Why Can Concept Lattices Support Knowledge Discovery in Databases?

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Abstract. Knowledge discovery should be understood as information discovery combined with knowledge creation. The creation of knowledge from information can be promoted by proper representations of information which make the inherent logical structure of the information transparent. Since concepts are the basic units of human thought and hence the basic structures of logic, the logical structure of information is based on concepts and concept systems. Therefore, concept lattices as mathematical abstraction of concept systems can support humans to discover information and then to create knowledge. The TOSCANA software even allows the navigation through a network of concept lattices and thereby information discovery in databases which may further lead to knowledge creation.

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1 Turning Information into Knowledge

In his book "InfoSense - Turning Information into Knowledge" [De99], Keith Devlin claims that, for living in the "knowledge society", we must develop a proper understanding of knowledge and information built on a solid scientific foundation. In particular, he advocates for a clear distinction between the notion of data, information, and knowledge. Such a distinction can be given briefly by the following equations (cf. [PRR99], p.36ff.; [De99], p.14f.):

$$\begin{aligned}\text{Data} &= \text{Signs} + \text{Syntax}\\ \text{Information} &= \text{Data} + \text{Meaning}\\ \text{Knowledge} &= \text{Internalized information} + \text{Ability to utilize the information}\end{aligned}$$

A comprehensive *scientific foundation of knowledge and information* has not been developed up to now. In [De97], Chapter 10, Devlin outlined how to approach a basic science of information. The main question is: What is information and how does it flow? (cf. [BS97]) Information may be derived from data when the data is joined with collective meaning understandable in a community to which the information might be addressed. One can say that information exists in the collective mind of a social group.

In their book "Working Knowledge", T. Davenport and L. Prusak gave the following characterization of knowledge:

> "Knowledge is a fluid mix of framed experiences, values, contextual information, and expert insight that provides a framework for evaluating and incorporating new experiences and information. It originates and is applied in the minds of knowers. In organizations, it becomes embedded not only in documents or repositories but also in organizational routines, processes, practices, and norms." ([DP98], p.5)

Thus, one can say that knowledge exists only in an individual person's mind. But, in a social group or culture, the knowledge of different individuals may become commonly embedded in various media and modes of behaviour so that knowledge creation and knowledge transfer can take place. *Turning information into knowledge* is best supported when the information with its collective meaning is represented according to the social and cultural patterns of understanding of the community whose individuals are supposed to create the knowledge.

The creation of knowledge from information can be promoted by proper representations of information which make the inherent logical structure of the information transparent. Here the term "logical" is meant in accordance with the understanding of *philosophical logic* as explained by Charles S. Peirce in his Cambridge Conferences Lectures on "Reasoning and the Logic of Things":

> "Logic is the science of thought, not merely of thought as a psychical phenomenon, but of thought in general, its general laws and its kinds." ([Pe92], p.116)

As a part of philosophy, logic is semantically related to the actual reality. Peirce saw the foundation for understanding the forms of thought in his categories of firstness, secondness, and thirdness: *Firstness* is the mode in which anything would be for itself, irrespective of anything else; *Secondness* is the mode in which a First would be related to a Second, irrespective of anything third; *Thirdness* is the mode in which a First and a Second would be mediated by a Third. According to his three categories, Peirce distinguishes three kinds of reasoning: abduction, induction, and deduction. The *Abduction* creates a hypothesis as a First out of a horizon of self-evidences; the *Induction* confirms a hypothesis by actual facts as a Second; the *Deduction* proves a hypothesis from valid premises by logical laws as a Third. Thus,

> "Deduction proves that something *must* be, Induction shows that something is *actually* operative, Abduction merely suggests that something *may be*." ([Pe98], p.216)

The evolutionary nature of logic is most clearly expressed in Peirce's *First Rule of Logic* which he formulates as follows:

> "reasoning tends to correct itself, and the more so the more wisely its plan is laid. Nay, it not only corrects its conclusions, it even corrects its premises." ([Pe92], p.165)

The property of *self-correction*, which already G. W. F. Hegel considered as constitutive for the dialectic process of growing reason [He86], is important for the reasoning in all sciences and humanities.

Concepts as basic structures of logic are most important for turning information into knowledge because humans first grasp realities by concepts wherefore concepts are the basic units of thought and knowledge. According to J. Piaget and Th. B. Seiler, concepts are epistemological structures by which humans act internally and externally in a process of assimilation and accommodation within their environment; humans reconstruct the relevant aspects of the environment for their thought and action by concepts which, in particular, yield the basis for the interpretation of the meaning of data ([Se01], p.164f.). Only the construction of complex concept systems and their systematic coordination allow a differentiated reconstruction of reality and leads to consistent concept orderings, the disposability of which is necessary for forming the ability of reasoning ([Se01], p.171).

Thus, for supporting humans in *turning information into knowledge*, the inherent conceptual structures and relationships of the information should be made disposable so that human reasoning is enabled to create knowledge out of the represented information. This means: the information should be presented in its logical forms and structures in such a way that it activates the horizon of self-evidences for abduction, the presence of actual facts for induction, and valid premises and laws for deduction. According to the purpose of the knowledge creation, the representation of the information has to be clear and appropriate so that it particularily allows critics and corrections, even with respect to the information itself.

2 Supporting Knowledge Discovery by Concept Lattices

Knowledge discovery might be understood to be more specific than knowledge creation, but both are activities of epistemological nature. Such activities are already discussed by G. W. Leibniz in his philosophy of science under the notion of "ars inveniendi", which has as counterpart the notion of "ars iudicandi". These notions have been activated in a modified manner in the 20th century by the logical positivism under the designations "context of discovery" and "context of justification". In accordance to these philosophical traditions, *knowledge discovery* shall be understood as information discovery combined with knowledge creation where the combination is given by turning discovered information into created knowledge.

Now, the general question "Why and how can knowledge discovery be supported mathematically?" leads to the question "Why and how can information discovery and knowledge creation be supported mathematically?" For answering this question, we take over arguments from [Wi01] which explain in general why mathematics can promote human reasoning: First, logical thinking as expression of human reason grasps the actual reality by the basic forms of thought: concepts, judgments, and conclusions (cf. [Ka88], p.6). Secondly, mathematical thinking abstracts logical thinking with its basic forms of thought for hypothetically developing a cosmos of forms for potential realities (cf. [Pe92], p.120f.). Because of this close relationship between logical and mathematical thinking, mathematics as a result of mathematical thinking is able to support humans in their reasoning about realities.

For realizing the mathematical support of human reasoning, the research group "Formale Begriffsanalyse" at the TU Darmstadt has started in the last years to develop a *"Contextual Logic"* as a mathematization of the traditional philosophical logic with its doctrines of concepts, judgments and conclusions [Wi00a,Pr00]. For the foundation of Contextual Logic it is most important to have a mathematization of concept which reflects the rich logical functionalities which concepts are able to unfold in contextual connections. Fortunately, such mathematization has already matured in Formal Concept Analysis and its applications since more than twenty years (see [GW99]): The contextual connections are mathematically framed by the notion of a *formal context* defined as a set structure (G, M, I) consisting of two sets G and M and a binary relation I between G and M (i.e. $I \subseteq G \times M$); the elements of G and M are called (*formal*) *objects* and (*formal*) *attributes*, respectively, and the relationship gIm (i.e. $(g, m) \in I$) is read: *the object g has the attribute m.* In the sense of Peirce's categories, within a formal context an object as a First and an attribute as a Second is mediated by the context-relation as a Third.

Formal contexts may be understood as mathematizations of *cross tables* representing actual relationships between objects and attributes as the cross table in Fig.1. This data table came up in a common research project of the research group "Formale Begriffsanalyse" and the ministry of civil engineering of the German province Nordrhein-Westfalen; the aim of the project was to develop a TOSCANA-system by which the architects in Nordrhein-Westfalen are enabled to find, for a specific task in building construction, all relevant paragraphs in laws, regulations, and standards [EKSW00]. In the cross table in Fig.1, paragraphs of laws, regulations, and standards (as objects) are related (which is indicated by the crosses) to special rooms of a hospital (as attributes); here a paragraph is related to a room if the paragraph has to be observed by an architect designing such a room. The abstraction of the table to the corresponding formal context converts the paragraphs to formal objects, the rooms to formal attributes, and the crosses to the formal relationships. In general, the cross table has to be distinguished from the abstracted formal context because the cross table is a logical structure representing actual relationships, while the formal context is a mathematical structure representing potential relationships. In spite of this distinction, the cross table and its corresponding formal context yield a typical example of the close relationship of logical and mathematical conceptions.

	consulting room	laboratory	residential room (bedroom)	toilet	wash- and bathroom	changing room
BauONW§16		×		×	×	
BauONW§17		×	×			
BauONW§26			×	×		
BauONW§27			×	×		
BauONW§28			×	×		
BauONW§29			×	×		
BauONW§33			×			
BauONW§40		×		×	×	
BauONW§42			×			
BauONW§44			×	×		
BauONW§45			×			
BauONW§46				×	×	
BauONW§50		×				
BauONW§51				×		
KhBauVO§7			×			
KhBauVO§9		×				
KhBauVO§10	×					
KhBauVO§13	×					
KhBauVO§17	×	×	×	×	×	
KhBauVO§20		×				
KhBauVO§22	×	×				
KhBauVO§23			×	×	×	
KhBauVO§25	×	×				
KhBauVO§27	×		×	×	×	
KhBauVO§28			×		×	
KhBauVO§29			×	×		
KhBauVO§30						×
KhBauVO§31		×				
BimSchG		×				
VGS		×				
LWG		×		×	×	
WHG		×		×	×	
DIN-N.f.Entwässerung		×		×	×	

Fig. 1. Cross table assigning hospital rooms to relevant paragraphs of building regulations

For the mathematization of concept, the formal context as a mathematization of the contextual background of the concept can now be presupposed: A *formal concept* of a formal context (G, M, I) is defined as a pair (A, B) where A is a subset of G and B is a subset of M such that A consists of all those objects of G having all attributes of B, and B consists of all those attributes of M applying to all objects of A; A and B are called the *extent* and the *intent* of the formal concept (A, B), respectively. This mathematization underlies the philosophical understanding of a concept to be a unit of thought consisting of an extension and an intension, which was already declared by the Logic of Port Royal in the 17th century [AN85]. Now, the subconcept-superconcept-relation can be mathematized as follows: A formal concept (A, B) of (G, M, I) is defined to be a *subconcept* of the formal concept (C, D) of (G, M, I) and (C, D) a *super-*

concept of (A, B) if the extent A is contained in the extent C or, equivalently, if the intent B contains the intent D.

By the contextual mathematization of concept, the *logical reciprocity* "the larger the extension the smaller the intension" gains conciseness and fruitfulness which have a lasting effect on mathematical thinking. Mathematically, the reciprocity can be captured by the following definition of the "derivation operators" of a formal context (G, M, I): For $X \subseteq G$ and $Y \subseteq M$, the *derivation* is defined by

$$X^I := \{m \in M \mid gIm \text{ for all } g \in X\} \text{ and } Y^I := \{g \in G \mid gIm \text{ for all } m \in Y\},$$

i.e., the derivation X^I is the set of all formal attributes applying to all formal objects of X and the derivation Y^I is the set of all formal objects having all formal attributes of Y. For $A \subseteq G$ and $B \subseteq M$, the pair (A, B) is a formal concept if and only if $A = B^I$ and $B = A^I$. The logical reciprocity finds its most concise expression in the following mathematical conditions: For $U, V \subseteq G$ and $U, V \subseteq M$, respectively,

$$(1) \quad U \subseteq V \text{ implies } U^I \supseteq V^I, \quad (2) \quad U \subseteq U^{II}, \quad (3) \quad U^I = U^{III}.$$

For the task of determining all formal concepts of a formal context (G, M, I), the condition (3) is basic, namely, (3) implies that (X^{II}, X^I) and (Y^I, Y^{II}) are formal concepts of (G, M, I) for all $X \subseteq G$ and $Y \subseteq Y^{II}$; important is the special case of the *object concepts* $(\{g\}^{II}, \{g\}^I)$ for $g \in G$ and the *attribute concepts* $(\{m\}^I, \{m\}^{II})$ for $m \in M$. The here appearing mathematical potential of the derivation operators cannot be estimated high enough; they represent mathematical connections which in general are investigated as set-theoretic dualities (also called *"Galois connections"*) and activated multifariously (also on the logical level).

The set of all formal concepts of a formal context (G, M, I) forms together with the mathematized subconcept-superconcept-relation the mathematical structure of a complete lattice, called the *concept lattice* of (G, M, I). The mathematical structure of a concept lattice becomes effectively accessible to human reasoning by (labelled) *line diagrams.* The line diagram in Fig.2 (cf. [Wi01]) represents the concept lattice of the formal context abstracted from the cross table in Fig.1. The small circles in the line diagram represent the formal concepts of the formal context where the circles of the object concepts are labelled by the designation of the corresponding object and the circles of the attribute concepts are labelled by the designation of the corresponding attribute. The ascending paths of line segments between circles represent the relationships of subconcept to superconcept. For instance, there is such an ascending path from the object concept of "KhBauVO§17" to the attribute concept of "toilet" which indicates that this object concept is a subconcept of that attribute concept. In general, such relationship between an object concept and an attribute concept exists if and only if the corresponding object has the corresponding attribute. More generally, for an arbitrary formal concept, its extent (intent) consists of all those objects (attributes) whose designation is attached to a circle of a descending (ascending)

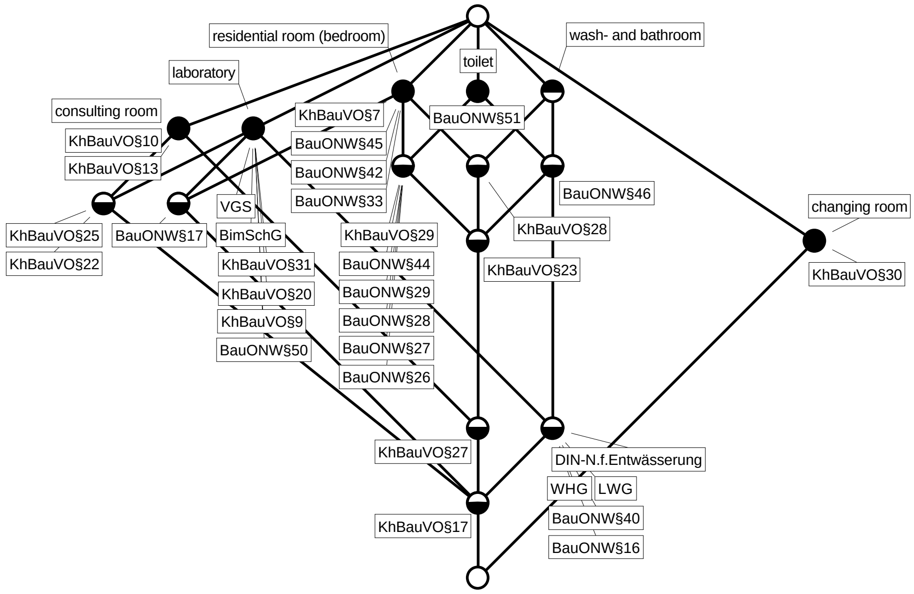

Fig. 2. Concept lattice of the formal context abstracted from the cross table in Fig.1

path of line segments starting from the circle which represents that concept. For instance, the circle on the very left represents the formal concept with the extent {KhBauVO§17, KhBauVO§22, KhBauVO§25} and with the intent {consulting room, laboratory}.

The line diagram in Fig.2 offers to the architects, who are able to read the diagram, the full information represented in the cross table of Fig.1; in addition, it supports the understanding of the infomation by showing how the objects and attributes form concepts and how the subconcept-superconcept-relation structures those concepts. Of course, a reader of the line diagram can only conceive the full information if he or she knows what the designations in the diagram mean. Such a reader, for instance, discovers immediatly the information that, for designing the water connections (needed in laboratories, toilets, wash- and bathrooms), the paragraphs "BauONW§16", "BauONW§40", "DIN-N.f.Entwässerung", "LWG", "WHG", and "KhBauVO§17" have to be observed. In activating and internalizing this information and joining it with the already present knowledge about the building project, an architect will create new knowledge which can then be utilized in that project.

As the example indicates, the logical connections in line diagrams of concept lattices can stimulate background knowledge for discovering new knowledge. This often produces also *critic and self-correction* of the present information and knowledge. In the mentioned research project, again and again, line diagrams enabled the building experts of the ministry to discover mistakes in the extensive data context which contributed to a considerable improvement of the data quality. An instructive case of critic and self-correction occured in a discussion of the line diagram in Fig.2: For testing the readability of such a diagram, a secretary of the ministry, not involved in the research project, was asked to join the discussion. After inspecting the diagram, the secretatry expressed her astonishment that, in the diagram, the paragraph "BauONW§51" is directly attached to the circle with the label "toilet", which means that only the toilets have to be designed for handicapped people; she could not understand why the wash- and bathrooms have not to meet requirements for handicapped people too. Even the experts became supprised when they checked again the §51 of the "Bauordnung Nordrhein-Westfalen" and saw that only toilets are mentioned in connection with handicapped people. Only after a comprehensive discussion the experts came to the conclusion that, by superior aspects of law, §51 also applies to wash- and bathrooms. Finally, by similar reasons, the consulting rooms and the residential rooms (bedrooms) were also included so that, in the cross table of Fig.1, three more crosses were added in the row headed by "BauONW§51" so that, in the line diagram of Fig.2, the label "BauONW§51" moved down to the circle with the label "KhBauVO§27".

To sum up, we explain the support of knowledge discovery by concept lattices as follows: The mathematization of the logical structures of concepts and concept hierarchies by formal concepts and concept lattices of formal contexts yields a close relationship between logical and mathematical thinking which, in particular, allows to activate a rich amount of mathematics to support human reasoning. Especially, the representation of concept lattices by (labelled) line diagrams enables an interplay between the mathematical analysis of relationships and the logical analysis of data and information, influenced by already existing background knowledge. Therefore *conceptual knowledge discovery*, i.e. conceptual information discovery and knowledge creation, can be performed by first looking under the guidance of some purpose for discoveries of information in (graphically represented) concept lattices and then creating new knowledge from the discovered information and appropriate preknowledge. These two steps should be repeated in a circular process which is open for critic and self-correction.

3 Exploring Databases by Conceptual Views

Up to now, we have seen how concept lattices can support knowledge discovery in data tables of restricted size. How can such a support be extented to large databases? Theoretically, the information coded in a database can still be represented in a cross table, but of enormous size so that it will be impossible to establish a line diagram of its concept lattice. Practically, for creating knowledge, one does not want to see all the information of a database at once; therefore it is sufficient to allow specific *views into the database* which can be combined in such a way that a navigation for discovering knowledge becomes possible. This can be done by methods of formal concept analysis which shall be first explained via an example.

In 1991, members of the research group "Formale Begriffsanalyse" started a project to develop a *retrieval system* for the library of the "Center of Interdisciplinary Technology Research" (ZIT) at the TH Darmstadt which was finished in 1996 [RW00]. It needed several experiments until a successful approach was found for the project. For instance, common retrieval methods turned out to be unsatisfactory because of the broad interdisciplinarity of the documents in the library. Therefore, a specific normed vocabulary was developed for satisfactory content extraction of the documents. In the average, 32 catchwords from the normed vocabulary were assigned to each document which yielded a very good substitute of an abstract for each document. These assignments, stored in a relational database, gave rise to a large cross table with 1554 documents as objects and 377 catchwords as attributes, within that the crosses indicate which catchword is assigned to which document.

From the established cross table, 137 *conceptual views* were derived with the help of experts for the field of content of those views. Each conceptual view is determined by a theme and a small number of catchwords representing that theme. For instance, the conceptual view with the theme "Informatics and Knowledge Processing" got the catchwords "Formalization", "Artificial Intelligence", "Expert Systems", "Knowledge Processing", and "Hypertext". The concept lattice of this view, shown in Fig.3 (cf. [Wi00b], p.360f.), is the concept lattice of the formal context represented by the five columns of the large cross table which are headed by the five listed catchwords. In Fig.3, there are no designations of objects, but the quantities of objects in the extent of the represented concepts, respectively. For instance, the 96 attached to the circle with the label "Artificial Intelligence" indicates that there are 96 documents in the library to which the catchword "Artificial Intelligence" is assigned.

Now, imagine a researcher who is looking for literature about expert systems dealing with trafic and who has chosen first the conceptual view "Informatics and Knowledge Processing". The diagram in Fig.3 gives him the information that there are 60 documents with the catchword "Expert System". To get more information about those 60 documents, particularily concerning trafic, the researcher could zoom into the circle labelled with "Expert System" with the conceptual view "Town and Trafic" to obtain the line diagram in Fig.4. The diagram informs that 9 of the 60 documents deal with "Trafic" and 4 with "Trafic" and "Mean of Transportation". Since there are only few documents left, the researcher might click on those numbers to get the titles of the documents, for instance, via 4 the titles "Digital Fate", "Evolutionary Paths in the Future", "Yearbook Labour and Technology 1991", and "Cooperative Media".

The retrieval system of the ZIT-library was implemented with the program *TOSCANA* which allows, in general, to navigate with prepared conceptual views in relational databases (see [VW95]). In [SWW98], TOSCANA-systems are discussed as *knowledge discovery support environments* and exemplified by a conceptual information system concerning flight data of Frankfurt Airport. The connection of conceptual knowledge discovery in databases and conceptual data analysis within TOSCANA-systems is further investigated in [HSWW00] and

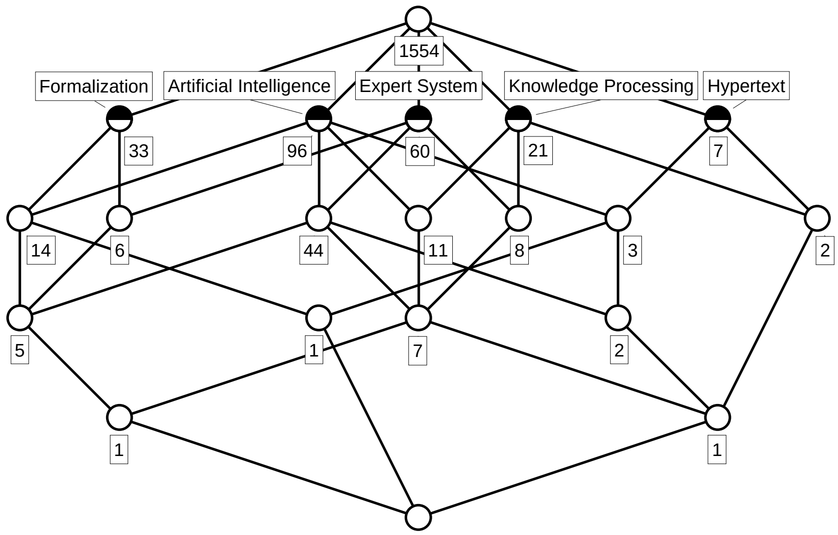

Fig. 3. Concept lattice of the conceptual view "Informatics and Knowledge Processing"

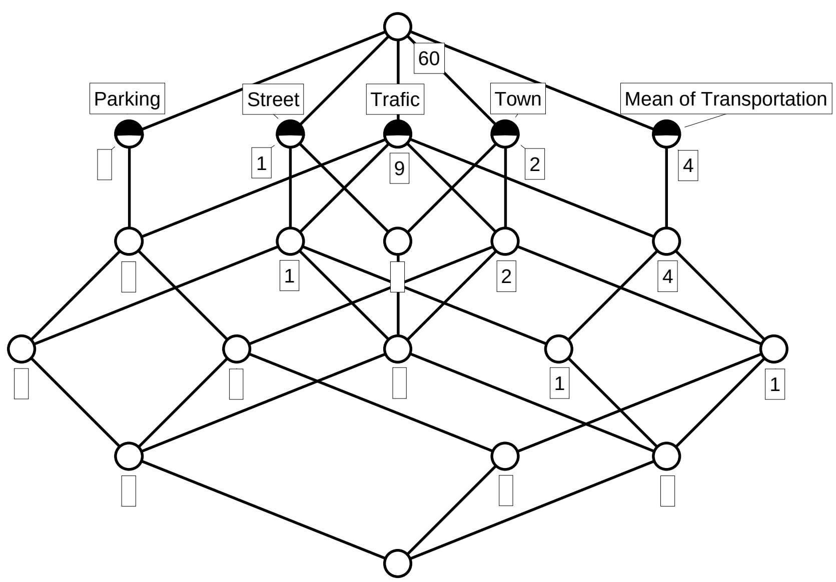

Fig. 4. Concept lattice of the conceptual view "Town and Trafic" restricted to "Expert System"

demonstrated on a large database of purchase transactions established for promoting the database marketing of a Swiss combine of retail trade. Another field of application of conceptual knowledge discovery by TOSCANA-sytems lies in the support of empirical theory building (see [SWW01]) for which a research project about the notion of "simplicity" in the music esthetics of the 18th century is an instructive example (cf. [MW99]).

In all TOSCANA-systems *information discovery* is primarily supported by line diagrams of concept lattices. Numerous experiences have shown that the network of information represented by a line diagram strongly promotes humans in discovering information. Of course, preknowledge is of advantage for the discovery process. Since the conceptual views represent expert knowledge, the navigation process induces also a learning process of the users increasing their knowledge and therefore contributing specifically to their information discovery and knowledge creation. The high transparency of the discovery process and the representation of its findings promotes the dialog between user and system and also human communication and argumentation which is important for knowledge creation.

Requirements basic for knowledge discovery support tools are stated by R. S. Brachman and T. Anand ([BA96], p.53); most of the content of these claims is already covered by the more explicit and detailed requirements described in [BST93]. TOSCANA-systems with their conceptual views (as queries) fulfill well all these requirements which are formulated as follows (cf. [SWW98]):

1. The system should represent and present to the user the underlying domain in a natural and appropriate fashion; objects from the domain should be easily incorporated into queries.
2. The domain representation should be extendible by the addition of new categories formed from queries; these categories (and their representative individuals) must be usable in subsequent queries.
3. It should be easy to form tentative segmentations of data, to investigate the segments, and to re-segment quickly and easily; there should be a powerful repertoire of viewing and analysis methods, and these methods should be applicable to segments.
4. Analysts should be supported in recognizing and abstracting common analysis (segmenting and viewing) patterns; it must be easy to apply and to modify these patterns.
5. There should be facilities for monitoring changes in classes or categories over time.
6. The system should increase the transparency of the knowledge discovery process and should document its different stages.
7. Analysis tools should take advantage of explicitly represented background knowledge of domain experts, but should also activate the implicit knowledge of experts.
8. The system should allow highly flexible processes of knowledge discovery respecting the open and procedural nature of productive human thinking; this means in particular to support intersubjective communication and argumentation.

As outlined in Section 2, the underlying domain represented in formal contexts and presented to the user by line diagrams of the corresponding concept lattices is mediated in a natural and appropriate fashion since formal concepts and their hierarchies are in close relationship to human thought and reasoning; in particular, objects from the domain are easily conceived by the concept lattices of the conceptual views which are used as queries in TOSCANA-systems. Also the second requirement about the extendibility of categorical structures is fulfilled by the great flexibility in forming and combining conceptual views; even in the process of discovery new insights may give rise to further conceptual views. The third requirement of meaningful data segmentations is satisfied because the conceptual views and their combinations yield an almost unlimited multitude of conceptual segmentations and with that a powerful repertoire of different views for exploring and analyzing data. This flexible repertoire supports analysts in recognizing and abstracting the interpretable patterns for which the fourth requirement asks.

Changes in classes or categories over time may be documented in specific conceptual structures so that they can easily be monitored in the sense of the fifth requirement. Concerning the sixth requirement, processes of knowledge discovery may be understood as developments in networks of conceptual views which yield increasing transparency of the processes and can be used for documenting the different phases of the processes. Background knowledge of domain experts enters the process of knowledge discovery via the conceptual views in which experts have explicitly coded formal aspects of their knowledge in structurally representing a certain theme, thereby opening possibilities for activating implicit knowledge as claimed by the seventh requirement. Overall, a TOSCANA-system offers a conceptually shaped “landscape” of structurally represented information allowing diverse excursions, during which a learning process yields an increasingly better understanding of what to collect and where to continue (cf. [Wi97]). The graphical representation of interesting parts of the landscape particularly supports intersubjective communication and argumentation.

To sum up, concept lattices with their line diagrams are indeed able to support knowledge discovery in databases. This is especially possible because of the TOSCANA software which allow to navigate with conceptual views in a highly flexible manner within a database. In this way humans can make accessible a rich conceptual landscape of knowledge.

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