

Can Paraconsistency Replace Non-Monotonicity?

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Abstract. In this position paper we reconstruct and evaluate three arguments by Jean-Yves Béziau put forward to show that paraconsistent logics may replace non-monotonic logics in the analysis of situations what the latter were created for, namely updating of databases by increment of information. As originally put forward they might seem too colorful or unserious and easily dismissible, but they really call into question the foundations of non-monotonic logic. We will show that non-monotonic logic may go off well of Béziau's challenging arguments, though.

Key words: Paraconsistency, non-monotonicity, belief-revision, conflicts, rationality.

1 Introduction

The question we are concerned here is whether non-monotonic logics can be replaced by paraconsistent logics in the analysis of database updating. This question has been raised by Jean-Yves Béziau in two very short passages of two papers. This problem is worth noting for it calls into question the foundations of non-monotonic logic. The many useful applications of non-monotonic logics relegates virtually any question about their philosophical adequateness, i.e. the question whether the basic elements of non-monotonic logics, be they axioms, universal concepts or notions, etc. have some desirable (meta)logical, epistemological or ontological features and whether other logics succeed in having such features making thus non-monotonic logics *philosophically* or *conceptually* redundant in spite of the impressive case made for them by their several *practical* virtues. It is a shame that some computer scientists just obliviate these problems, when conceptual clarification might improve practical developments.

Let us illustrate this problem with an example from other logics and other features. Many-valued algebraic semantics are used in studying several logics (for

example, L_n , LP, Post's, Kleene's and Bochvar's logics, etc.) even though these logics are "really" two-valued given Suszko's reductive theorem (cf. [4]). In spite of the philosophical and conceptual mistake (taking by logical truth-values what are not logical truth-values), many-valued semantics are preferred in name of several practical or of application virtues, for example, many-valued semantics for ?n are truth-functional and then, for all practical purposes easier than their corresponding usually non-truth functional bivalent semantics. Béziau's question is similar to this: Does paraconsistent logic may replace non-monotonic logic, for the former already considers or gives more adequate explanations for the cases what the latter were created for?

As we shall expound in due course, Béziau thinks that this question deserves an affirmative answer. We do not. We will argue that (1) even though the foundations of non-monotonic logic are not philosophically as well developed as that of paraconsistent logic, they can be so; (2) that there are situations where neither a paraconsistent logic nor a non-monotonic logic alone and separately can deal with, and (3) that the particular development of non-monotonic logics, the earliest ones built from classical logic, does not say anything about the capability of these logics to accommodate inconsistent situations.

The structure of this paper goes as follows. After introducing in section 2 the non-monotonic stance towards conflicts (conflicts either between facts and inferences or conflicts between inferences), in section 3 we reconstruct as strongly as possible Béziau's arguments for the paraconsistent replacement of non-monotonic logics. We aim at reconstructing Béziau's thinking faithfully, so we will assume that our reconstruction actually is the position he is supporting. In section 4 we deal with those arguments, showing that none of them is successful and that paraconsistent and non-monotonic logics cannot do the job of each other and that sometimes it would be better use a combination of them.

2 Conflicts, Defeasibility and Paraconsistency

Non-monotonic logics were designed mainly to deal with situations where conclusions need to be retracted and inferences blocked given some increments of information. For example, consider the data base

(Normally) birds fly. Tux is a bird.

The point made by non-monotonists is that it could be irrational to wait for additional information to draw an undefeasible conclusion, so one can conclude that Tux flies. But with further information, in this case that Tux is a penguin (and that penguins do not fly), the conclusion must be retracted. Thus, non-monotonic logics are logics dealing with conflicts between plausible conclusions drawn from expandable information and "hard facts", some of which possibly newly acknowledged.

Conflicts may arise also between (at least) two potential defeasible conclusions. Consider the example known as 'Nixon Diamond' (because of a particular representation of it that we will not use here). Suppose a database containing

information to the effect that a given individual, Nixon, is both a Quaker and a Republican. Quakers generally are pacifists, whereas Republicans, by and large are not. The question is what defeasible conclusions are warranted on the basis of these data, and particularly whether we should infer that Nixon is a pacifist or that he is not pacifist.

There are at least two different stances with regard to conflicts of this last kind. The two attitudes are the following ones:

- (1) Infer nothing about Nixon's pacifism until having enough information.
- (2) Since there is no obstacle, infer something, for example
 - (2a) Infer that Nixon is a pacifist;
 - (2b) that Nixon is not pacifist;

Those inferring according to (1) are called "cautious" or "skeptical" agents, while those opting for either option in (2) are called "bold" or "credulous" agents. The difference between these basic attitudes comes to this. In the presence of potentially conflicting defeasible inferences and in the absence of further considerations, a credulous agent always commits to as many defeasible conclusions as possible, whereas a skeptical agent refrains from assenting to potentially conflicting defeasible conclusions.

Even though a credulous agent wants to draw as many inferences as possible, it is usually subject to a consistency constraint, i.e. for him it is not an option

- (2c) Infer B and infer $\neg B$.

This seems rather strange, since inconsistent data bases are as well known as their incomplete cousins. Part of the problem is to know what the informational content of a contradiction is and what can be inferred from it. Graham Priest distinguishes in [9] three basic approaches on this:

- *The cancellation view.* $\neg B$ cancels the content of B . If an inference is thought to be valid when the content of premises contains that of the conclusion, then nothing (with content) can be inferred from $\{B, \neg B\}$;
- *The complement view.* According to this, $\neg B$ has whatever content B does not have. Hence $\{B, \neg B\}$ has total content and entails everything;
- *The intermediate view.* The content of $\neg B$ depends of the content of B , but their relation is neither of the previous. On this view $\{B, \neg B\}$ has partial content. Hence, contradictions entail some things but not others.

Skepticism may be motivated either by a cancellation view or by a complement view. In the first case an agent does not infer anything because there is nothing (informative) to be inferred. In the second case the agent does not infer a contradiction for it would trivialize the data base. First non-monotonic logics inherited the complement view from classical logic, but it has changed in combining non-monotonic and paraconsistent logics, where not everything follows from a contradiction. Then a credulous agent can be in fact more credulous by being paraconsistent and drawing conflictive but not trivializing inferences.

Well, there is a view of negation, the intermediate view, which does not make contradictions necessarily explosive. It may be even *theoretically* considered that contradictions can appear in a database, but is there any reason to think that

it might *actually* happen? There are a number of examples and we will consider just one of them, known as “Preface Paradox”, introduced in [5]. A person, as a result of thorough and painstaking research, writes a book in which (s)he claims A_1, \dots, A_n . (S)he has very rational reasons to believe these claims. But (s)he is aware that no factual book has ever been written which did not contain some falsehoods. The inductive evidence for this is overwhelming. So (s)he quite rationally believes $\neg A_1 \vee \dots \vee \neg A_n$ too (or equivalently $\neg(A_1, \dots, A_n)$). Clearly this belief set is inconsistent, yet (s)he believes it and is paradigmatically rational. Seemingly it is paradoxical till one is ready to give up classical logic and accept that one can have contradictory rational beliefs.

Moreover, an inconsistent set of beliefs can be rationally acceptable because it scores highly on many other criteria of rational acceptability besides consistency, for example simplicity, a low degree of *ad-hocness*, fruitfulness, explanatory power, unifying power. As Priest [9] puts it, “consistency is no longer a necessary condition for rational belief; at best, it is one of a list of (potentially conflicting) desiderata.”

3 Béziau’s arguments

Jean-Yves Béziau has claimed that paraconsistent logics can be seen as serious alternatives to non-monotonic logics. According to him [3]:

It is clear that a logic like e. g. *linear logic* is far to be as challenging as paraconsistent logic. Informal motivations for linear logic are based on a few attractive and funny examples involving cigarettes and food, but they are not connected with a serious philosophical analysis (much the same as the penguin case for non-monotonic logic).

And in [2] he says:

Paraconsistent logic can be seen as an alternative, for example, to non-monotonic logic. Non-monotonists reject monotonicity because they think that there are experiences (most of the time involving birds) which show that monotony is wrong and in particular leads to some contradictions. But one who thinks the paraconsistent way would reject the principle of non contradiction and not monotony. (...) It seems to us that the future shall give the preference to paraconsistent logic taking in account the progress of genetical [sic] biology which already produces chicken without feathers, and in the future we may have flying pigs. In such an absurd world, it will make no sense to reason by default, because everything could be true by default.

At first sight Béziau’s ideas on the relation between paraconsistency and non-monotonicity seem quite naïve and blatantly wrong. He may be wrong, but it is not blatantly so and he is not naïve. Let us reconstruct his case in a less colorful way.

3.1 First Argument: Non-monotonic Logics are not based on any serious Philosophical Analysis

Béziau asks for a consideration about whether paraconsistent logic may replace non-monotonic logic given that what causes the introduction of non-monotonicity is the addition of new information contradicting or implying the negation of either original data or consequences of those data, which leads to retractions and withdrawals. In the above quotations we find three arguments against non-monotonicity favoring the paraconsistent way. The first one has as conclusion that non-monotonic logics are not based on any serious philosophical analysis, but are motivated by rather “toy examples”.

Note that Béziau’s contention is not based on a point of view regarding monotonicity as a necessary element of logicity. What he is asking is why the phenomena studied by non-monotonic logics require a more radical departure from tradition than the challenge to the principle of necessary falsity of contradictions. Paraconsistency just requires abandoning the *ex falso quodlibet*, but it keeps logical consequence being reflexive, transitive, and monotonic. There seems to be no explanation of why non-monotonic logics motivating examples require such divergence.

One possibility is that in studying those examples we have not exhausted all the traditional resources, and probably we have not even tried. Until we have an answer to these questions we only can say that the new approach makes things easier, but not that older, more familiar approaches cannot handle those phenomena. But making things easier is not making them clearer conceptually.

3.2 Second Argument: Paraconsistency at Work

Béziau’s main criticism is in the colorful but distracting second quotation, so let us propose the following reconstruction of his argument. If we infer some B from the database and then B arise as new information, neither B need to be withdrawn nor the inferences from B retracted. Maybe both B and B could be considered equally plausible but their conjunction is not explosive, i.e. not everything could be inferred from it. In terms of belief revision theory, in the presence of contradictions Béziau opts for expansion using a paraconsistent logic instead of contraction or revision. Béziau thinks that it is not rational to exclude a contradiction if it is logically possible, i.e. if it is not a trivializing contradiction.

3.3 Third argument: Default is Useless in an Inconsistent World

Béziau’s third criticism is directed towards a particular non-monotonic logic, perhaps the most well-known and influential one: Reiter’s Default Logic (see the basics of this logic in [1]). It is well-known that once the set of axioms of a default theory is inconsistent, the default extension will collapse into triviality immediately. Now, Béziau’s argument goes as follows. Suppose we live in an inconsistent or dialetheic world, i.e. a world such that some (but not all) contradictions are true. Then, default logic says that this world would be trivial,

which is unacceptable for it is false: The world is just dialetheic, not trivial, some contradictions are true but not all sentences are. Thus, default logic is useless in an inconsistent but non trivial world.

4 Answers to Béziau

4.1 The Philosophical Import of Non-Monotonic Logics

Béziau shows deference to paraconsistent logic because according to him it is closely connected to the quest for logicity, and he may be right in thinking that this is a harder problem than those motivating non-monotonic logics. Nonetheless, the systems resulting from the study of those examples have raised several philosophical debates, independently of whether its motivating examples are good or not. Actually Béziau is aware of that. He thinks that logicity lies beyond any particular principle or Tarskian condition, and a few lines after attacking non-monotonic logics by being based on no serious philosophical analysis he admits that the emergence of non-monotonic logics counts as evidence for that broadening of logicity. But there are other very important philosophical issues raised by non-monotonicity. Let us mention some of them but without trying to exhaust them:

- The analysis of actual agents' ways of handling information and the constraints and levels of rationality some logics impose on them;³
- Traditional debates on what is the relation between non-monotonic logics and classical logic: Are they rivals, complementary?⁴ If change of logic is change of subject⁵, where does the change of subject between classical logic and non-monotonic logics take place?
- Why are these logics so effective? It comes from design? Even if a more traditional logic could do the job of non-monotonic logics, what make those other logics so hard to use directly?
- If non-monotonic logics are indeed logics, what is the core of logicity? Where does logicity lie?

Béziau surely is aware of all that, too. How then his first criticism should be understood? We think that his first argument is not directed towards non-monotonic logic, for he appreciates it at least as a pure part of logic worth of investigation, and even more as crucial evidence for the idea that the relation (or operator) of a logical structure does not need to satisfy any special condition. Rather, he is talking to “working non-monotonists”: A conceptual clarification of non-monotonic logics and their place in the logical landscape is needed, not a simple complaint on how well they work in some practical cases. We agree. In that sense non-monotonic logics are not philosophically well motivated. But it should be stressed that such serious philosophical bases for non-monotonic logics can be

³ Cf. [7].

⁴ See [6] on this.

⁵ Cf. [10]

given, even though non-monotonists are not generally worried about providing them.

4.2 Logical Possibility and Rationality

The answers to Béziau's second argument come from several sources, but we are going to reconstruct some given in [9], and although they were not expounded against Béziau, they could have been so.

According to our reconstruction, Béziau claims that it is not rational to exclude a contradiction from a database if the contradiction is logically possible, i.e. if it does not lead to triviality. Béziau is right in saying that sometimes it would be worth or even necessary to draw an inference whose conclusion is a non-trivializing contradiction or to add a contradiction to the database. However, in general *logically possible* and *rational* are not equivalent; it is logically possible that we were hot cakes, but it is not so rational to believe that.

Leaving aside its many problems, Bayesianism seems to be right in claiming that a belief is not an all-or-nothing matter, but comes by degrees which operate according to the laws of probability. If a belief A is assigned the probability $P(A)$ and information B arrives, A is assigned a new probability, $P'(A)$, determined by conditionalization: $P'(A) = P(A/B)$. Thus, even if we tolerate inconsistencies and have a paraconsistent probability theory as that of [8], some of the contradictory beliefs may have a significantly greater probability and then it would not be accurate to maintain both A and $\neg A$.

On the other hand, just like an inconsistent set of beliefs can be rationally acceptable because it scores highly on many other criteria of rational acceptability besides consistency (simplicity, a low degree of ad-hocness, fruitfulness, explanatory power, unifying power), it may not be rationally acceptable by the same reasons. If we always revise by adding on the new information, we are like to lose simplicity or unity, besides consistency. Thus, Béziau is right in saying that sometimes it would be worth or even necessary to draw an inference whose conclusion is a non-trivializing contradiction. However, that contradiction might be itself retractable: In face of new information consistency might be restored, and to model that we need a non-monotonic logic and not only a paraconsistent logic.

4.3 Non-monotonicity and Paraconsistency

The third criticism is a rather unfair remark on a technicality. Reiter's default logic trivializes in the presence of contradictions, so default logic is useless in Béziau's "absurd" world. However, default logic can be suitably modified in order to make it paraconsistent and work as a defeasible paraconsistent logic. The question amounts to know whether such a logic does a better work than either a paraconsistent or a non-monotonic logic alone, and above we have suggested that it may be useful when we need to infer a contradiction which in its turn may be retracted in the light of new information. The answer seems to be again complementation rather than replacement.

5 Conclusions

We have reconstructed and evaluated three arguments by Jean-Yves Béziau put forward to show that paraconsistent logics may replace non-monotonic logics. The first one is that non-monotonic logics are based on no serious philosophical analyses, but in rather few and funny examples. Second, that if inconsistency is the main reason to make retractions then paraconsistency can do the job without a major modification of the consequence operator. Third, that non-monotonic logics would be useless in an inconsistent world, for they would lead to triviality. We have argued that these arguments deserve close attention in spite of their apparent lack of seriousness and of being blatantly wrong. We have showed that non-monotonic logics have a philosophical import, even though if non-monotonists are not worried about that. For example, these logics demand a rethinking of what logic is, what its subject matter is or the connections between logicity and rationality. We have also shown that Béziau is right in pointing out that the consistency constraints in many non-monotonic logics are excessive, but it does not follow that paraconsistent logics alone can do the job of non-monotonic logics. The logical possibility of a contradiction does not imply that it could be regarded also as rational or undefeasible. Often a combination of both kinds of logics is needed for a more accurate modeling of phenomena, and there seems to be no great technical difficulty on doing that.

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