Integrated use of linear programming and multiple criteria methods in an engineering design process

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Abstract

In an aeronautical sector company, where engineering design process steps and activities are developed, a decision aiding methodology was required to support the processes.
We have proposed the integrated use of linear programming and multiple criteria methods, which can be used to orient the conceptual design of alternative functional and physical solutions and to cope with complex design problems.
Linear programming has been used in relation to some case studies, to generate design alternatives that satisfy the set of the initial requirements, while multiple criteria methods have been proposed to interact transparently with the client, in relation to some aspects that a linear programming model cannot include, to evaluate, compare and select alternatives in order to identify and formalize new expectations that the first solutions have not be able to resolve. The iterative use of the two approaches, in a cyclic procedure of mutual learning, allows the requirements to be defined more clearly and a final satisfying solution to be reached.

Introduction

Engineering design is an iterative decision-making process which is developed to devise a component, product, process or system that meets the customerôs needs (Eggert, 2005). A collaboration with the Marketing and Business Development Department (MBDD), of a company that designs and produces aircraft for civil and military use, has allowed us to understand their main conceptual design activities (Norese et al., 2008a; Norese & Liguigli, 2009) and to propose an integrated use of linear programming and multiple criteria methods in order to aid engineering designers (by means of a logical framework that was used also in Alenia Spazio, 2004 and Norese et al., 2008).

An aircraft is a complex system, but it can also be seen as a component of an even more complex structure, a System of Systems (SoS), in which different systems communicate and work together to achieve specific targets. In an SoS, integration and synergic work may vary from a simple collaboration, in which the single components work alone, to a situation in which the single components are not able to work in an autonomous way, when extrapolated by the SoS.

The MBDD supports product development by managing the relationship with the client in the initial engineering design process phase. The clientôs needs have to be identified, in order to decide whether and how a specific legacy aircraft has to be updated in relation to these needs, or to understand what kind of aircraft has to be designed or (at least partially) re-designed, in order to guarantee its integration in the new SoS that the client perceives as possible or essential for the future.

The client's involvement in the initial phase of the design process is analyzed in the literature in "front end" models of the product development process (see, for instance, Smith & Reinertsen,1992; Reinertsen,1999). Some authors have focused on the concept phase of the process where, through the involvement of the client, it is possible to obtain meaningful improvements (Clark & Fuijmoto, 1991) and to resolve ambiguity and uncertainties in the customerôs requirements that may cause orientation difficulties (Smith & Reinertsen,1998).

In aeronautics, a partial and apparently limited re-design requires years of work (five years on average) and therefore uncertainty concerning the evolving nature of the clientôs requirements is normally present, with an evident impact on the engineering design process. The MBDD asked our research group for suggestions and methods, in order to improve the interaction with the client (who has to understand every step of the process and freely propose his point of view), to reduce time and guarantee quality of the results, which can be solutions and/or a better definition of the needs, objectives, priorities and future scenarios of aircraft use.

We proposed the integrated use of two kinds of Operation Research methods, in relation to some case studies, and the cyclic use of the method application outputs, as new inputs for the other method application (Belton & Stewart, 2002). Linear programming (LP) can be used to analytically define the constraints and aspirations of a client, generate the widest set of design alternatives that satisfy the initial requirements (admissible solutions) and calculate optimal solutions, in relation to specific objectives. Multiple Criteria Decision Aid (MCDA)[1] models can be developed and MCDA methods activated iteratively, in order to transparently interact with the client. Alternative solutions, produced by means of the previous LP application, can be analysed and evaluated, in relation to aspects that an LP model cannot include, such as the perception of a risk (of using a too innovative technology, or to generate new complexity in the future maintenance problems, and so on) in relation to a specific solution. Some clientôs requirements can be identified and formalized when a

[1] More details can be found on the Euro Working Group MCDA website: www.cs.put.poznan.pl/ewgmcda/

solution is not compatible with expectations that were not clear enough before the MCDA analysis and therefore not included in the LP model. At this point, new functional and organization limits may be included in the LP model and the consistency of each solution should also be tested for these new constraints, in the new cycle that it started, and so on until an acceptable solution is reached.

The first section of the paper focuses on the iterative nature of the engineering design process and offers a synthetic overview of the methods, theories and tools that are used by designers.

In the second section, the problem, as perceived by the MBDD, is presented and, in the third section, a set covering model is proposed for the generation of design alternatives.

In the fourth section, some multiple criteria approaches are described, in relation to the evaluation of design alternatives, and the integrated use of two methods is proposed to support communication with the clients, in order to better define their needs and expectations. The possible development of the procedure, in relation to more complex projects and decision contexts, is analysed in the conclusions.

Engineering design process

Several theories and various tools are proposed in engineering design to aid designers in different ways: to understand stakeholdersô needs, improve quality, address variability and uncertainty in the design process or generate alternatives for designers.

The engineering design process, as described by Eggert (2005), is structured in five steps: definition of the problem, gathering pertinent information, generating multiple solutions, analyzing and selecting a solution and, finally, testing and implementing the solution. A procedure of identifying and formally listing the customerôs requirements is usually present in problem definition, in order to define product functions and features. These activities are included in the first step of the described process, but in some cases problem definition is complicated and can be completed only when pertinent information is gathered. And generating and analyzing multiple solutions, with the involvement of the client and some areas of the enterprise, is a way to obtain relevant information on the product design and functional specifications.

Once at least the structural components of the design have been identified, above all with inputs from testing, manufacturing and marketing teams, the design team generates alternative conceptual solutions that are oriented in different ways to achieve predefined goals (i.e. requirements that have to be satisfied).

Considering costs, quality and risk, as the main selection criteria, the most promising alternatives are selected for a further analysis (Dean & Unal, 1992), which enables a complete study to be made of the solutions and elaboration of the final design specifications that best fit the requirements. A prototype is therefore constructed and functional tests are performed to verify and, when necessary, to modify the design.

In the conceptual phase of the design process, it may be necessary to go back to a previous step at any point in the process. The chosen solution may prove to be unworkable for different reasons and may require specification redefinition, new solution generation, the collection of more information or, in the worst situation, redefining the problem. This is a continuous and iterative process.

Several tools are commonly used to aid designers. Methodologies and theories that have been proposed in the literature, usually offer a more analytically rigorous support for engineering designers. *Concurrent engineering* may be the most practical methodology to improve the design process. The approaches that are most frequently suggested to obtain input from stakeholders in the design process are the Pugh Method (Pugh, 1990), Quality Function Deployment (Akao, 1997) and the Analytical Hierarchy Process (Saaty 1980; 1994), which always incorporate subjective judgments. Others are used to generate alternatives for designers, such as TRIZ (Altshuller, 1988) and the C-K Theory (Hatchuel & Weil, 2009).

Problem Statement

The problem definition step, in the engineering design process, is critical when the client has to face an evolving situation and cannot clearly communicate needs that are no well defined. This criticality is often present in aeronautics, where many years are required to create a new aircraft, but also to innovate some elements of a legacy system.

The problem definition step is developed in the MBDD by anticipating some activities that pertain to the successful steps of the process (as described in Eggert, 2005) and using them to acquire essential, but latent or fragmented, knowledge elements.

Clearly understanding the point of view of the client, at a functional level but also in organization terms, is essential to identify and structure the requirements that orient the design. The MBDD arrives at a complete problem definition through a procedure that involves the organization of a client in a comparative analysis of some promising draft solutions. These solutions are elaborated in the MBDD, in relation to general technical requirements, and then the strength and weakness elements of the solutions are discussed with the client or, more precisely, with some organization-client key actors (for example, a pilot or whoever is in charge of maintenance).

Even if the innovation is related to a single aircraft component, the future use of the aircraft in an integrated SoS has to be analyzed. Various types of aircraft, but also satellites and maritime or ground systems can be involved, in order to achieve an assigned target in missions of various kinds (i.e. military, civil or a combination of the two situations). Innovation is often required in order to specifically facilitate coordinated work and communication in the SoS.

The MBDD procedure includes two subsequent stages: in the first one, some ñfunctionally acceptableò solutions are identified or elaborated, in relation to the functionalities that are required. In the second stage, the clientôs attention is focused on these solutions, in order to evaluate the associated costs (which are not only monetary), their economic sustainability and specific benefits and risks, as proposed in (Office of Aerospace Studies, 2002). This analysis orients the elaboration of a better solution for the client, but at the same time defines the overall problem and identifies pertinent information and/or information sources. A representation of how the cyclic procedure develops is proposed in Figure 1,with indications on the main activities that are included.

In the last few years, some clients have required the use of an Operations Research tool, in order to facilitate comparisons of the solutions in a multiple criteria analysis. Having found the tool very interesting, the MBDD asked to our research group for a method to help the generation of ñinterestingò and acceptable solutions, in order to reduce time and guarantee the completeness of the acceptable solution set. We analyzed their use of the tool and the weak and strong points of their applications. We then proposed the integrated use of LP and MCDA models in a procedure that fits the MBDD approach to the problem, but also improves the interaction with the client, who can propose his point of view (in terms of limits of the solutions and opportunities that have to be stressed), in a simple but formal language, and who can almost immediately analyze all the new solutions that are consistent with the new vision.

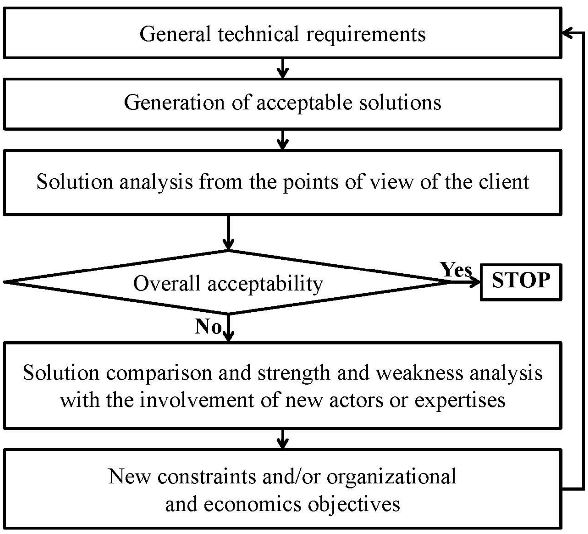

Figure 1: Cyclic procedure

Linear programming application

The request of a client can be very generic and the real needs are not always easy to understand. In order to reduce uncertainty, a request can be expressed in terms of mission types that the new system (or asset) has to face, as a component of an SoS.

From a technical point of view, an asset is a system that guarantees specific functionalities. The assets may be component parts of a single aircraft that have to be integrated to complete a mission, or to be integrated with other assets in other aircraft or in systems that operate on the ground. The assets may also be specific kinds of aircraft (or other resources) that have to be activated together in a specific mission. In all these situations, the integrated assets can be seen as an SoS and both the performances of the assets and the relationships between them allow the missions to be performed.

The purpose of the analysis is to define a mathematical model in which the variables are the different assets that can be activated to accomplish a mission. The functionalities that have to be guaranteed (or guaranteed at a required level), in relation to the ñnatureò of the mission, can become the constraints of the model.

The different objectives, in relation to a specific decisional problem, can be: minimize the costs, maximize the effectiveness, minimize the risks of a mission and so on. A combination of assets that is acceptable because it guarantees the Required Level of Performance (RLP) for each functionality (i.e. for each constraint of the model) becomes an admissible solution, which is called architecture (of the SoS). The optimal solution is an admissible solution that minimizes (or maximizes) the objective. If a single admissible solution does not exist, the need for a technological innovation (i.e. at least a partially new asset) is underlined. A new product, or an improvement in a legacy system, satisfies the clientôs needs if all the missions that the client had proposed to describe his needs can be faced with a minimum cost.

The problem can be represented by a linear programming model, if all the constraints and the objectives are linear functions. If there is only one objective, the oldest and most famous method of Operations Research, the Simplex method (Dantzig, 1963), can be used to obtain the optimal solution. If there is more than one objective, the multi-objective linear programming methods (Ehrgott & Wiecek, 2005) can be used.

At the start of the model setting, the assets that have to be included in the model and a list of functionalities, i.e. the constraints of the model, are defined in relation to the (generic or specific) request of the client and above all using the Universal Join Task List (UJTL) Report[2].

A complete list of about 720 functionalities, in terms of ability to perform a task, is proposed in the UJTL Report, in relation to the strategic, operational and tactical level of mission in a military context. The UJTL was developed for the U.S. Armed Forces, but it has been used by several other countries and international military organizations, such as NATO. The MBDD has structured and adapted the Report to facilitate its use with the clients. The MBDD synthesizes all the coordination, monitoring and controlling functionalities for military missions in the Mission Management macro functionality and Find-Fix-Track is the code that is used to indicate the set of functionalities which, at different levels of detail, allow the area of interest to be patrolled, in order to indentify and trace the target. Using this framework, **xx** main functionalities, that have to be guaranteed in a military mission, are always present as model constraints. When the mission requires a specific and not usual functionality or for non military missions, the UJTL Report is used directly as a check list.

If the adopted objective is to minimize the number of assets that have to be involved in the proposed missions, the mathematical problem can be re-formulated in terms of a *set covering* problem, which consists in finding the

[2] Report available on the www.dtic.mil website

minimum number of service centers (in our model, the assets) so that the request for each service (the guarantee of a required level of a specific functionality) is covered (Tadei & Della Croce, 2001).

In this mathematical model, the performance p_{ij} of the j-asset for the i-functionality is compared with S_i, the RLP that has to be guaranteed for the i-functionality, in order to define the covering matrix $[t_{ij}]$, in which the elements t_{ij} are equal to 1, if $p_{ij} \geq S_i$, or equal to 0 otherwise.

The *set covering* problem can be formulated in the following way:

$$Min \sum x_j \qquad j = 1,\ldots, m$$
$$\sum t_{ij} x_j \geq q_i \qquad i = 1,\ldots, n \qquad x_j=\{0,1\}$$

where x_j has a value of 1 when the asset is included in the solution (which in this case, is an SoS architecture), otherwise it is equal to 0.

The value of the redundancy, for each functionality with redundancy (i.e. a critical functionality that requires more than one asset that is able to satisfy this task, in an SoS architecture), is equal to q_i. For the others, q_i is equal to 1.

We used Xpress-MP, version 2007 (Mosel 2.0.0, IVE 1.18.01, Optimizer 18.00.01), produced by DASH Optimization, to treat models with a single mission or multi scheduled missions that are included in the model. For a multi missions model with 18 variables and 210 constraints, the application has provided six admissible and three optimal solutions in 0.15 seconds.

The model structure and the linear programming application to the problem were tested in relation to some previous military cases, where the solutions and their characteristics were well known for the MBDD. We spent a great deal of time defining and modifying the constraints, in order to have a better fit of some specific requirements, but the immediate calculation of the solutions facilitated convergence towards a good model. The same procedure was then applied to a new case, in relation to the surveillance of a critical sea canal. The model development and PL application were accepted by the MBDD as effective steps of a procedure that can support communication with the client.

At this point attention was focused on the tool that should be used to understand why a solution is not adequate enough.

Multiple criteria approaches

The U.S. Air Force Center of Expertise for Analyses of Alternatives (Office of Aerospace Studies, 2002) suggested a multiple criteria approach in which all the aspects that are related to the effectiveness have to be analyzed and then synthesized in an overall judgment, in a transparent way. The different costs (which are not necessary monetary) of each solution have to be identified and synthesized in an overall cost. Every solution can be graphically shown in a two axe diagram (see Figure 2) where, as is natural, the most effective solution is also the most expensive. One or more acceptability thresholds can be introduced to facilitate a decision that is not easy to make.

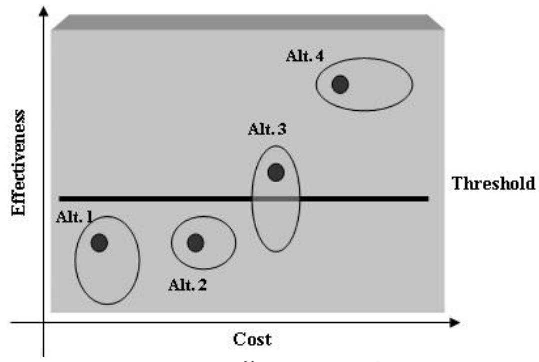

Figure 2: Cost/effectiveness analysis

The MBDD adopted a different approach when a client suggested the use of a multiple criteria method for the comparison of the alternatives. The first application was not totally satisfactory. Some other experiments allowed the MBDD to realize that the correct approach has to involve structuring the evaluation model in macro aspects that can be analysed separately by the organisation actors in charge of each specific aspect. At the same time, the MBDD elaborated a way of translating each personal judgment into an analytical function.

When we analysed the procedure they were using, we noticed that the results were very interesting, in relation to the first aim (improve communication with the client in order to understand his point of view and adequately model his requirements), but very poor as far as the second (transparently arrive at the decision and analytically document the decision motivations) was concerned. In fact, their analytical functions did not result to be consistent with either the original judgements or with the logical structure of the method. At the same time, their need to artificially construct evaluation functions, after the application of the method and in order to explicitly document the process, was analysed together with them, to show them that the wrong method had been adopted.

There are many multiple criteria methods that can be used to aid decision making (see for instance, Belton & Stewart, 2002). It is necessary to choose among the various methods in relation to the specific requests of the decisional problem. In this case, they essentially are: transparency of the process that elaborates a conceptual solution; an objective way of expressing each evaluation and, finally, a treatment of the uncertainty that affects data and judgments.

We proposed the analysis of two methods, Analytic Hierarchy Process (AHP) (Saaty, 1980; 1994) and ELECTRE III (Roy, 1978; 1990), which were tested in relation to the examined case studies. AHP is the method that is already used by the MBDD with its SW tool, Expert Choice, which supports model structuring by means of an easily visualization of the organizational aspects, scenarios, problem dimensions and model criteria. A sensitivity analysis facilitates the identification of model weakness elements and the consequent improvement or re-structuring of the problem and/or the multiple criteria model. An analytical evaluation of the solutions is not required. Comparative judgments are used both to assess the solutions and calculate weights for the compensatory synthesis procedure.

In the ELECTRE III method, unlike AHP, the alternatives have to be evaluated in relation to all the criteria in an explicit and (as much as possible) objective way. Thresholds are introduced when uncertainty is present in some evaluations, to limit the negative effect of the uncertainty on the results. Criteria can have different degrees of importance and, in this case, coefficients of relative importance of the criteria have to be introduced.

ELECTRE III starts by comparing each solution with each of the other solutions. A fuzzy outranking relation, based on the two principles of concordance and discordance, is modelled in phase I of the method through the computation of a concordance index, a discordance index and an outranking degree. The method uses the latter result in the second fuzzy relation exploitation phase, in order to construct two complete pre-orders through a descending and an ascending distillation procedure. Outranking relation modelling offers some interesting advantages, in comparison to other multiple criteria methods: each criterion can use a different ordinal or cardinal scale, since a unique specific scale (such as the cost-benefit analysis monetary scale or the 0-1 utility scale of the multi attribute utility theory) is not necessary and the outranking relation is not compensatory (or partially not compensatory).

A weak point of ELECTRE III is its software package, which does not pay any attention to dialogue with the decision maker, which is essential in model structuring and parameter definition and when the results require a collective analysis. A new product, which is more suitable and includes several multiple criteria methods, is currently being developed in the Decision Deck project[3]. This weak point is related to the original nature of the method which was invented to be used when a problem was well structured, i.e. when:

- a set of solutions is identified, or elaborated, and tested in terms of completeness, admissibility and comparability, and
- a family of evaluation functions (i.e. criteria) which has been created to represent all the different aspects of the problem at hand contains a sufficiently small number of criteria to be a basis for discussion (legibility condition) and to be considered by all the actors as a sound basis for the continuation of the decision aid study; its coherence (exhaustiveness, cohesiveness and redundancy) has to be verified by operational tests (Roy & Bouyssou, 1993; Roy, 1996).

For this reason, ELECTRE III is not normally used until the problem (and/or the model) is structured and only when these conditions are satisfied does it become a powerful method to transparently compare solutions, in relation to all the different criteria, and to rigorously synthesize evaluations that are associated to the consequences of each decision.

Therefore, our proposal was: the AHP would be used in the problem definition step, when pertinent information has to be identified together with the client, by means analysis and selection of conceptual solutions, while the ELECTRE III would be used at the end, in relation to the defined problem, when a decision has to activate the subsequent design process phases.

In the examined cases, the aircraft and the other systems were under production, or at least in the final phases of the production process, and the nature of the missions was clear, since the MBDD knows the decision context very well. Therefore, the principal aspects of the evaluation problem were easily identified (SoS performance in relation to the operational scenarios of the missions, technical effectiveness in relation to the operational management process and life cycle costs) and their disaggregation into organizational and functional-economic components was visualized through the SW Expert Choice and its multilevel decision tree.

In order to support interaction with the client, three models were elaborated during some simulation sessions in the MBDD: an AHP-Expert Choice model that is sufficiently general to be used in different decision situations, with a decision tree that is articulated in five levels and twenty-six elementary components for the comparison of the solutions, and two models for ELECTRE III, with twelve criteria for the first case study and fifteen criteria for the second one. Different decisional scenarios were hypothesized, in order to analyze which impact could have on the result the importance that the criteria assumed.

The AHP-Expert Choice model and the results of some applications were then analyzed to understand their potentiality to facilitate communication between the MBDD and the different clients. The ELECTRE III results were examined in terms of robustness and reliability, and the models in terms of formal validity and consistency with the internal procedures of the company.

Conclusions

A client's involvement in the initial phase of an engineering design process is always important and has to be carefully managed. The temporal horizon to produce an innovation in the aeronautic sector always involves a difficult definition of the clientøs needs and some risks in translating the needs into formal requirements. The analysis and comparison of some draft solutions is an effective approach to understand the clientøs point of view and the general structure of his/her preference system. However, this approach requires time to elaborate understandable technical solutions, analyse them with the client and elaborate new solutions for a new collective analysis, in a learning cycle.

Complexity and uncertainty elements can have a negative impact on the problem definition in some decision situations, above all when different, and sometimes conflicting, points of view require the involvement of some specific competences, from the clientøs organization, as a not easy, but almost obligatory course of action.

A structured procedure can support the acquisition of the different points of view and their translation into mathematical models and then into product requirements, and can prevent, or at least control, ambiguous specifications by an activity that has the aim of verifying the overall consistency of the models.

The opportunity to produce conceptual solutions in a short time (a solution requires only few seconds of

[3] www.decision-deck.org

calculation time), with the guarantee of technical acceptability and specific performance levels in relation to an objective, makes communication possible and effective in the engineering design process.

Mathematical models that use an intelligible language introduce a positive psychological effect, in terms of clear thinking structure and perception of the logical progress. At the same time they facilitate the traceability of the process steps and results.

The integrated use of linear programming and multiple criteria methods can make the active collaboration phase with the client more rigorous (no acceptable solutions are lost and the evaluations can be documented and used consistently) and efficient, because all the structured and partially structured indications can be introduced into the models and transformed, by means of the methods, into information for the decision process.

The MBDD is planning to test the new approach with its clients and our group will be involved in analysing the criticalities and opportunities.

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