Consensus dynamics in a dolphin social network

Abdelmalik Moujahid[1], Blanca Cases[1] and Francisco Javier Olasagasti[1]

[1] Computational Intelligence Group. Dept. of Computational Science and Artifitial Intelligence, University of Basque Country.

Abstract. In this work we investigate the consensus between individuals in a social network of bottlenose dolphins by simulating the OCR (Opinion Changing Rate) model recently proposed by Pluchini et al. in ref [1]. This model is a social adaptation of the Kuramoto one in which the concept of *opinion changing rate*, i.e, the natural tendency to change opinion, transforms the usual problem of opinion consensus into a class of synchronization. We study the emergence of synchronized groups of individuals both in terms of natural frequency rates and central positions in the network.

Key words: Social networks, Synchronization, Community structures

1 Introduction

The study of complex network has attracted a lot of attention in the scientific community in recent years [2-5]. Indeed, many natural, technological, biochemical and social systems can be conveniently modeled as networks made of a large number of highly interconnected units. In general terms a network can be represented formally as a graph: a set of generically called nodes (vertices) connected by links representing some relationship. Recent studies have revealed that such systems are all characterized by a number of distinctive topological properties: relatively small characteristic distances between any two nodes, high clustering properties, power-law degree distribution and presence of community structure.

In social networks, the nodes are people, and ties between them are friendship, political alliance or professional collaboration. The structure of interaction network describing who is interacting with whom, how frequently and with which intensity, reflects the importance of topology in social dynamics. On the other hand, consensus is a key aspect of social group dynamics. Everyday life presents many situations in which it is necessary for a group to reach shared decisions. Consensus makes a position stronger, and amplifies its impact on society. So the analysis of this social network under a particular topology from numerical simulation of opinion dynamics models is an important issue to understand the social group dynamics.

In this work, we deal with the problem of consensus formation in animal social network with known community structure simulating the OCR (Opinion Changing Rate) model proposed in ref [1]. The network we study was constructed from observation of a community of 62 bottlenose dolphins living in Doubtful Sound, New Zealand [9. Ties between dolphin pairs were established by observation of statistically significant frequent association. The paper is organized as follow. First, we review the main features of the Kuramoto and OCR models. Then we describe a dolphin social network in terms of their natural divisions using betweenness-based algorithm of Newman and Girvan [7]. In the second part, we discuss the results of numerical simulations of the OCR model on the network. Also, we investigate the influence of particular individuals in maintaining the cohesion of communities.

2 From Kuramoto model to the OCR model

Originally, the Kuramoto model was motivated by the study of collective synchronization, a phenomenon in which a large number of coupled oscillators spontaneously locks to a common frequency, despite the differences in their natural frequencies [6,8]. The dynamics of the Kuramoto model is given by:

$$\dot{\theta}_i(t) \quad = \quad \omega_i + \frac{1}{N}\sum_{j=1}^{N} K_{ij} sin(\theta_j - \theta_i), \tag{1}$$

where $\theta_i(t)$ denotes the phase of the oscillator i at instant t and ω_i its natural frequency. The frequencies ω_i are distributed according to some probability density $g(\omega)$. K_{ij} represents the coupling force between units. The original model studied by Kuramoto assumed mean-field interactions $K_{ij} = K, \forall i, j$. The dynamics of this model depends only on two factors: the coupling force K whose effect tends to synchronize the oscillators, and the frequency distribution that drive them to stay away each from other by running at different natural frquency. When the coupling is sufficiently weak, the oscillators run incoherently, whereas beyond a certain threshold collective synchronization emerges spontaneously. The existence of such a critical threshold for synchronization is very similar to the consensus threshold found in the majority of the opinion formation models. Based on this concept, Pluchini et al.[1] define the OCR model as a set of coupled ordinary differential equations governing the rate of change of agents' opinions. The dynamics of a system of N agents is given by:

$$\dot{x}_i(t) = \omega_i + \frac{K}{d_i} \sum_{j=1}^{N} A_{ij} \alpha sin(x_j - x_i) e^{-\alpha|x_j - x_i|}, \qquad (2)$$

where $x_i(t) \in]-\infty + \infty[$ is a real number that represents the opinion of the ith agent at time t. The ω_i's corresponding to the natural frequencies of the oscillators in the Kuramoto model represent here the so-called *natural opinion changing rates* (ocr), i.e., the intrinsic inclinations of the agents to change their opinions. The values ω_i's are distributed in a uniform random way with an average ω_0. According to this, we can simulate conservative individuals with values of $\omega_i < \omega_0$, flexible ones with $\omega_i \simeq \omega_0$ and more flexible ones with $\omega_i > \omega_0$.

$K \geq 0$ is the coupling force, d_i is the degree of each agent and A_{ij} is the adjacency matrix . The exponential factor in the coupling term ensures that, for opinion difference higher than a certain threshold, controlled by the parameter α (we typically adopted $\alpha = 3$), opinions will no more influence each other. This is perhaps the main contribution of the OCR model with respect to the Kuramoto model.

Thus, to study the opinion dynamics of the OCR model we solve numerically the system given by equation (2) for a given distribution of the ω's and for a given coupling force K. As reported in [1], in order to measure the degree of opinions coherence, we use an order parameter related to the standard deviation of the opinion changing rates defined as $R(t) = 1 - \sqrt{\frac{1}{N}\sum_i(\dot{x}_i(t) - \dot{X}(t))^2}$. Here $\dot{X}(t)$ is the average over all agents of $\dot{x}_i(t)$. Values of R approaching unity would imply a high degree of opinions coherence, while low values indicate a incoherently regime.

3 Dolphin social network

Bottlenose dolphins communities have been described as a fission-fusion societies and therefore individuals (or agents) can make decisions to join or leave a group. Two social groups (or clusters) were identified in this population. The community structure of this network, obtained using betweenness-based algorithm of Newman and Girvan [7], is shown in Fig. 1, and the distribution of agents in each group is reported in Table1.

Table 1. List of agents in each final group as resulting from using the Newman and Girvan algorithm [7] for a partition in two clusters.

Cluster	Agents in each cluster
1 (21)	2,6,7,8,10,14,18,20,23,26,27,28,32,33,40,42,49,55,57,58,61
2 (41)	1,3,4,5,9,11,12,13,15,16,17,19,21,22,24,25,29,30,31,34,35,
	36,37,38,39,41,43,44,45,46,47,48,50,51,52,53,54,56,59,60,62

In order to measure the quality of a particular division of a network into communities, we have used the measure known as *Modularity Q* introduced in ref [7]. Given a particular partition of a network into n groups (or clusters), it is possible to define a $n \times n$ size symmetric matrix e whose element e_{ij} is the fraction of all edges in the network that link vertices in group i to vertices in group j. According to this, the trace of this matrix $Tre = \sum_i e_{ii}$ gives the fraction of edges in the network that connect vertices in the same group, and therefore a good division into groups should have a high value of this trace. On the other hand, the sum of any row (or column) of the matrix e, namely $a_i = \sum_j e_{ij}$, give the fraction of edges connected

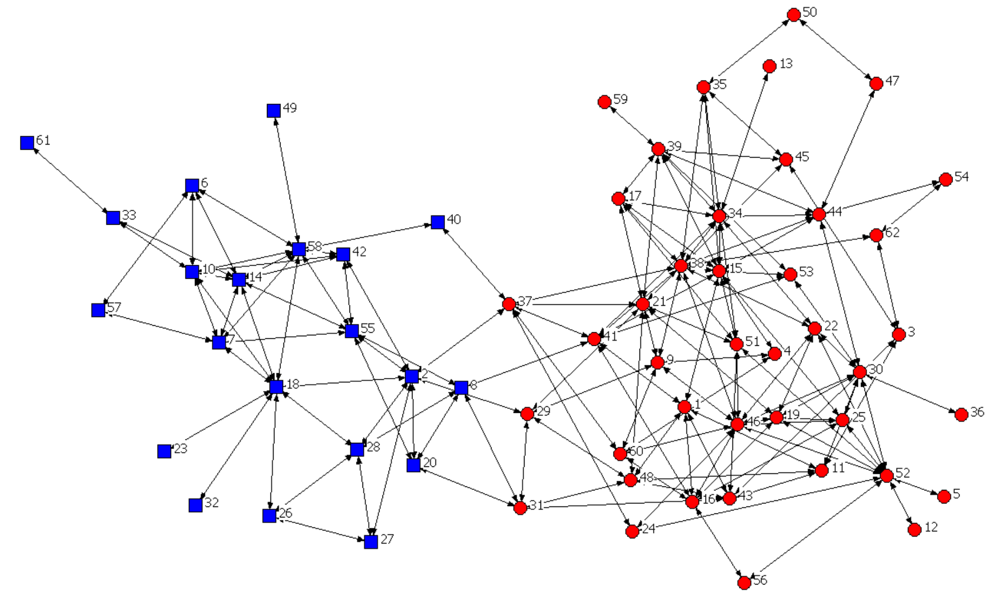

Fig. 1. Community structure in the bottlenose dolphins of Doubtful Sound, extracted using the Newman and Girvan algorithm [7]. The square and circles denote the primary split of the network into two groups.

to group i. So, the expected number of intra-group edges is just $a_i a_i$. Finally, the modularity Q is given by:$Q = \sum_i (e_{ii} - a_i^2)$.

Values of Q approaching unity,which is the maximum, would imply a strong community structure. If we take the whole network as a single group, or if the network is a random one, $Q = 0$. For the partition into two groups reported in Table 1., the modularity $Q = 0,38$.

This network is made of $N = 62$ vertex and $l = 159$ edge; each vertex represents an individual and each edge represents association between dolphin pairs occurring more often than expected by chance [19].

4 Numerical results

In this section, we integrate the system of equation 2 over the bottlenose dolphins network. In the model variable $x(t)$'s represent decisions of the 62 agents, and the ω's their natural decision changing rate *dcr*. For the numerical results we fix the coupling force $K = 2.2$ and the ω's are randomly chosen from a uniform distribution in the range $[-0.5, 0.5]$ with average $\omega_0 \simeq 0$. We set $x_i(t = 0) = 0, \forall i$, i.e, in the initial state all dolphins share the same decision changing rate values. The results are presented in Fig. 2. Panels (a) and (b) show the decision changing rate ($\dot{x}(t)$) and the order parameter $R(t)$ overs 100 time steps in a logarithmical scale for the abscise axe.

As it can be appreciated, as soon as we start the simulation, the system enters in a short unstable transient regime in which agents tend to synchronize their activities due to the coupling force (see Fig. 2(a)). This regime is characterized by maximum values of the order parameter $R(t)$ (see Fig. 2(b)). Immediately after, the system rapidly clusterizes resulting in two final clusters in which agents share common *dcr* values. This situation reflect community structure present in bottlenose dolphins society. The distribution of agents in each final clusters is reported in Table 2.

Table 2. Distribution of agents shown in Fig. 2(a) as resulting from the simulation of OCR model over the bottlenose dolphins community for $K = 2.2$. Elements in parentheses represent the numbers of agents in each final clusters.

Cluster	Agents in each cluster
1 (21)	2,6,7,8,10,14,18,20,23,26,27,28,31,32,33,42,49,55,57,58,61
2 (41)	1,3,4,5,9,11,12,13,15,16,17,19,21,22,24,25,29,30,34,35,36,
	37,38,39,40,41,43,44,45,46,47,48,50,51,52,53,54,56,59,60,62

As it can be verified, the distribution of agents in Table 2. is almost the same as obtained in Table 1. Only two agents, the vertices $31(\omega = -0.29)$ and $40(\omega = 0.20)$, have exchange clusters. In the dolphin network this two vertices fall in the boundary between the communities of the network (see Fig 1). Therefore, depending on their natural decision changing rate, can joint or leave a particular group. In this case, the modularity is $Q = 0.3799$ for this split into 2 clusters.

It is important to stress that, in this region of the coupling strength, agents in each final group, tend to maintain a synchronized regime despite their different natural decision changing rates. This is due to the strong influence of the community structure presents in the network which affect notably the decision of agents to join or leave a group. While a subsequent increases in the value of the coupling strength force a completely synchronized regime.

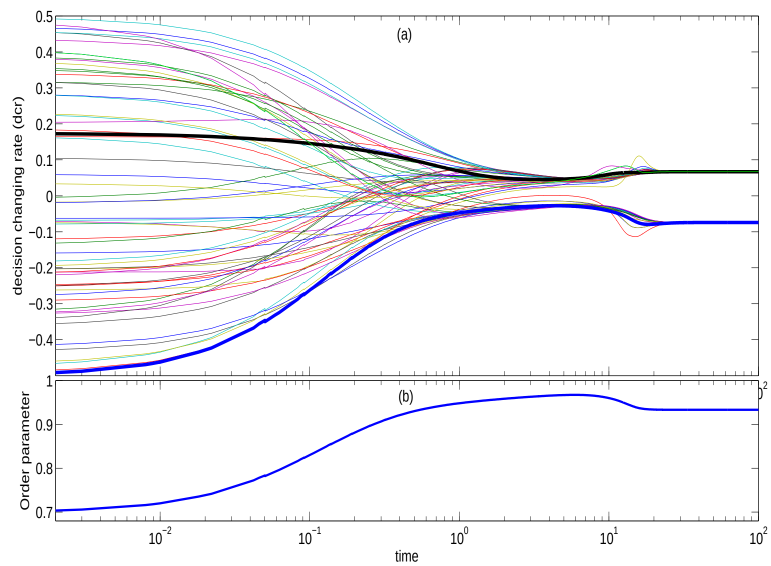

Fig. 2. Decision dynamics of clusters synchronization in the OCR model on the Dolphins Network for $k = 2.2$ and $\omega \in [-0.5, 0.5]$.

In the other hand, centrality measure (betweenness) [10] for each individual of the network show that vertices $2(\omega = -0.5)$ and $37(\omega = 0.17)$ have high betweenness values. Betweenness is a measure of the influence of individuals in a network over the flow of information between others. So, this two individuals represent a potentially information brokers in this dolphin society. In Fig. 2, panel (a), we have represented, in bold markers, the *dcr* variables associated to this two individuals. According to their natural decision changing rate, vertices 2 $(\omega = -0.5)$ and $37(\omega = 0.17)$ represents more conservative and flexible individuals respectively.

To test how central individuals influence the other members of society in the decision-making process, we have considered two situation concerning the vertex 2 (the same analysis can be realized for the vertex 37).

The first one is when this central agent have natural decision changing rate set to zero value ($\omega(2) \simeq 0$) simulating a flexible agent. Results are shown in Fig. 3. As it can be appreciated, two new single groups formed by vertices 29 and 48 merge. It correspond to individuals with high absolute value of ω: $\omega(29) = 0.41$ and $\omega(48) = -0.47$. For this split into 4 clusters, the modularity is $Q = 0.3822$. Details of that Distribution of agents is reported in Table 3.

Table 3. Distribution of agents shown in Fig. 3(a) as resulting from the simulation of OCR model over the bottlenose dolphins community for $K = 2.2$ and $\omega(2) = 0$.

Cluster	Agents in each cluster
1 (1)	48
2 (21)	2,6,7,8,10,14,18,20,23,26,27,28,32,33,40,42,49,55,57,58,61
3 (39)	1,3,4,5,9,11,12,13,15,16,17,19,21,22,24,25,30,34,35,36,
	37,38,39,40,41,43,44,45,46,47,50,51,52,53,54,56,59,60,62
4 (1)	29

The second situation simulates a more flexible individual that tends to anticipate the others members of society, i.e, $\omega(2) = 0.5$, which represents a maximum value of ω. Simulation results are represented in Fig. 4 over 200 time steps.

Table 4. Distribution of agents shown in Fig. 4(a) as resulting from the simulation of OCR model over the bottlenose dolphins community for $K = 2.2$ and $\omega(2) = 0.5$.

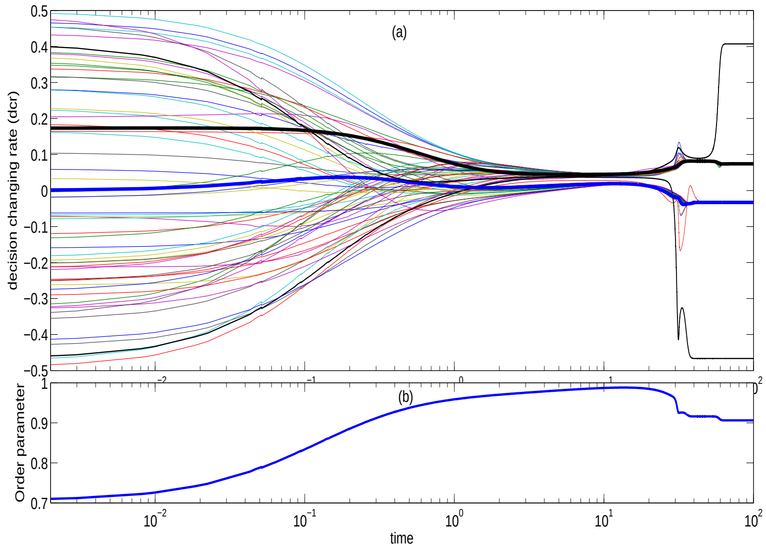

Fig. 3. Decision dynamics of clusters synchronization in the OCR model on the Dolphins Network for $k = 2.2$ and $\omega(2) = 0$. Central individuals are represented in bold marker.

Cluster	Agents in each cluster
1 (1)	48
2 (5)	7,10,14,33,57
3 (3)	8,20,31
4 (6)	18,23,26,27,28,32
5 (38)	1,3,4,5,9,11,12,13,15,16,17,19,21,22,24,25,30,34,35,36, 37,38,39,41,43,44,45,46,47,50,51,52,53,54,56,59,60,62
6 (1)	29
7 (7)	2,6,40,42,49,55,58
8 (1)	61

In Fig. 4(a) we can see that the system reaches a transient completely synchronized regime ($R(t) = 1$ in Fig. 4(b)) in which all individuals run a the same dcr value, generally the average of all

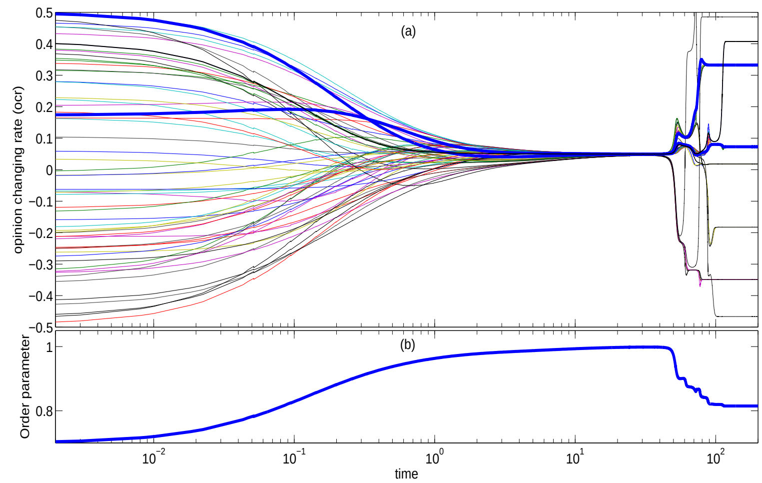

Fig. 4. Decision dynamics of clusters synchronization in the OCR model on the Dolphins Network for $k = 2.2$ and $\omega(2) = 0.5$. Central individuals are represented in bold marker.

the ω. After, we observe that the group containing initially the individual 2 (see Tables 2) is divided into several subgroups. As in the previous situation, individuals with high absolute value of ω run alone in single groups. Details of that distribution are reported in Table 4. Compared with the initial configuration of Table 2, we see that the group 2 is quite stable. Apart from vertex 40 that leave this group to join another one (see Table 4). Moreover, the modularity Q ($Q = 0.3189$) shows a significant decrease, indicating that the partition obtained, does not correspond to the natural partition of the network.

It is important to stress that changing the distribution of the natural decision changing rate, the evolution of each individual can change, but qualitatively the behavior of the system is the same.

5 Conclusions

This work provide evidence that network topology is fundamentally important in decision-making dynamics allowing individuals to join or to leave particular groups depending on their positions in the network. On the other hand, the natural decision changing rate of central individuals is determinate in clusters formation process.

Acknowledgements The Spanish Ministerio de Educacion y Ciencia supports this work through grant DPI2006-15346-C03-03

References

1. Pluchino A. et al. Int. J. Mod. Phys. C 16, 515 (2005)
2. Réka Albert and Albert-László Barabási, Reviews of Modern Physics, 74, 47-97 (2002)
3. Newman M.E.J., SIAM Review 45, 167-256 (2003)
4. Dorogovtsev S.N. and Mendes J.F.F., Advances in physics 51, 1079-1187 (2002)
5. Strogatz, S.H., Nature 410, 268-276 (2001)
6. Juan A. Acebrón et al. Reviews of Modern Physics, 77 (2005)
7. Newman M.E.J. and Girvan M., Physical Review E 69, 026113 (2004)
8. Strogatz, S.H. (2003), SYNC:the emerging Science of Spontenous Order (Hyperion, New York)
9. David Lusseau et al., Behav. Ecol. Sociobiol. 54, 396-405 (2003)
10. Freeman L.C., Sociometry 40, 35-41 (1977)