Fuzzy Approach to Landslide Susceptibility Zonation

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Abstract. The paper addresses a landslide-prone area on Fruška Gora Mt. in NW Serbia. It proposes a model of relative landslide susceptibility based on fuzzy sets. Having a variety of spatial attributes (proven statistically significant) at disposal, as well as present landslide inventory map, we conducted systematic analysis through (i) assigning fuzzy memberships to attribute categories, and (ii) combining the memberships by means of fuzzy operators. The performance defined by Area Under Curve parameter of the Receiver Operating Characteristics curve, led to preference of Frequency Ratio method for assigning memberships, and Fuzzy Gamma Operator for combining those memberships in 2-level experimenting configuration. Results are also well related with previous investigations with different approaches.

1 Introduction

Landslides and alike mass movements are one of the most widespread hazardous phenomena [1]. They seem to be among the top seven natural hazards, and advancing [19] in the world of growing needs for urbanization, land exploitation, and yet unstable climate conditions. Accordingly, there has been a significant ascent of interest in landslide assessment topics, resulting in more frequent multidisciplinary case studies and rising number of scholars per investigation [11].

Common notion of landslide hazard is broadly misinterpreted in relation to its conventional definition, which regards the hazard quantitatively as a function of frequency of hazardous phenomena over specified area or volume [23]. Nevertheless, even such precise scientific formulation is not entirely straightforward, since literal hazard assessment appears to be feasible only for the limited areas with excellent data coverage [4]. Entire range of problems is encountered in this framework, including the input data quality, lack of evidence on previous occurrences or triggering events, lack of consistent evaluation of the modeling results [4]. Therefore, most of the studies actually address landslide susceptibility as non-temporal variant of the landslide hazard, which evaluates the landsliding potential in the relative scale.

Practice of landslide zonation had been illustrated in versatile techniques in various case studies, yielding more or less reliable results depending on the complexity of the terrain and suitability of the approach [5]. Thereto, the principle assumption imply

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that future landslide occurrences stand in relation with the present ones [3], while central – multi-criteria modeling idea couples different input thematic data (geological, geomorphological, environmental maps), relate them to the referent map of present landslides, and processes a single output – hazard/susceptibility map. Techniques of relating referent landslide map with the inputs are numerous: heuristic (expert-based), deterministic (physically-based), statistical and probabilistic, artificial intelligence based (neural networks, decision trees, machine learning algorithms, data mining), fuzzy logic based, and so forth. All those equally face the non-linearity of the problem, and strong dependence on the referent landslide data, the entire input data feature space, for that matter.

Weather using ordinary fuzzy sets, or fuzzy measures, or even combining fuzzy with other statistical or classification approaches (Dampster-Shafer, K-means, Neural Networks) the ultimate advantage is seen in logics, which provides a substantial possibility for standardization of the analysis under the consideration [12]. Thus, the procedure tends to be repeatable, adjustable and reliable. When it comes to the landslide assessment analysis in particular, a number of researchers have applied fuzzy approach to handle the non-linearity, which is common in multi-criteria framework. Interestingly enough, Himalayan terrains were addressed in many investigations with fuzzy theory background, starting from standard fuzzy set approach [15], [21], [6], through combinations of neural-fuzzy [13] and risk-oriented fuzzy approach [14]. Most of these studies agreed that plausible susceptibility models could be obtained by applying advanced operators, with preference toward Cosine Amplitude method for obtaining memberships. Very similar conclusions with analogue methodology were inferred over Iranian case studies [22], and in Turkey [7], China [24] and so forth. The latter is also interesting in respect of harmonizing expert-based and fuzzy-driven solutions, inferring that one does not exclude another, but supports it. Finally, Regmi et al. [20] conducted one of the most consistent researches, where many different fuzzy configurations were put to test. Detailed elaboration of the choice of fuzzy operator type, optimal fitting of gamma operator as a method of preference, and some suggestions on handling multi-type landslide cases, can be found in this research. In addition, most of the researchers encourage the usage of the fuzzy method in other, similar or entirely different ambients, worldwide.

Herein we will concentrate on fuzzy logic approach, and compare results with some of earlier works that involved heuristic, statistical and machine learning techniques over the same area, using similar datasets. Thus, the primary objective is to investigate whether the fuzzy logic approach enhances the susceptibility model and to which extent. Optimization of the procedure, in accordance with the characteristics of the dataset, was also one of the foci, in order to reach the best performance of the model.

Organization of the paper goes as follows: in Chapter 2, a brief overview of all implemented techniques is presented; Chapter 3 follows with very basic description of study area; data acquisition and preparation is regarded in Chapter 4; results of susceptibility model and comparative analysis are presented and discussed in Chapter 5; Chapter 6. concludes the paper. Appendix 1 contains very detailed modeling parameters of attributes, which were used in the procedure.

All parametric calculations, were performed in MS Excel sheets, and spatial attributes were prepared and visualized with ArcGIS 9+ packages (ranged, calculated, cross-tabulated etc.), as well as final models.

2 Methods

2.1 Feature Selection

It is recommendable to filter the set for features which are of no relevance to the analysis, if nothing, for the sake of hardware and time expenditure, thus the filtering procedure is not to be overlooked. In parlance of the latter, a statistical significance tests needs to be run prior to the dataset utilization.

Chi-Squared statistic, parameter X^2, is a significance criterion, which relates the frequencies of observed independent variable instances φ_o within the dependent variable classes, and their expected frequencies φ_e, in the following fashion:

$$X^2 = \sum_{i=1}^{q} \sum_{j=1}^{n} \frac{\left(\phi_{o_{i,j}} - \phi_{e_{i,j}}\right)^2}{\phi_{e_{i,j}}} , \qquad \textbf{(0)}$$

where q is the number of classes within a dependent variable, and n within the independent variable. In our case, the former represents landslide inventory classes, while the latter disclose the classes of a particular terrain attribute, since X^2 needs to be paired with every single attribute separately. The given terrain attribute disapproves the hypothesis of being statistically independent from the landslide inventory classes only if it exceeds the critical X^2 threshold, defined by the level of confidence (in respect with the normal distribution) and degrees of freedom (defined by reduced product of q and n, $(q-1)(n-1)$). In effect, this method reveals the relation of an attribute and the referent landslide inventory, but the ranking among multiple attributes is rather relative, primarily due to the measurement scale dependence of X^2 [2].

2.2 Fuzzy Set Theory

Concepts of fuzzy logic have a very long tradition in spatial analysis framework. Main purpose of fuzzy logic is to deal with vague information and with data that contain some kind of uncertainty [25]. When using fuzzy set theory or fuzzy logic, each object or statement is given value from interval <0,1> indicating its membership to the given set. Each object can be member of several sets with different membership values. This concept is very helpful for categorization of data and for decision making, because unlike Boolean logic it produces results valid with specific degree of truth. That helps with finding not only the perfect match for a given criteria, but rather shows how much each of possibilities meet given criteria. At some specific situations, when modeling physical geographical crisp sets, Boolean logic fail to provide correct and quality results because of natural substance of the phenomena at hand. In such cases, fuzzy set theory and fuzzy logic provides solutions for dealing with imprecise and vague data, which would be hard or even impossible to process by any other means.

2.3 Fuzzy Memberships

Membership value is determined by membership function. Membership function is a function that maps all given elements to interval of values <0,1>.

$$\mu_A : U \to \langle 0,1 \rangle \,, \qquad \textbf{(0)}$$

where μ_A is a membership function, U is a set of elements. Then for each $x \in U$, $\mu_A(x)$ is membership value of the element x to the set A [25]. For purpose of this paper, we use two functions for computing the fuzzy membership values: Frequency Ratio and Cosine Amplitude.

Frequency Ratio gives proportion of landslide cells in the specific category for each of input layers. It can be described as ratio of relative frequency of landslide cells in a category (an attribute class) to the relative frequency of all landslide cells in the area:

$$\mathrm{FR} = \frac{N_{\mathrm{cell}}(L_i)/\mathrm{N}_{\mathrm{cell}}(C_i)}{N_{\mathrm{cell}}(L)/\mathrm{N}_{\mathrm{cell}}(C)} \,, \qquad \textbf{(0)}$$

Where $N_{cell}(L_i)$ is the number of landslide cells in the category i, $N_{cell}(C_i)$ is the total number of cells in the category i, $N_{cell}(L)$ is total number of landslide cells and $N_{cell}(C)$ is the total number of cells. If the result is higher than 1 it shows higher density of landslide cells in the category then overall in the dataset. Results lower than 1, points to categories that have density of landslide cells lower then density in the dataset. To transform FR to membership values those outputs have to be normalized by dividing each FR by maximal FR in the given group of classes. Then the membership values are from the interval <0,1> and the higher the number is, the higher is the influence of this category on landslide occurrence.

Another method for determining the membership values of categories to the set of categories important for landslide occurrence is Cosine Amplitude method:

$$\mathrm{CA} = \frac{N_{\mathrm{cell}}(L_i)}{\sqrt{N_{\mathrm{cell}}(C_i) \cdot N_{\mathrm{cell}}(L)}} \,. \qquad \textbf{(0)}$$

In this case, the membership value is calculated as ratio between number of landslide cells in the category and the square root of its product with the total number of landslide pixels in the dataset. Unlike FR the output values do not have to be normalized because they already fall in interval <0,1>.

2.4 Fuzzy Operators

Several fuzzy operators exist for combining membership functions. Best-known operators are AND and OR, but both of them suffer with problem that one of combined sets have significant impact on result of such combination while the other sets do not have such influence. In case of operator AND minimum of all values is the one that defines output and in case of OR operator it is the maximum value. Because of this reasons we use other operators such as Fuzzy Algebraic Product, Fuzzy Algebraic Sum, Gamma Operation and Weighted Average. All of them are described in detail [2] so only short review is given here.

In Fuzzy Algebraic Product and Fuzzy Algebraic Sum the outputs are defined as:

$$\mu_{\text{product}} = \prod_{i=1}^{n} \mu_i \ , \qquad \textbf{(0)}$$

$$\mu_{\text{sum}} = 1 - \prod_{i=1}^{n} (1 - \mu_i) \ , \qquad \textbf{(0)}$$

respectively, where n is number of membership function to be combined and μ_A is the i-th membership function. Fuzzy Algebraic Product tends to produce output function lower or equal to the lowest function given, while Fuzzy Algebraic Sum is complementary to the former, so it provides output function higher than all the inputs but never higher than 1.

Gamma Operation is defined by:

$$\mu_\gamma = (\mu_{\text{sum}})^\gamma \cdot (\mu_{\text{product}})^{1-\gamma} \ . \qquad \textbf{(0)}$$

The exponent γ, which is a number from <0,1> interval, allows optimization of the membership combination. Setting it to the extremes of the interval give either Fuzzy Algebraic Sum (γ=1) or Fuzzy Algebraic Product (γ=0).

Weighted Average is defined as:

$$\mu_w = \frac{\sum_{i=1}^{n} w_i \cdot \mu_i}{\sum_{i=1}^{n} w_i} \ , \qquad \textbf{(0)}$$

where w_i is a weight of membership function, indicating importance of the membership function on result and n is a number of membership functions to be combined. Weight system in this equation allows more interaction from user to the calculation, because it allows emphasis of certain values.

2.5 Performance Evaluation

Performance metrics involved Receiver Operating Characteristics (ROC), which is a cut-off independent performance estimator [9]. It involves contingency table inspection (derived by area cross-tabulations of attribute vs. landslide inventory). ROC values are created by plotting the cumulative True Positive Rates (TPR=sensitivity) versus False Positive Rate (FPR=1–specificity) for every model, resulting in a set of ROC curves. The performance is evaluated by the Area Under the Curve (AUC) relative to the entire plot area, so that an AUC equal to 1 has the best performance, while an AUC as low as 0.5 results in a very poor performance [10]. In addition, TPR is a good measurement of performance in the landslide assessment framework, since it takes into account instances that are not classified as landslides in the model but actually are landslides, which is more dangerous underestimation than false alarms.

3 Case Study

The study area encompasses the NW slopes of the Fruška Gora Mountain, in the vicinity of Novi Sad, Serbia. The site (N 45°09'20", E 19°32'34" – N 45°12'25", E 19°37'46") spreads over approximately 100 km^2 of hilly landscape, but with interesting dynamics and an abundance of landslide occurrences. As judged in some previous investigations over this area [16], [17,], [18], the landslide process is chiefly governed by geological and morphological attributes, while the triggering mechanism could be assigned to excessive rainfall, but moderate seismic activity typical for this mountain, could also be an option.

4 Dataset

Dataset included geological, geo-morphometric, hydrological and environmental attributes, obtained from different resources, converted to raster grid format with 30 m cell resolution. It also included landslide inventory map.

- Geological data were assembled by using geological map 1 : 50 000, photo-geological map (Remote Sensing based interpretation of geological structures and geodynamic processes and forms) 1 : 50 000, and field survey data. For the purpose of this research, a segment of geological map was digitized and simplified to *geo-unit* attribute. Geological structures, which were used to make a *buffer geo-structures* were extracted from photogeological map. In addition, the *buffer geo-boundaries* was created by choosing only the boundaries between the units with significant difference in hydrogeological function.
- Model of the terrain surface was created from digitized contour maps at 1 : 25 000, first by calculating Triangulated Irregular Network (TIN) and then substituting it with the Digital Elevation Model (DEM) of 30 m resolution,

by means of TIN-to-raster data conversion. Given the terrain morphology, various geo-morphometric attributes were created as first order derivates of DEM: *aspect*, *elevation*, *slope angle*, *slope length*, *profile* and *planar curvature*.

- Hydrological attributes are represented by *topographic wetness index* (*TWI*) as the second order derivate of DEM, and *buffer stream* calculated after automatic generation of drainage pattern using DEM.
- *Land cover*, as an environmental attribute, was desirable in order to delineate deforested and cultivated areas as more convenient for the development of landslides than vegetated areas. The attribute was created by Landsat TM band ratioing (particularly red and near infrared bands, due to the authentic spectral behavior of vegetation). Several vegetation indices were considered, and Normalized Difference Vegetation Index (NDVI) seemed like the optimal solution, due to its simplicity and accuracy. Since the area of the interest is not very populated, urban influences were not considered. Classification of NDVI into land cover categories was semi-supervised, i.e. visual, but aided by K-means classification to four different entities (Appendix 1).
- *Landslide inventory map* was essential requirement to make a susceptibility assessment evaluated for performance. The map was created by extracting landslide forms from photogeological map. Subsequently, it was simplified to binary attribute (TRUE and FALSE landslide categories). It is important to mention constrains of such map, since it considered only earth slides [23] of rotational, translational and complex type, with two stages of the activity (dormant and active). This is understandable regarding the scale of the study (1 : 50 000) and the nature of the dominating landslide phenomena within the area of interest. According to this binary map, total of 10% of the area fall into landslide category (about 10 km^2).

Apparently, dataset involved continual numeric data, but categorical attributes as well. The methodological approach required ranging of continual attributes to categorical data, prior to their processing, and several solutions were regarded. Finally, ranging by means of Natural break cut-offs was the method of choice, which was applied to all continual attributes. Different continual attributes were ranged by appropriate number of intervals (Appendix 1), due to differences in pixel frequencies among attributes. In favor of selected approach of preparing the data, feature selection parameter proved that all attributes had statistical dependence to referent landslide inventory, having the values significantly higher than critical (Appendix 1).

5 Results and Discussion

Given the categorized (ranged) raster attributes and the referent landslide inventory map, we first calculated the memberships of each category in each attribute. Two parallel variants of the experiment were driven: EXPERIMENT 1 used Cosine Amplitude, while EXPERIMENT 2 used Frequency Ratio to obtain the memberships.

Both experiments had exactly the same course, thus the following manipulations took place in each.

5.1 Susceptibility Model

In order to combine memberships by different operators we undertook a small intervention to exclude too many extreme membership values (0 and 1) by replacing them with close approximations (0.0001 and 0.9999). We proposed 2-level fuzzy combinations based on *a priory* knowledge of the phenomena (Fig. 1), i.e. the pairs of attributes of similar origin were grouped together. Continual Susceptibility Model was obtained after the second level combination. The final susceptibility model was generated by ranging the continual values into five standard categories of relative susceptibility: Very Low – VL, Low – L, Moderate – M, High – H, Very High - VH [8]. Regarding the distribution of the pixels in Continual Susceptibility Model, it was justifiable to adopt the quantile interval cut-offs for afore mentioned categorization. Only the highest susceptibility class VH was regarded for performance evaluation (AUC) against the referent landslide inventory (Table 1). This was instructed by the fact that determined landslides should be marked as a priority zone (preferably as VH class).

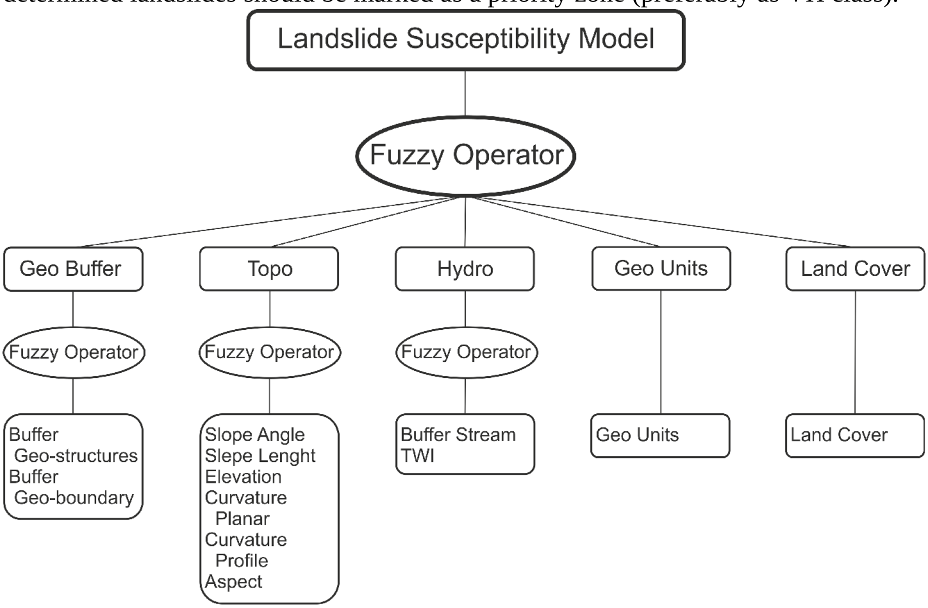

Fig. 1. Flowchart of the experiment configuration.

To remain consistent, we kept the same type of the operator at both combination levels. Initial results in both experiments gave preference to Fuzzy Gamma Operator, so we directed further fitting toward optimization of parameter γ. Cases of γ=0 (Fuzzy Product) and γ=1 (Fuzzy Sum) were already regarded, so we tested several choices within that interval (0.25, 0.5, 0.75). It turned that the best performance (AUC) was achieved by γ=0.5, making it a parameter of choice for our final susceptibility model. Finally, EXPERIMENT 2 gave slightly better performance over EXPERIMENT 1, meaning that Frequency Ratio could be preferred over Cosine Amplitude for assigning memberships.

Table 1. Performance evaluation of different fuzzy experiments configurations (1-Cosine Amplitude, 2-Frequency Ratio memberships), and other landslide susceptibility models (shaded)

Model	AUC	TPR
EXPERIMENT 1 (Weighted Average)	0.65	0.37
EXPERIMENT 1 (Gamma Operator, γ=0.5)	0.70	0.53
EXPERIMENT 2 (Weighted Average)	0.71	0.56
EXPERIMENT 2 (Gamma Operator, γ=0.5)	0.72	0.58
AHP	0.67	0.48
CP	0.72	0.60
SVM	0.85	0.77

Distribution of relative susceptibility classes goes as follows: VL – 53%, L – 14%, M – 12%, H – 11%, and VH – 10%. Dominance of the VL class characterizes the terrain as mostly stable, while similarly as in the referent inventory map, the most adverse zones occupy about 10% of the area. Furthermore, a majority of the actual landslide instances fall into the VH and H classes (37% and 23% of all landslides, respectively), while M, L and VL classes occupy mostly non-landslide instances (75% of non-landslide instances in total for all three classes).

Highest overall performance in EXPERIMENT 2 (AUC=0.72) could be acknowledged as plausible, which is also supported visually (Fig. 2a-b), since VH class corresponds very well with the spatial trends of landslide scarps. Apparent influence of intermediate layer *Geo Buffer* caused several outliers by underestimating some landslide scarps. A considerable drawback is relatively low TPR in both experiments (Table 1) which is inconvenient for any hazard-related analysis, since the model tends to underestimate actual landslide instances (claiming class other than VH for an actual landslide instance). However, the actual performance is somewhat better, since we regarded only VH class for cross-tabulation. Thus, H or even M class could be fair replacements for VH class, as they buffer-out around it, which if included in cross-tabulation might reduce the number of False Negatives, thus increasing TPR.

5.2 Comparison

In order to determine the true practicality of our results, we related proposed model to other available results including: Analytical Hierarchy Process (AHP) model [16], Conditional Probability (CP) model [18], and machine learning with Support Vector Machine (SVM) model [17]. When comparing the best fuzzy-based result with the other models the same policy of comparing only VH class holds, due to compatibility issue. Namely, some of the comparison models, such as SVM, are discrete in their nature and cannot follow (standardized) relative susceptibility categorization (VL–VH).

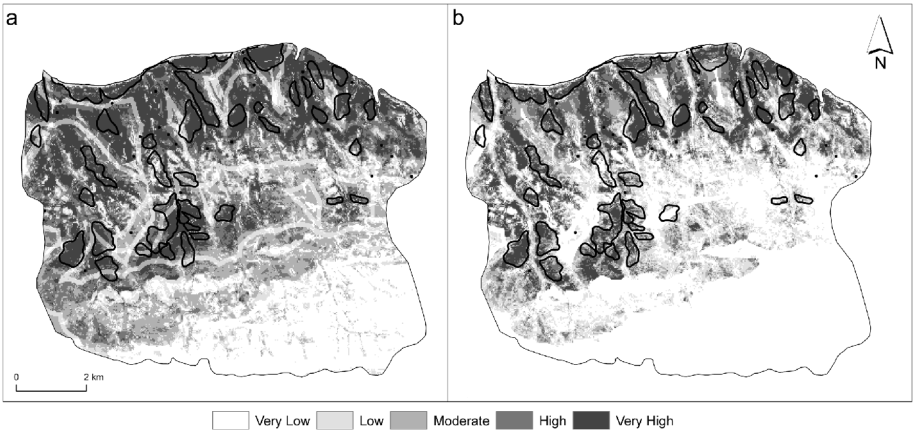

Fig. 2. Landslide susceptibility models based on EXPERIMENT 1 (CA, γ=0.5) **a)**, and EXPERIMENT 2 (FR, γ=0.5) **b)**. Bold contours outline the landslide scarps from the landslide inventory. Legend depicts relative susceptibility classes.

Expectedly, SVM approach outperformed fuzzy-based models by far (Table 1). Ease of handling continual and categorical data most likely enables such dominance of SVM model over other results. On the other hand, fuzzy approach turned practically as successful as statistical one (CP model), but with more subjectivity involved in the modeling procedure (in ranging the input intervals, but also in selecting the operators and numbers of combination levels). It outperformed AHP model, not as much in the overall performance (AUC) as in considerably higher TPR, giving itself a slight preference for safer assessment (Table 1).

6 Conclusion

In present paper, we regarded fuzzy set approach in the landslide susceptibility framework, having different input attributes and referent landslide inventory at disposal. Subjectivity in ranging input attributes was inevitable, due to incapability of the approach to handle continual numerical variables (in the stage of assigning memberships). Another subjective intervention regarded proposing the number of levels for fuzzy combination, and grouping the attributes with similar origin at level 1. We proposed two configurations of generating memberships of input attribute categories, EXPERIMENT 1 (CA) and EXPERIMENT 2 (FR), and led further optimization toward the choice of fuzzy operators for combination task. The best performance was reached with Fuzzy Gamma Operator with γ=0.5. The resulting Landslide Susceptibility Model turns plausible, and seems improved when compared to some previous models designed for the same study area, particularly heuristic one.

Further refinement, left for the future work, should involve combining of fuzzy approach with some other techniques. The latter primarily address merging with heuristic expert decisions, while fuzzyfication in machine learning approach is also to be challenged. Another improvement could be recognized in reducing the subjectivity in experiment design, and configure the experiment structure on statistical basis or information theory basis.

To conclude, our research came up with suitable model, while the procedure remained simple, semi-automated and re-operable in GIS environment. The resulting map could serve preliminary levels of risk or disaster management, landscape (regional) planning, route selection, insurance management and so forth.

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Apendix 1– Table of Attributes

Input attributes, their class memberships in EXPERIMENT 1 configuration (μ_{FR}) and EXPERIMENT 2 configuration (μ_{CA}), and their statistical dependence (X^2) on landslide inventory (dependent variable)

attribute name (type, group) categories	μ_{FR}	μ_{CA}	X^2 ($X^2_{critical}$)
buffer geo-structure (continual, geo-buffer)			1949.6
0 - 134	0.051	0.781	(27.9)
134 - 276	0.092	0.770	
276 - 426	0.141	0.805	
426 - 582	0.260	1	
582 - 755	0.261	0.812	
755 - 942	0.178	0.445	
942 - 1159	0.024	0.077	
1159 - 1418	0	0	
1418 - 1758	0.262	0.197	
1758 – 2305 m	1	0.568	
buffer geo-boundary (continual, geo-buffer)			306.3
0 - 94	0.275	1	(27.9)
94 - 218	0.038	0.630	
218 - 342	0.107	0.499	
342 - 458	0	0.372	
458 - 589	0.040	0.295	
589 - 726	0.484	0.429	
726 - 878	0.579	0.313	
878 - 1050	1	0.332	
1050 - 1244	0.458	0.105	
1244 – 1749 m	0.682	0	
buffer stream (continual, hydro)			2381.6
0 - 94	0.550	0.643	(27.9)
94 - 212	0.900	1	
212 - 324	0.780	0.809	
324 - 432	0.453	0.431	
432 - 543	0.346	0.305	
543 - 660	0.218	0.165	
660 - 797	0	0	
797 - 966	0.024	0.002	
966 - 1173	0.362	0.100	
1173 – 1542 m	1	0.214	
TWI (continual, hydro)			4947,6
7.5 - 9.3	0	0	(26.1)
9.3 - 10.3	0.172	0.261	
10.3 - 11.4	0.696	1	
11.4 - 12.8	1	0.955	
12.8 - 14.3	0.811	0.575	
14.3 - 16.2	0.821	0.442	
16.2 - 18.3	0.781	0.320	
18.3 - 20.8	0.506	0.176	
20.8 - 22.5	0.131	0.079	

aspect (categorical, topo)			1091.0
flat	0	0	(26.1)
N	0.594	0.701	
NE	0.552	0.688	
E	1	1	
SE	0.889	0.490	
S	0.278	0.140	
SW	0.645	0.638	
W	0.494	0.571	
NW	0.414	0.433	
elevation (continual, topo)			7515.7
78 - 102	0.660	0.619	(27.9)
102 - 138	1	1	
138 - 173	0.828	0.838	
173 - 209	0.530	0.499	
209 - 248	0.158	0.147	
248 - 287	0.141	0.118	
287 - 329	0.018	0.013	
329 - 376	0	0	
376 - 426	0	0	
426 – 540 m	0	0	
slope angle (continual, topo)			4453.1
0 - 4.2	0.300	0.243	(18.5)
4.2 - 9.5	1	1	
9.5 - 14.8	0.473	0.403	
14.8 - 21.1	0.119	0.086	
21.1 - 40.1º	0	0	
slope length (continual, topo)			1346.8
0 - 60	0.435	1	(27.9)
60 - 181	0.591	0.964	
181 - 353	0.937	0.960	
353 - 602	1	0.667	
602 - 981	0.650	0.261	
981 - 1506	0.301	0.080	
1506 - 2196	0.178	0.033	
2196 - 3094	0.427	0.061	
3094 - 4392	0.187	0.019	
4392 – 6499 m	0	0	
plan curvature (continual, topo)			989.4
concave	0	0	(18.5)
-	0.657	0.333	
flat	1	1	
-	0.626	0.419	
convex	0.149	0.059	
profile curvature (continual, topo)			1214.0
concave	0	0.009	(18.5)
-	0.414	0.287	
flat	1	1	
-	0.741	0.366	
convex	0.081	0	
geo-units (categorical, geo-units)			8319.3
al' - Danube's inundation plane	0.100	0.099	(29.6)
al - aluvium	0.211		
dl - deluvium cover	0.807	1	
t - terrace sediments	1	0.785	

l - loess	0.338	0.334	
Pl - clay	0.858	0.847	
M_2 - marlstone	0.133	0.083	
M_1 - limestone, sandstone	0.469	0.880	
Se - ultra-mafic rocks	0	0	
J - limestone	0	0	
Pz - schists	0.002	0.003	
land cover (categorical, land cover)			6316.2 (16.2)
water	0	0	
arable land	1	1	
grass land	0.992	0.852	
forest	0.132	0.168	