Total Variation Regularization Method for 3D Rotational Coronary Angiography

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Abstract. 3D rotational coronary angiography plays an important role in the field of diagnosis and treatment planning of coronary artery disease. Due to the cardiac motion, only limited number of projections can be used to reconstruct coronary arteries for each heart phase, which makes the reconstruction problem ill-posed. To reduce the under-sampling artifacts, we apply an iterative method that makes use of total variation regularization. Some different reconstruction algorithms are compared and our method outperforms the others in the experiments.

1 Introduction

Coronary X-ray angiography is a very important imaging method in the field of diagnosis and treatment planning of coronary artery disease. 3D image information offers great advantage for quantitative analysis of vessel properties. Moreover, the successive 3D reconstructions can be used to determine the temporal dynamics of the arteries [1].

The projection data for 3D reconstruction of the coronary arteries reconstruction is acquired on an X-ray C-arm system. Simultaneously, the electrocardiogram (ECG) is recorded. After the ECG gating only few number of projections are available for each cardiac phase, which leads to severe angular under-sampling and renders the reconstruction problem ill-posed. In addition, several heart beats occur during the data acquisition which causes the data inconsistency. As a result, the standard reconstruction methods like filtered back projection yield unsatisfactory results with many artifacts. One way of tackling this problem is to first estimate the motion of the arteries and then perform a motion-compensated reconstruction. But it is still a challenging problem to get an accurate motion model, especially for the non-periodic case [2].

According to the theory of compressed sensing [3, 4], one can solve the ill-posed problem by first finding a sparse representation for the images and then applying the L1 norm minimization method in the transformed domain. Pan's group [5] adopted total variation as the sparsifying transform for reconstruction of static objects. Their method can reduce the under-sampling artifacts but they did not investigate the performance of the algorithm for moving objects, e.g. coronary arteries. We apply the total variation regularization method for 3D rotational coronary angiography. The scheme appears to be robust against both under-sampling artifacts and motion artifacts.

2 Materials and Methods

In tomography, the goal is to reconstruct an object from line-integral projection data. A discrete version of the projection process can be represented as

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{1}$$

where

$$\boldsymbol{A} = (a_{ij})$$

is a real m_n system matrix representing the projection operator, $\boldsymbol{x} = (x_1, ..., x_n)$ is a real vector representing the object, and $\boldsymbol{b} = (b_1, ..., b_m)$ is the corresponding projection data. Then the optimization problem can be described as

$$\min_{\boldsymbol{x}} ||\boldsymbol{x}||_{TV} \ s.t.\ ||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_2^2 < \alpha \tag{2}$$

The inequality constrain is used to describe the data inconsistency which can come from many factors, including the heart motion, system noise, X-ray scatting. Thus, it is impossible to always find an image that is perfectly consistency with the data. As a result, we only require that the image yields the projection data that are within the L2 distance of the actual projection data. $||\cdot||_{TV}$ is the total variation norm, which is the L1 norm of the image gradient [6]. It is well known that the constrained problem (2) can be transformed to an easier unconstrained optimization problem

$$\min_{\boldsymbol{x}} ||\boldsymbol{x}||_{TV} + \beta||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_2^2 \tag{3}$$

The unconstrained problem (3) is still hard to solve due to the high dimensions. The size of the system matrix $\boldsymbol{A}$ is usually very large. Large memory should be used to store the matrix and a lot of time is needed for computation. To overcome these problems, we apply the forward backward splitting method [6]. The objective function of (3) consists of two convex functions. The idea of the forward backward splitting method is to optimize the two parts of the objective function individually. The algorithm can be described as

- *Step 1*: Do N_{ART} iteration steps of the standard ART.
- *Step 2*: Do N_{TV} iteration steps of the gradient descent update for minimizing $\min_{\boldsymbol{x}} \mu||\boldsymbol{x}||_{TV} + ||\boldsymbol{x} - \boldsymbol{v}||_2^2$, ($\boldsymbol{v}$ is calculated from step 1)
- *Step 3*: Repeat step1 and step 2 until $||\boldsymbol{x}^{(t)} - \boldsymbol{x}^{(t+1)}||_2^2$ is less than a certain value or the maximum iteration number is reached.

In order to do a reproducible scientific research, we adopted a dataset with periodic cardiac motion from CAVAREV [7] in our experiments. The dataset can be downloaded for free. CAVAREV offers an evaluation method but does not provide the ground truth. Thus classical evaluation schemes like MSE (mean squared error) can not be used to evaluate the results. But since the main goal of C-arm CT imaging of highly contrasted cardiac vasculature is to find the size and location of vessels, the evaluation method offered by CAVAREV seems to be more suitable. The method is based on the spatial overlap of the vasculature reconstruction with the ground truth. The Dice similarity coefficient (DSC) is calculated at each projection image with different parameter for the quality assessment [7]. The measure for the reconstruction is the max value of DSC at all projection images. The DSC value ranges from 0 to 1. The value 0 stands for no spatial overlap while the value 1 stands for a perfect match. To further evaluate, we compared our method to some other reconstruction algorithms: standard ART, ECG-gated FDK, PICCS [8] and L1 minimization methods [9]. In the experiments, we set the gating window to 0.06 s that only 15 projection images were used to do the reconstruction. N_{ART} and N_{TV} was 4 and 10 respectively. μ was chosen as 0.005 and the maximum iteration number was 200. The parameters for the other methods were set as in the referenced paper.

3 Results

The reconstructed transaxial slices from different methods are listed in Fig. 1. The max DSC values of the reconstructions from different methods at different heart phases are in Table 1. Due to the angular undersampling, the reconstructions from standard ART and ECG-gated FDK include many streak artifacts. PICCS and L1 minimization method reduce the artifacts dramatically. The streak artifacts are nearly invisible in the results of TVR. Table 1 shows that TVR outperforms the other methods at all three different heart phases, since the max DSC value of TVR is larger than the one of the others.

4 Discussion

The PICCS, L1 minimization and TVR are optimization based reconstruction methods. The differences between those methods are the regularization terms. From the view of compressed sensing, the regularization terms can be seen as a sparsifying transform. The algorithms just apply the L1 minimization method in different domains. A more sparse representation can reduce the number of unknowns (more coefficients are zero or very small.), making the ill-posed problem easier to solve. For coronary arteries, the total variation norm can give a more sparse representation than L1 norm. In the experiments, the reconstruction results from ECG-gated FDK are used as the prior image which contain many streak artifacts. Thus PICCS may not offer a more sparse representation than total variation norm. As total variation norm gives the most sparse representation for coronary arteries in the experiments, our method outperforms the others. Since a more sparse representation of the image can increase the reconstruction quality. Some other representations (wavelet, DCT) will be investigated.

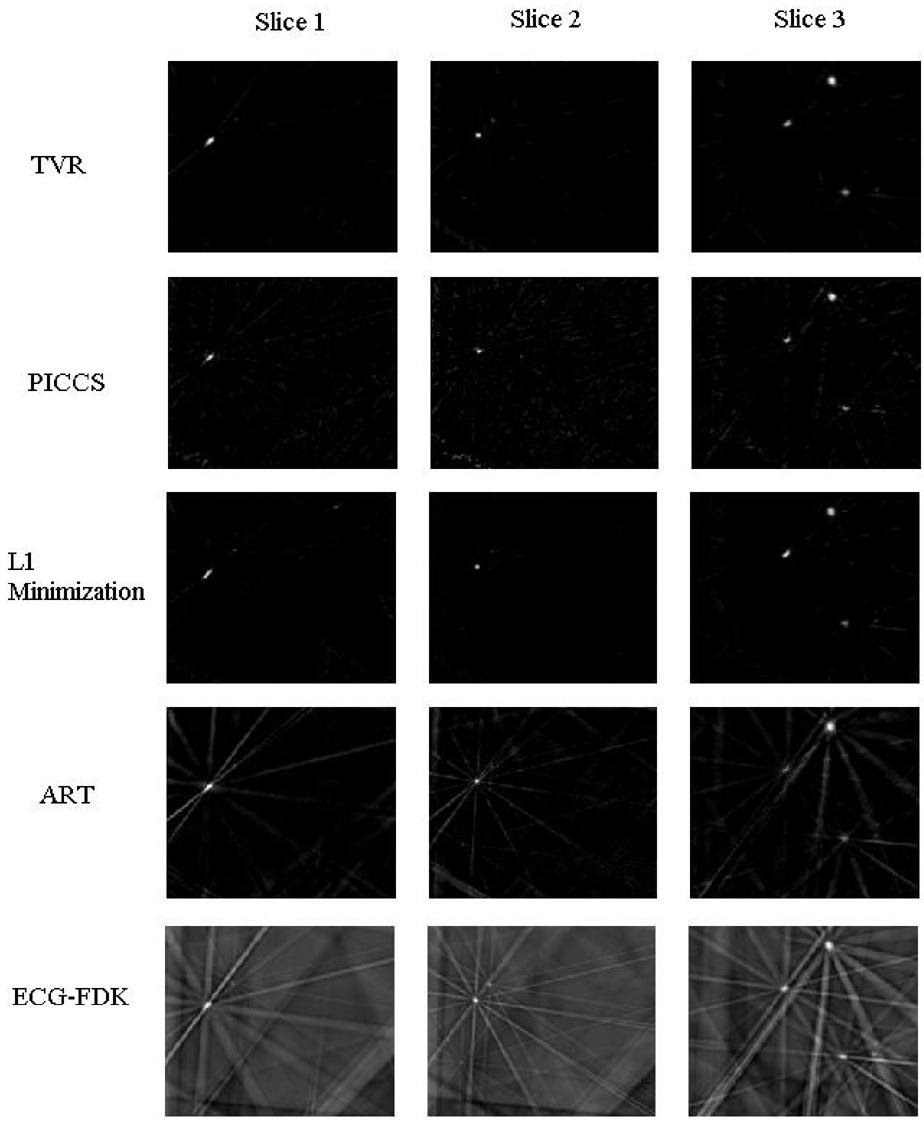

Fig. 1. Reconstructed transaxial slices from different methods. The streak artifacts are nearly invisible in the slices from TVR.

Table 1. Max DSC values of different reconstruction methods at different heart phases. High DSC value indicates a high overlap between the ground truth and the reconstruction result.

Heart Phase	0%	40%	90%
TVR	0.721	0.772	0.785
PICCS	0.595	0.613	0.726
ART	0.510	0.545	0.554
L1 Minimization	0.684	0.723	0.730
ECG-gated FDK	0.484	0.534	0.555

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