microworlds, where students tend to 'play' in quite a few occasions. The teacher's presence is necessary to decide when and whether this should end. With appropriate prompts and good management skills teachers with their authority could bring students back on-task, something that may be difficult for a system to achieve easily.

#### 4.8 On supporting collaboration

There is a substantial amount of research on computer-supported collaborative learning. Here we would like to emphasise only the need to support the teacher during classroom sessions with students collaborating in groups. A computer-based facilitator can notify the teacher about students who finish part of the tasks so as they can help others, or provide information about the dynamics of different groups (e.g. dominating students), suggestions about more productive groupings, or even intervene to help maintain a balance so as to ensure that the positive effect of collaboration is achieved.

### 5 Further research

The strategies presented here need to be operationalised further before implementing appropriate intelligent support in microworlds. Studies with low communication bandwidth between teachers and students (such as the ones presented in [19, 24]), especially if designed to promote interventions that take into account constructivist principles (such as the teaching experiments described in [12]) can help in deriving more information on the effectiveness of such strategies and specific ways of implementing them. Although the exact approach and some of the prompts will be, inevitably, activity-specific, a general framework such as the one presented here, could allow activity designers or teachers to specify, for different activities, which responsibilities they would like to devolve to an intelligent computer-based facilitator and how it could help them fulfill them.

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