# Estimation of the permeability of granular soils using neuro-fuzzy system

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## Abstract

Determination of the permeability coefficient is crucial for the solution of several geotechnical engineering problems such as modeling of underground flow, determination of the hydraulic properties of leachate water in waste disposal areas, calculation of the compressibility, and so on. Constant head permeability test, which is usually performed for the determination of the permeability, is easy to apply; however, it is not easy to obtain undisturbed sand specimens from field. Therefore, the tests are usually employed on specimens having similar relative densities to those from the field. An alternative approach to permeability tests for granular soils is the prediction of permeability levels in terms of a number of particle size distribution and shape parameters. Although these methods are capable of making reasonable predictions for permeability coefficient, they have certain limitations. In this study, the approximation ability of neuro-fuzzy systems is utilized for the prediction of the permeability coefficient. Permeability test results on 20 different types of granular soils are used to generate a database to train adaptive neuro-fuzzy inference system (ANFIS), which is considered to predict the results of eight different permeability tests. It is concluded that ANFIS structure is superior in the prediction of permeability tests considering particle shape and grain size distribution information.

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## 1. Introduction

Permeability is considered one of the most important parameters in soil mechanics. Basically, it is defined by the quantity of water passing through a soil medium in a certain period, and is determined by in-situ and laboratory tests. In common practice, the permeability coefficient is usually obtained by constant-head permeability test, and is utilized in filtration-drainage, settlement, and stability calculations. These problems are extremely important for environmental aspects such as waste water management, slope stability control, erosion, and structural failure related with the ground settlement issues. In this respect, empirical equations are utilized to predict these parameters; however, these equations have certain limitations and uncertainties. In addition to the incorporation ability of the past experiences regarding to these obscure parameters, neuro-fuzzy systems enable the engineers to predict the unknown parameters belonging to these problems with its superior approximation abilities.

Although constant head permeability tests don't take much time to perform, the relationship between permeability parameter and a number of grain size distribution parameters have been investigated by several researchers (Seelheim, 1880; Hazen, 1892; Slichter, 1898; Freeze and Cherry, 1979; Carrier III, 2003; Chapuis, 2004). In Table 1, the empirical equations presented by a number of researchers are given. In addition, advantages, disadvantages and limitations of these studies are also included in the table. These formulas are capable of estimating the permeability with a reasonable precision. Therefore, an alternative and more precise technique is developed in this study using adaptive neuro-fuzzy inference methodology. Using the database obtained by several permeability tests, which include a number of particle shape and grain size distribution parameters, permeability coefficient of sands are modeled with the methodology. It should be mentioned that, because grain size distribution is the main factor affecting the void ratio and relative density of soils, regarding to the knowledge that global void ratio is primarily effective on the permeability of granular soils, the training and testing databases have similar gradations.

## 2. Materials and Methods

The conceived ANFIS model comprises following input parameters: a) $D_{10}$ (the diameter which finer material is equal to 10% of the total by weight), b) $D_{60}$ (the diameter which finer material is equal to 60% of the total by weight), c) mean roundness, d) mean sphericity, e) global void ratio (e), f) maximum void ratio ($e_{max}$), and g) total fractal dimension. $D_{10}$ and $D_{60}$ parameters are extracted from the grain size distribution of the sands. The roundness, the sphericity and total fractal dimension parameter definitions are given in another study (Sezer et al., 2008). Soils used in this study consist of 100 % sand, and the physical properties of the sands are given in Table 2. The void ratios of the sands are extracted from the specimens prepared under Standard Proctor densification level. Sands are arranged in similar gradations to compare their permeability coefficients in relation with their shape characteristics. Three groups of the sands (tabulated as B, L and A in Table 2) are crushed materials and two of them (R and S) are natural materials. The sands are separated into four grain size distributions, and labeled as 1, 2, 3, and 4, which representing coarse, medium, fine and well gradation, respectively. As can be derived from Table 2, three of the sands of each origin (1, 2 and 3) are poorly graded and the last sand group (4) is well graded. In order to ensure 100 % coarse material inclusion, the soils in this study are washed under Sieve No.120. Moreover, gravel sized or bigger particles are not included; therefore, all the grains are passed the No.4 sieve. Detailed explanation on the origin of the soils can be found in elsewhere (Sezer, 2008).

Table 1. Empirical equations manifested for permeability prediction of soils

| Researcher / Organization | Equation | Limitations, Advantages / Disadvantages |
|---|---|---|
| Hazen (1892) | $k = C_H d_{10}^2$ | Effective diameter changes between 0.1 and 30 mm (Hazen, 1892; Carrier III, 2003). |
| Kenney et al. (1984) | $k = (0.05 \sim 1)\,\mathrm{d}_5^2$ | D=0.074-25.4 mm and $C_U$=1.04-12. |
| Breyer-(Kresic, 1998) | $k = 6\times10^{-2}\times\frac{g}{v}\times\log\left(\frac{500}{C_u}\right)\times d_{10}^2$ | $C_u$= 1~20, $d_{10}$= 0.06~0.6 mm. |
| Slichter (1898) | $k = \frac{g}{v}\times n^{3.287}\times d_{10}^2$ | best suited for soils with $d_{10}$ =0.01 ~5 mm (Vukovic & Soro, 1992) |
| Chapuis (2004) | $k = 1.5\times d_{10}^2\times\frac{e^3}{1+e}\times\frac{1+e_{\max}}{e_{\max}^3}$ | N/A |
| NAVFAC (Chapuis et al. , 1989) | $k = 10^{1.291e-0.6435}\, d_{10}^{10^{0.5504-0.2937e}}$ | e=0.3~0.7; $d_{10}$=0.10~2.0 mm; $C_u$=2 ~12; and $d_{10}/d_5$ >1.4 |
| Terzaghi- (Odong, 2007) | $k = 0.0084\times\frac{g}{v}\times\left[\frac{n-0.13}{(1-n)^{1/3}}\right]^2\times d_{10}^2$ | The selected average value of 0.0084 is actually a classification coefficient typically ranging between 0.0061 and 0.00107. |
| USBR- (Vukovic and Soro, 1992) | $k = 0.048\times\frac{g}{v}\times d_{20}^{0.3}\times d_{10}^2$ | Gives the best results when $C_u$ is lower than 5 (Cheng and Chen, 2007) |
| Alyamani and Şen (1993) | $k = 1.5046*\left(I_0 + 0.025*\left(d_{50}-d_{10}\right)\right)^2$ | The method is more accurate for well-graded sample (Odong, 2007). |
| Kozeny-Carman (1956) | $k = 0.083\times\frac{g}{v}\times\left[\frac{n^3}{(1-n)^2}\right]\times d_{10}^2$ | $d_{10}$<3 mm., for granular soils, the inertia term is not taken into account (Carrier III, 2003). |

For investigating the effect of particle shape on the permeability of soils, constant head permeability tests are employed on the soil specimens in accordance with ASTM D2434-68 standard. The combination permeameter is utilized to perform permeability tests on specimens of 31.65 $cm^2$ cross-section and of 10-11 cm in length, which were prepared at Proctor density level.

Permeabilities of granular soils (k) are computed in accordance with D'arcy's Law:

$$Q = k\frac{h}{\ell}a \qquad (1)$$

where Q is the discharge, a is the cross sectional area of the specimen, h is the hydraulic load on the specimen and $\ell$ is the length of the specimen. The test apparatus is given in Figure 1.

Table 2. Physical properties of the soils used in this study (Sezer, 2008).

| Origin | Sand type | $C_u$ | $C_c$ | Gravel % | Sand % | Silt-Clay % |
|---|---|---|---|---|---|---|
| Limestone (L) | 1 | 1.55 | 0.94 | 0 | 100 | 0 |
| | 2 | 1.61 | 1.03 | 0 | 100 | 0 |
| | 3 | 1.67 | 0.90 | 0 | 100 | 0 |
| | 4 | 8.00 | 1.22 | 0 | 100 | 0 |
| Basalt (B) | 1 | 1.55 | 0.94 | 0 | 100 | 0 |
| | 2 | 1.61 | 1.03 | 0 | 100 | 0 |
| | 3 | 1.67 | 0.90 | 0 | 100 | 0 |
| | 4 | 11.82 | 1.18 | 0 | 100 | 0 |
| Andesit (A) | 1 | 1.55 | 0.94 | 0 | 100 | 0 |
| | 2 | 1.61 | 1.03 | 0 | 100 | 0 |
| | 3 | 2.33 | 0.76 | 0 | 100 | 0 |
| | 4 | 6.00 | 1.03 | 0 | 100 | 0 |
| River Sand (R) | 1 | 1.55 | 0.94 | 0 | 100 | 0 |
| | 2 | 1.61 | 1.03 | 0 | 100 | 0 |
| | 3 | 1.67 | 0.90 | 0 | 100 | 0 |
| | 4 | 8.50 | 1.98 | 0 | 100 | 0 |
| Shore sand (S) | 1 | 1.55 | 0.94 | 0 | 100 | 0 |
| | 2 | 1.61 | 1.03 | 0 | 100 | 0 |
| | 3 | 1.82 | 0.92 | 0 | 100 | 0 |
| | 4 | 6.68 | 1.04 | 0 | 100 | 0 |

The permeability test results and the corresponding empirical equation outcomes are given in Figure 3. The coefficient of permeability (k) of coarse grained sands is higher, in comparison with medium and fine sands. Least coefficients are observed for well graded soils. Analyzing the results given in Figure 3, it can be concluded that the given formulas are capable of estimating the *k* parameter to a reasonable degree. Nevertheless, the outcomes of different equations are still far from the equality. The Breyer formula is not taken into account for the predictions in coarse and fine sands, where USBR formula is not used for the permeability estimation of well graded sands (Vukovic and Soro, 1992). The investigations, which are graphically demonstrated in Figure 3, indicate that the outcomes of Chapuis and Slichter methods are in harmony with the test results. Investigating the test results on medium sands (Figure 3b), it can be stressed that NAVFAC formula rearranged by Chapuis et al. (1989) and Kozeny-Carman (1956) methods are quite successful. Furthermore, the test results given in Figure 3c indicate that Chapuis method best predicts the permeability coefficient of fine sands.

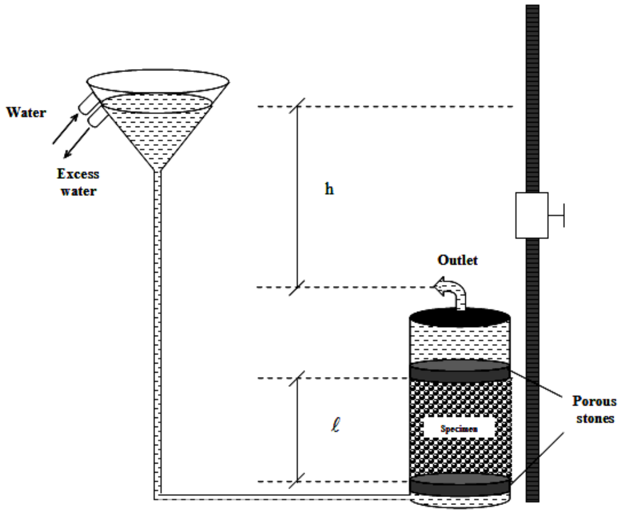


**Fig. 1.** The combination permeameter- constant head situation**.**

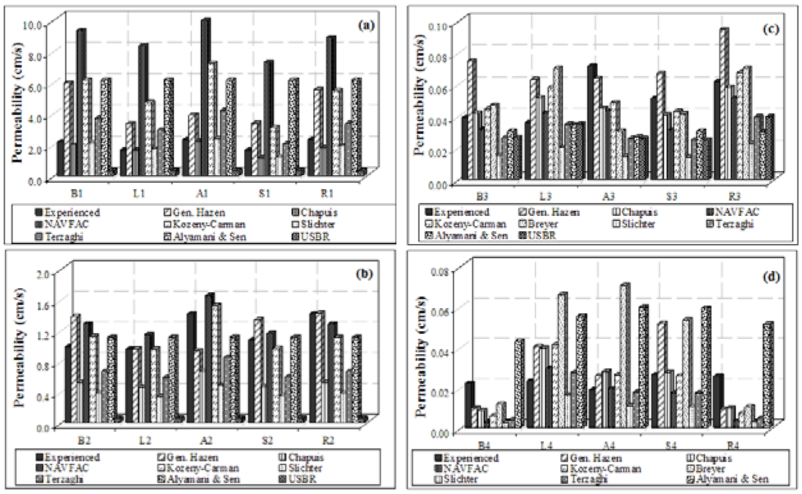


Figure 3. Permeability test results and the outcomes of the empirical formulae on a) Coarse b) Medium c) Fine d) Well graded sands

For all poor graded sands, the USBR method underestimated the permeability test results. Alyamani & Sen method has underestimated permeability in coarse grained sands, while the method overestimates the permeability coefficients in fine grained sands. Nevertheless, despite the good predictions in poor graded sands, Chapuis (2004), Alyamani & Sen (1993) and USBR methods give similar permeability coefficient predictions. In this manner, great care should be taken for the use of these formulas in permeability prediction. Moreover, for well graded soils, the estimates are far from proposing a formula (Figure 3d).

As can be seen from Figure 3, the differences in the results and the limitations on the empirical formulae encourage seeking for a new alternative approach for the permeability prediction. In this investigation, ANFIS is considered to be a possible and plausible alternative to traditional techniques.

## 3. Adaptive neuro-fuzzy inference system and the model

Fuzzy sets have the advantage of use of the "partial belonging concept", instead of crisp belongingness. The membership value of a data point is the measure of the belongingness of the point to any set. The Adaptive Neuro-Fuzzy Inference System (ANFIS) is emerged as a powerful technique which couples Artificial Neural Network (ANN) and Fuzzy Inference System (FIS) methodologies (Jang and Sun; 1995). The NN-based learning technique and a fuzzy inference methodology which using membership function and a fuzzy rule-base philosophies, is used to establish nonlinear relationships between input and output spaces (Jang, 1993; Jang and Sun, 1995). Therefore, the ANFIS is capable of organizing self-structures in terms of the rules and membership functions with the help of input and output data patterns.

In this study, the model is setup using backpropagation learning algorithm and Sugeno-type fuzzy inference system. Details of Sugeno type neurofuzzy structure can be found elsewhere (Jang, 1993).

Consider a Sugeno type of fuzzy system based on the rules:

Rule 1. If $x$ is $A_1$ and $y$ is $B_1$, then $f_1=p_1x+q_1y+r_1$

Rule 2. If $x$ is $A_2$ and $y$ is $B_2$, then $f_2=p_2x+q_2y+r_2$

Premises results can be computed by :

$$w_1 = \mu_{A_1}(x)\mu_{B_1}(y); \quad w_2 = \mu_{A_2}(x)\mu_{B_2}(y) \tag{2}$$

The weighted average may be calculated as:

$$f = \frac{w_1 f_1 + w_2 f_2}{f_1 + f_2} \tag{3}$$

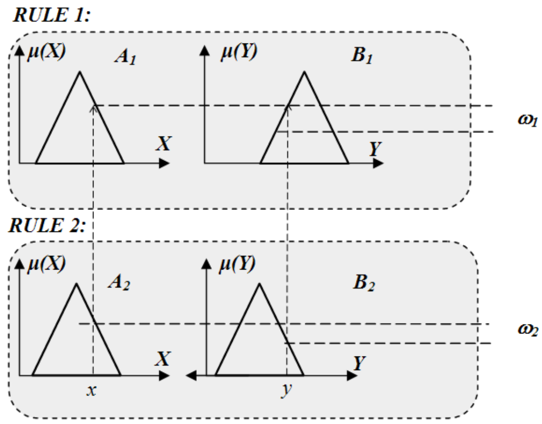


Figure 4. Sugeno-fuzzy model with two rules.

Definition of this parameter into diverse phases can be formulated by:

$$\overline{w}_i = \frac{w_i}{w_1 + w_2} \tag{4}$$

Then output can be obtained as:

$$f = \overline{w}_1 f_1 + \overline{w}_2 f_2 \tag{5}$$

Figure 5 demonstrates the corresponding ANFIS architecture to the rules in Figure 4. The circular and square nodes represent the fixed and learnt nodes, respectively.

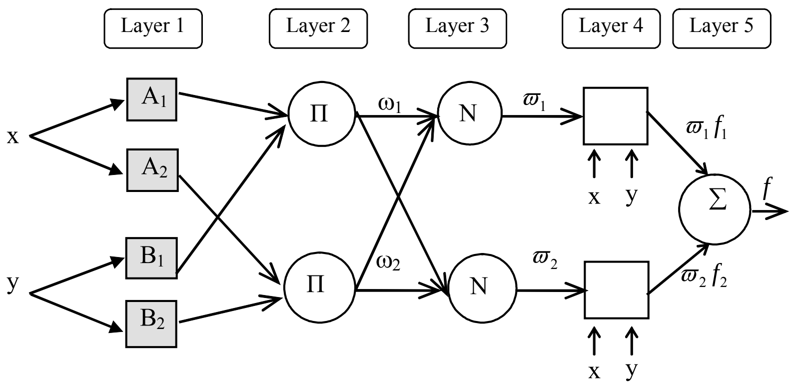


Figure 5. The ANFIS architecture for a two rule Sugeno system

Since the lack of space, no further explanation on the ANFIS calculation will be given here. Detailed explanation is given in Jang (1993). As mentioned before, the input parameters of the model are: $D_{10}$, $D_{60}$, r, s, e, $e_{max}$ and $D_{tot}$, while the output is the permeability coefficient (k). Focusing on the three dimensional surface graphs of the two selected input and one output, it is possible to conclude that, there are nonlinear relationship among the parameters, and coefficient of permeability is highly effected by high levels of mean sphericity and void ratio (Figure 6). Similar behaviors are observed at other three-dimensional surface plots; nevertheless, no other surface graph is presented here for space problem.

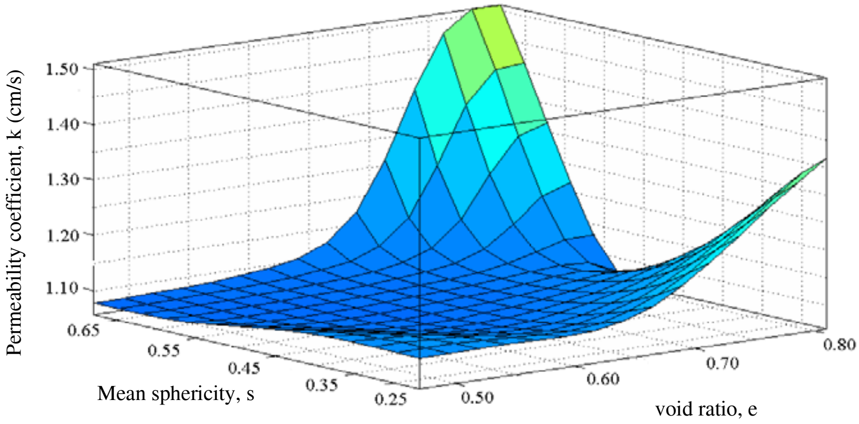


Figure 6. The surface graph of the output against two input parameters.

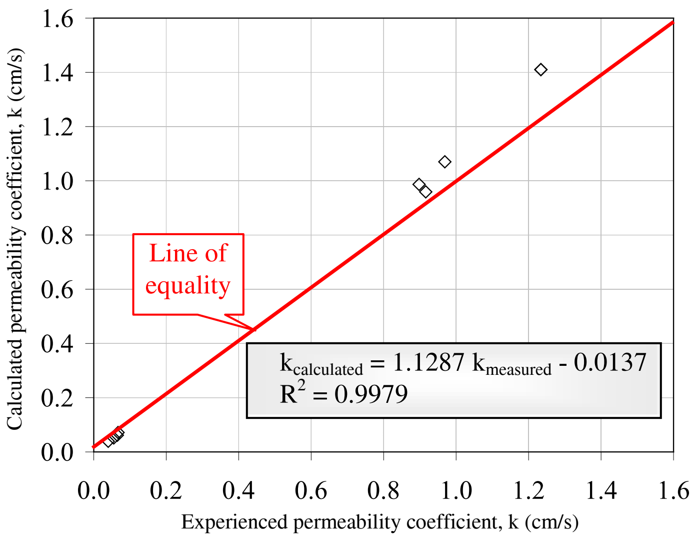


Figure 7. The comparison of calculated and experienced permeability coefficients.

The testing of model is employed on data having similar grain size distribution. The calculated and permeability test results are given below. Extremely high $R^2$ value of 0.9979 approves the permeability coefficient modeling capability of the model (Figure 7).

## 4. Conclusions

In this investigation, ANFIS methodology is applied on uniform and well graded sands in order to propose an alternative approach for the permeability coefficient estimation. Regarding to the 28 tests on prepared sands, permeability tests are conducted to find out the hydraulic conductivities of these soils. Empirical permeability coefficients are discussed in summary for the evaluation of alternative methods for permeability prediction in the literature. Moreover, graphs illustrating the success of these equations are drawn to question the success of these methods. It is concluded that, these methods are capable but not sufficient for correct prediction of k parameter, in terms of their narrow prediction level and limitations in application of grain size distribution. Adaptive neuro-fuzzy inference system is used for k parameter prediction, using a number of particle shape and grain size distribution parameters. It is also derived that, ANFIS is successful in permeability prediction of soils having similar gradation. It is clear that the modeling ability of the ANFIS model strictly depends on the data, i.e. grain size distribution and the shape of the sand grains. On the other hand, it should be noted that larger database development is essential for accurate prediction of *k* parameter for a wide-range grain sizes (including gravels and sands of different median sizes).

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