A Reconstruction of the Hierarchical Relations Between Incompatibility-Entailment, Committive, and Permisive Consequences*

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Abstract

In this paper we propose to investigate the mutual relations among Brandom's three dimensions of semantic inferential articulation, namely, incompatibility entailment, committive, and permissive consequences. Brandom (Unpub.) argues (1) that incompatibility entailment implies committive consequence, and (2) that committive consequence in turn implies permissive consequence. However, under a closer scrutiny of this hierarchy, we see that a number of points are in need of further clarification. First, we question Brandom's claim that the hierarchy is strict. We show that, under reasonable assumptions, all three inferential relations are equivalent. Secondly, we discuss the soundness of Brandom's proofs of (1) and (2). We discuss both results against the background of the main pillars of Brandom's philosophy of language.

1 Introduction

One of the main argumentative lines in Brandom's philosophy of language is the reconstruction of "autonomous discursive practices" in terms of propositions, assertions, and declarative sentences (cf. BSAD, p. 117).¹ The latter is a syntactic notion, and we will not deal with it here. The other two notions, namely propositions and assertions, are treated by semantics and pragmatics, respectively.

On the semantic side, the notion of semantic content of a proposition is defined in terms of its specific inferential articulation (cf., for instance, MIE, pp. 133ff).² Such inferential articulation is understood in a normative way: "Content is understood in terms of proprieties of inference, and those are understood in terms of the norm-instituting attitudes of taking or treating moves as appropriate or inappropriate in practice" (MIE, p. 134). Proprieties of inference are normative statuses, and as such they are grounded on the practice of giving and asking for reasons.

Thus, according to Brandom, the content of an expression is conferred by the intentional states that ground its use in language. "For it is the practical inferential proprieties acknowledged by such attitudes [of treating an inference as correct or incorrect] that make noises and marks mean what they mean" (MIE, p. 174). Nevertheless, Brandom's explanatory strategy consists in explaining simultaneously both the content of

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¹In this paper, BSAD is an abbreviation of Brandom (2008); MIE is an abbreviation of Brandom (1994)

²This inferential articulation can be conceptualized in terms of the notion of incompatibility (cf. BSAD, ch. 5).

expressions and the content of intentional states. It doesn't come as surprise, therefore, that semantics and pragmatics are explained by the same account of the practice of giving and asking for reasons.³ This leads us to consider this practice in more detail.

In the practice of giving and asking for reasons, the fundamental move is that of making an assertion. To make an assertion is to make a particular move:

According to the model [of the game of giving and asking for reasons], to treat a performance as an assertion is to treat it as the undertaking or acknowledging of a certain kind of *commitment*... Doxastic commitments are essentially a kind of deontic status for which the question of *entitlement* can arise. (MIE, p. 142, emphasis in the original)

Being committed and being entitled are normative statues, which are grounded on the participants' mutual attributions of these statuses to each other. In Brandom's framework, the ability to attribute commitments and entitlements are conditions of possibility for the practice of assertion. Besides these fundamental abilities of attributing commitment and entitlement, a story must be told as to how they combine in order to give rise to the inferential practices which eventually constitute the contents of assertions. Indeed, "the inferential articulation [of assertions], in virtue of which they deserve to be understood as propositionally contentful, consists in consequential relations among the particular doxastic commitments and entitlements" (MIE, p. 142). Thus, not only are the abilities to attribute commitment and entitlement fundamental, but also the abilities to *relate* commitments (entitlements) to other commitments (entitlements). How the ability to attribute commitments and entitlements, and the ability to interrelate these attributions, combine in order to underwrite the required inferences is spelled out in terms of the dimensions of authority and responsibility of assertions. Brandom claims that:

In producing assertions, performers are doing two sorts of things. They are first *au-thorizing* further assertions (and the commitments they express), both concommitant commitments on their part (inferential consequences) and claims on the part of their audience (communicational consequences). In doing so, they become responsible in the sense of answerable for their claims. That is, they are also *undertaking* a specific task *responsibility*, namely the responsibility to show that they are *entitled* to the commitment expressed by their assertions, should that entitlement be brought into question. (MIE, p. 173, emphasis in the original)

There is both an *intra*personal and an *inter*personal dimension to the inheritance of commitments (entitlements). Both of them are essential to the reconstruction of the practice of giving and asking for reasons (as it is clear from the quote above). But it is only the intrapersonal dimension of the inheritance that we will focus on, the "inferential consequences," as Brandom calls them. This doesn't mean, as we will see, that we don't need to take the other dimension into account.

There are three different notions of inferential relations, based on the notions of commitment and entitlement. They are defined as follows (cf. BSAD, pp. 120f; MIE, pp. 168–9):⁴

Committive consequence: We say that *p* committive entails *q*, denoted as $p \rightarrow q$, if and only if whenever a speaker *S* is committed to *p*, *S* is committed to *q*.

³A further question that would be interesting to ask is whether this account allows for a difference between literal meaning and speaker meaning, and if so, how they could be reconstructed.

⁴The symbolic convention is ours. It is also worth noticing that although it is acknowledged that inferences can have more than one premisse, we will restrict ourselves in this paper to the single-premisse case.

This kind of inference corresponds to examples like "A is to the West of B, so B is to the East of A; This monochromatic patch is green, so it is not red" (MIE, p. 168).

Permissive consequence: We say that p permissive entails q, denoted as $p \Leftrightarrow q$, if and only if whenever S is entitled to p, S is entitled to q.

This kind of inference corresponds to examples like "the claim that this is a dry, well-made match can serve as a justification entitling someone to the claim that it will light if struck" (MIE, p. 169).

The third inferential relation is based on the notion of *incompatibility*. We say that p is incompatible with q if and only if commitment to p precludes entitlement to q, and commitment to q precludes entitlement to p. The notion of incompatibility defines the semantic content of propositions and performances —the content of p is the class of all q such that p and q are incompatible (cf. BSAD, ch. 5)—, and gives rise to the notion of incompatibility entailment:

Incompatibility entailment: We say that *p* incompatibility entails *q*, denoted as $p \ominus q$, if and only if for all *r*, if *r* is incompatible with *q*, *r* is incompatible with *p*.

This kind of inference corresponds to examples like "If my first pet (in fact, let us suppose, a fish) *had been* a donkey, it *would have been* a mammal" (BSAD, p. 122).

Brandom (Unpub., p. 23–25) argues that one can use the dimensions of authority and responsibility of assertions (cf. above) in order to determine the following connections between the three inferential relations:

If p incompatibility entails q, then p committive entails q.	(1)
If p committive entails q, then p permissive entails q.	(2)

In the next section, we will reconstruct Brandom's hierarchy of semantic inferential relations, as discussed in Brandom (Unpub.). In section 3, we will show that the hierarchy is actually different, as both the reciprocal of (1) actually follows from (2), and the reciprocal of (2) follows from (1). After an assessment of the consequences of this result, we will go on in section 4 to challenge Brandom's proofs of (1) and (2). This will trigger a number of considerations, summarized in section 5.

2 The Hierarchy of Inferential Relations

Now we take up the discussion about the mutual connections between Brandom's three semantic inferential relations. It is on the basis of the analysis of the notion of commitment in terms of its dimensions of authority and responsibility as described in §1 that claims (1) and (2) can be proved. To this effect, we can reconstruct the notion of assertion in the following way. Suppose that *S* is a speaker and *p* a sentence. What *S* is doing when he asserts *p* is undertaking a commitment, which can be spelled out along the two dimensions of responsibility and authority as follows: We say that *S* is *committed to p* if and only if:

Responsibility: S has to respond to any challenge to *p*.

Authority: S authorizes others to be committed to p. This means that S has to be prepared for making his own any challenge to p addressed to other people. Or, conversely, that other people can discharge responsibility to show entitlement to p by deferring it to S.

In the model of giving and asking for reasons, a challenge is a fundamental notion. In line with Brandom's contention that assertion is the fundamental speech-act (cf. MIE, p. 172), a challenge needs to be reconstructed in terms of assertion, otherwise assertion would not be the basic speech-act Brandom wants it to be. Therefore, one "can challenge an assertion only by making an assertion incompatible with it" (MIE, p. 178).⁵

With this reconstruction at hand, we will first prove that $p \odot q$ entails $p \Box q$, that is, claim (1), and next we will proceed to prove claim (2).

Proof of (1): Suppose that $p \odot q$ and that *S* is committed to *p*. We will show that *S* is committed to *q* by showing (i) that *S* has to respond to any challenge to *q*; and (ii) that *S* authorizes others to assert *q*:

(i) Suppose q is appropriately challenged with r, which is incompatible with q. Then p is appropriately challenged with r as well, because $p \odot \rightarrow q$ and therefore r is incompatible with p. In other words, if $p \odot \rightarrow q$ and r is a challenge to q, then r is a challenge to p. Furthermore, since S is committed to p, he has to respond to r. We have just shown that S has to respond to any challenge to q.

(ii) By assumption, S authorizes others to be committed to p. This means that he will make his own any challenge to p addressed to other people. But since any challenge to q is a challenge to p, he also has to make his own any challenge to q. \Box

This proves that $p \odot q$ entails $p \Box q$. Now, we will prove claim (2), that is, that $p \Box q$ entails $p \diamond q$.

Proof of (2): Suppose that $p \Box \rightarrow q$ and that *S* is not entitled to *q*. This means that if *S* were to assert *q*, he could not fulfill his responsibility to show entitlement to *q*. This would not be the case if *S* were entitled to *p*, since he could discharge his responsibility with respect to *q* by asserting *p*. The reason is that we have assumed that $p \Box \rightarrow q$.⁶ Therefore, *S* is not entitled to *p* either. By counterpositive, it follows that if *S* is entitled to *p*, he is entitled to *q*, and we are done. \Box

Brandom claims that the order is "strict"—i.e., that the 'if... then' in (1) and (2) are not reversable. Although he doesn't give any explicit reason for this claim, we can give the following charitable explanation. The reason why this hierarchy is not reversable is the following. These three inferential relations are intended to be recognizable in terms of more familiar inferential relations, such as deduction and induction. Committive and permissive consequences relate to these familiar notions in the following way. The former corresponds to a generalization to the material case of *deductive* inferences, and the latter to a generalization to the material case of *inductive* inferences (cf. BSAD, p. 121; MIE, pp. 168–9). The third inferential relation, namely incompatibility entailment, is characterized by Brandom as a "counterfactual-supporting, *modally* robust inferential relation" (ibid.; cf. §3). For ease of presentation, we will call this latter relation, the *robust inference*. These corresponding inferencial relations, namely robust, deductive, and inductive inferences, are, purportedly, *strictly* ordered by strength. This means that the inferential relations to which they each correspond must also be ordered by such a strict order. We will challenge this picture in the next section.

⁵The definition of challenging p by asserting an incompatible q assumes that p and q are recognized as incompatible by both speaker and challenger, and not only that they are so at the level of normative, social statuses.

⁶It comes as no surprise that inheriting entitlement depends on inheriting commitment. For there are only four ways in which a speaker can discharge his responsibility of showing entitlement to a claim, and only one of them is inferential, namely, that a commitment follows from another.

A few words on the relevance of this hierarchy, and a closer examination of it, are in order. As we showed in §1, the semantic content of an assertion is understood in terms of its inferential articulation. In ch. 5 of BSAD, Brandom gives an analysis of the inferential articulation along the dimension of incompatibility entailment. But, as he acknowledges (cf. BSAD, footnote 5, p. 123), this is just one dimension. The other two dimensions are to correspond to generalizations to the material case of deductive and inductive inferences. That we are dealing with generalization to the material case means, on the one hand, that those inferences are grounded on the meaning of the (logical and non-logical) expressions in the sentences involved —at least for the case of deduction-, and, on the other hand, that the present analysis of inferential articulation is dealing with inferences that occur in our everyday-use of sentences and expressions, which pressumably include to a large extent inductive inferences. The hierarchy can shed light on how these different inferential practices are organized with respect to each other, and therefore it shows how all the dimensions of the semantic content of expressions are organized, thus giving a more fine-grained analysis of the notion of semantic content.

3 A Closer Scrutiny

Brandom claims (cf. Brandom, Unpub., p. 25) that the 'if...then' in (1) and (2) are not reversable. However, under a closer scrutiny, we can see that the claim is not sound. It can be proved that (2) implies that the converse of (1) holds. In other words, from the fact that $p \Box \rightarrow q$ implies $p \diamond \rightarrow q$, we can prove that (3) holds:

If p committive entails q, then p incompatibility entails q. (3)

This means that if (1) and (2) hold, incompatibility entailment is *equivalent* to committive consequence. The proof of (3), given (2), is the following:

Proof of (3): Suppose that $p \Box \rightarrow q$. We need to show that $p \odot \rightarrow q$. Let *r* be such that it is incompatible with *q*. We shall prove that *r* is incompatible with *p*. Consider the notion of *p* being incompatible with *r* paraphrased as follows:

- (i) If S is committed to p, S is not entitled to r.
- (ii) If *S* is committed to *r*, *S* is not entitled to *p*.

(i) If, on the one hand, we assume that *S* is committed to *p*, by the assumption that $p \Box \rightarrow q$ it follows that *S* is committed to *q*. Since *r* is incompatible with *q*, *S* cannot be entitled to *r*.

(ii) On the other hand, if we assume that *S* is committed to *r*, since *r* is incompatible with *q*, then *S* cannot be entitled to *q*. Moreover, since $p \Box \rightarrow q$ implies that $p \diamond \rightarrow q$ (this is the assumption that (2) holds), then *S* cannot be entitled to *p*. \Box

We have just shown that r is incompatible with p. Therefore, if $p \Box \rightarrow q$, then $p \odot \rightarrow q$. In view of (1), we have that $p \Box \rightarrow q$ if and only if $p \odot \rightarrow q$.

As we said above, this hierarchy has consequences for the way in which we can recognize these three inferential relations in terms of more familiar inferential relations. We noted above that committive consequence is intended to correspond to deductive inferences, and that permissive consequence is intended to correspond to 'robust' inference. The third inferential relation is characterized as follows: The fact that the *properties* of being a donkey and being a mammal stand in the relation of incompatibility entailment means that every *property* incompatible with being a mammal is incompatible with being a donkey. ... We could say: "Necessarily anything that is a donkey is a mammal." (BSAD, pp. 122, emphasis in the original)

The notion of incompatibility entailment, therefore, has an intrinsic modal ingredient. The question now is whether, under the light of the equivalence between commitive consequence and incompatibility entailment, we can keep the promise that committive consequence corresponds to deductive inference. The problem at stake is whether the notion of deductive consequence is the same as, or equivalent to, the notion of robust inference. There are a number of caveats triggered by this consequence, but for reasons of space we cannot develop them here.⁷

What does come as surprise is that, under the assumption of symmetry of the incompatibility relation,⁸ we can prove that (1) implies the reciprocal of (2), that is (4):

If p permissive entails q, then p committive entails q. (4)

Proof of (4): We will prove that if $p \Leftrightarrow q$ then $p \odot q$. The desired result will follow by (1). Suppose that $p \Leftrightarrow q$ and that *r* is incompatible with *q*. The latter assumption implies that if *S* is committed to *r*, *S* cannot be entitled to *q*. Since entitlement to *p* implies entitlement to *q*, it follows that *S* cannot be entitled to *p* either. By symmetry (see footnote 8), it follows that *r* is incompatible with *p* and we are done.

The consequences of (4) are obvious. It implies that all three inferential relations are equivalent. This destroys all chances to maintain the characterization of these inferences in terms of deduction and induction. We cannot hold that committive consequence corresponds to deduction and that permissive consequence corresponds to induction.⁹

4 The Analysis of Commitment

Brandom's derivation of claim (1) seems to conflate two different notions of committive consequence. One of these notions occurs at the attitude-level, whereas the other occurs at the status-level. On the one hand, there is a notion of committive consequence in terms of the individual ability to attribute a commitment as a consequence of attributing another commitment —i.e., an intrapersonal inheritance of commitment. On the other hand, there is a notion of committive consequence brought about by the social normative status conferred to the individual by the attributions of the members of a linguistic community. Therefore, the notion of commitment to a sentence, as obtained as the result of a committive consequence at the attribute-level, is in principle different from the notion of a commitment to a sentence, as obtained as the result of

⁷There is an issue concerning the concrete definition of robust inference, and the modal vocabulary in terms of it. Moreover, the failure of the principle of extensionality in modal logic requires a more fine-grained analysis of the equivalence between (classical) deduction and robust inference.

⁸The relation of incompatibility is symmetric by definition. What is actually meant is something stronger. It says that if one denies that p and q are incompatible, one is denying both that (i) If S is committed to p, S is not entitled to q, and (ii) If S is committed to q, S is not entitled to p. In other words, symmetry means that (i) implies (ii) and viceversa. We take that this is what Brandom has in mind when he imposes the condition of symmetry over the incompatibility relations that are "suitable as semantic primitives" (cf. BSAD, p. 123).

⁹This claim was a bit odd from the begining, under the light of (2). The reason is that (2) implies that any deductive inference corresponds to an inductive inference. In other words, since $p \Box \rightarrow q$ means that there is a sound deduction from p to q, (2) implies that there is a plausible induction from p to q. This doesn't make much sense. Deduction and induction are categorically different concepts, not concepts that differ in 'strength'.

a committive consequence at the status-level. Brandom calls the former an *acknowl-edged* commitment,¹⁰ and the latter a *consequential* commitment (cf. MIE, pp. 194ff). Both of these notions are at play in Brandom's reconstruction of the propositional conceptual content of a sentence (cf. MIE, p. 174). But we shall show that the gist of the proof of claim (1) relies on a conflation of these notions of commitment.

It is important to disentangle the individual and the social aspects in the game of giving and asking for reasons. The game consists of a number of participants, each of whom has the ability to keep score of each other's attributions of commitments and entitlements, as well as treating certain inferences as good (cf. MIE, pp. 182ff). These are individual abilities, as is manifest in Brandom's insistence that this game is "doubly perspectival", as opposed to, say Baseball, where there is one single official score (cf., particularly, MIE, p. 185). On the one hand, S's assertion that p consists in undertaking a commitment, which, among other things, implies a self-attribution of commitment — an acknowledged commitment. On the other hand, what commitments follow from S's assertion that p are evaluated from the perspective of each of the participants, including S himself. But these commitments need not be the same for everyone. This creates a difference between consequential and acknowledged commitments.

To make the point clearer, consider the case where *S* is committed to incompatible claims. To refrain from considering *S* as irrational, one needs to make a difference between *S*'s own commitments and what *S* is committed to according to the community —i.e., there are commitments that are not incompatible according to him, but that are so according to the community. For instance, suppose that, according to *S'*, $r \square \rightarrow q$, but this is not so for *S*. Furthermore, suppose that *p* and *q* are incompatible (according to both *S* and *S'*). If *S* commits himself to *p* and *r*, he need not be attributing himself incompatible commitments. But according to *S'*, *S* is also committed to *q*. This results from *S'*'s committive consequence from *r* to *q*. Therefore, according to *S'*, *S* is making incompatible commitments, since *p* and *q* are incompatible and *S'* is attributing *S* commitment both to *p* and *q*. In this example, *S* is not committed to *q* in this sense that it is not one of his *acknowledged* commitments. On the other hand, *S* is committed to *q* from the point of view of *S'* —the *consequential* commitment.¹¹

That this difference is important can be seen from Brandom's insistence that "this is one of the benefits of this sort of approach over causal-functional accounts of intentional states" (MIE, p. 196). The reason is that one can attribute oneself incompatible commitments —i.e., commitments that are not incompatible for oneself, but that are so according to the community—, and thus one can be mistaken, that is, one can be 'ilogical', and yet rational.

The difference between the status- and the attitude-level is also important to the extent that Brandom claims that:

The way in which the collaboration of attitudes adopted from two socially distinct perspectives —attributions of commitments to oneself and by others— is required to institute discursive commitments is the central theme of this work. It is in terms of the social-perspectival character of discursive deontic statuses that the notion of *objectivity* is to be made intelligible (MIE, pp. 166f).

The two levels of acknowledged and consequential commitments are conflated in the proof of (1). Recall that Brandom is discussing *intra*personal inheritance of commitments. This means that the discussion is centered around the practical ability of

¹⁰Actually, an acknowledged commitment is a commitment self-attributed, regardless of whether it was self-attributed as part of making an assertion, or as a consequence of an intrapersonal inheritance of commitment.

¹¹There is also the issue as to whether S' can be mistaken in his assessment that $r \Box \rightarrow q$. Moreover, we are not told a story as to how to compute consequential commitments out of the individual scores kept by the participants (cf. below).

each participant to be disposed to treat himself as committed to q as a consequence of treating himself as committed to p, and to project this inference on the rest of the participants. Therefore, the discussion is conducted at the level of *acknowledged* commitments. *Consequential* commitments, since they are essentially social, cannot be the focus of the discussion. Yet, the proof of (1) introduces the social dimension of authority and responsibility. Along the latter dimension, for instance, Brandom shows that if $p \odot q$ (for *S*), and if *S* undertakes the commitment that *p*, then *S* has to respond to any challenge to *q*. The missing step is to go from the social status of 'having to respond to a challenge to *q*' back to *S*'s individual self-attribution of commitment to *q*. But this step is not allowed in as much as we stick to a difference between the social- and the attitude-level of commitments.

A similar analysis can be conducted to the proof of (2). In other words, the proof of (2) also relies in a conflation of the two levels of *entitlement*.

5 Conclusion

First, we should note that the entailment from $p \Leftrightarrow q$ to $p \odot q$ relies on the assumption of the symmetry of the incompatibility relation. This assumption seems to have consequences for the metamathematical results on the incompatibility relation (cf. BSAD, ch. 5). In as much as one feels compelled to keep these results, one must stick to this assumption and, therefore, to the entailment in question.

Secondly, the idea that incompatibility entailment, committive consequence, and permissive consequence correspond to robust inference, deductive inference, and inductive inference respectively, must be abandoned —at least if robust inference is meant to be something different from deduction. There are no three different notions of inference, but just one. This result might have consequences with respect to the socrekeeping dynamics described in ch. 3 of MIE.

Thirdly, in view of the equivalences obtained in §3 and an analysis of the notion of commitment, it will be interesting to measure the extent to which different participants of the game of giving and asking for reasons can have different inferential attitudes and different attributions of incompatibilities. *Prima facie*, this discussion seems to be at odds with the possibility of maintaining a difference between the status- and the attribution-level of commitments, as discussed in §4.

Fourth, it follows from our discussion in §4, although we cannot argue for it in detail for reasons of space, that the appropriate level for the discussion of the hierarchical relations between the three semantic inferential relations, and therefore for the discussion of semantic content, is the *status*-level. But unlike the inferential relations at the attribution-level, Brandom doesn't spell out when someone is committed to a sentence at the status-level. Is *S* committed to *p* if all the participants, including himself, attribute him commitment to *p*? But since there are differences between these two levels, this requirement is too strict. How many participants then should attribute *S* commitment to *p* in such a way that *S* is committed to *p* at the status level? Half of them? Most of them? Only the relevant ones?

In this paper, we have explored the internal logical relations between the definitions offered by Brandom of these three kinds of entailment relations. From a different, external perspective, we suspect that Brandom may be misguided in focusing on incompatibility-entailment rather than on committive consequence as the most interesting notion of entailment/consequence from the point of view of his pragmatist inferentialism. Indeed, (under a different formulation) committive consequence may do better justice than incompatibility-entailment to the pragmatist and in particular *constructivist* intuitions and assumptions that are at the heart of his inferentialist philosophy (cf. his references to Dummett in MIE). Under a different formulation, committive consequence may define a constructivist logic, perhaps similar to intuitionistic logic, in which case the hierarchy might end up being: committive consequence (roughly similar to intuitionistic logic) \Rightarrow incompatibility-entailment (classical logic) \Rightarrow permissive consequence (inductive logic). But for now, this shall remain as a suggestion for future work.

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