Dynamics of haze in hierarchical systems: mathematical approach to the social system description

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ABSTRACT: *Dynamics of haze on a base of multilevel hierarchical system theory (MHSR) was investigated. The hierarchical haze is out of the boundary of the world hierarchical pyramid. Two approximations are proposed. We can not know the full law or connection of the higher strata so only some influence of this higher strata to lower strata can be studied. State of hierarchical hazy is a state of dynamic equilibrium without the hard boundary. The lower strata can be obtain the mathematical image of the sway (higher strata). If this image is complete the hazy vanishes. It is the same as our system increases the its own level.*

Keywords: Social system evolution, trajectory, generalized space, control, optimization

INTRODUCTION

The modern state of the theory of social phenomena can not be considered as satisfactory. From any completed system of knowledge should be followed the opportunities of positive conclusions and forecasting of the development the systems objects. Unfortunately, the modern social theories do not allow to make of serious enough and argued of a prediction for the future behaviour of systems. This imperfection of the theories of social systems is realized enough for a long time (Jackson 1985). Recent events around the world indicate complete insolvency of the traditional view on the social world state and evolution. The new reality demands a new approach to the social process evolution and control.

The classical "descriptive" approach does not satisfy the main idea of quantitative description. This approach only analyzes the past situation. But now we need another description, which can give us the prediction, the control parameters and characteristic of the critical points (Kile 1995, Groumpos et all. 1997, Miklachevich 1998. 1999a, 1999b).

We believe it is necessary to search for prospect on ways of consecutive development hard systems methodology (Checkland) and methodology of system engineering.

In this way the bases problem consists of the possibility that processes and social groups evolution of mathematical description are amenable. Another point of view states that all social groups include indescribable elements ("the spirit"). The main argument on the first point of view is existence of some good examples of successful description of social state and interaction like economics or demography.

Starting from this supposition we can hope that all social groups can be mathematically described and we can give quantitative prognosis.

Present work is devoted to introduction of concept of space of condition and trajectory of development concerning social systems.

BASE IDEAS AND BACKGROUNDS

Actually there are some difficulties with the mathematical approach to social problems. The traditional sociological variables are not very good for the quantitative description because the sociological variables do not satisfy condition of the theory of measurement (Miklashevich 2000). It is possible to hope that application of physical methods will help to solve the main problem of determination and controlling of the social and biological systems evolution. According to our breakdown the problem of dynamics of hierarchical hazy in the social system development has a two aspects.

Two ways of approximation

The first aspect is connected with multilevel hierarchical system theory. The multilevel hierarchical system theory (MHST) is developed on the base of general system theory (Bertalanffy et all 1977, Mesarovich et all 1970, Mesarovich and Takahara Y. 1975). We are studying the social and business development at present paper as a tightly bounded processes which are interdependent. According to the MHST interactions between all systems are divided into two big classes: interaction of control and interaction of collaboration. These two interactions correspond to interactions between systems which are situated at different levels and interactions between systems which are situated at the same level. All systems together are constructing the hierarchical world. This hierarchical world has the graphics images (Novikava et all. 1990, 1991, 1995b, Buka 1997). The mathematical images of large scale systems are well known [Chestnut (1995), Kile (1995), Novikava et all (1995a, 1997)].The sample of that kind of hierarchical world is the hierarchical State model. All strata have diverse characteristics in concrete States statutes (symbol images) which must be connected with their history and with sway strategies in their space. The states are changing diverse details of their own constructions on all strata and these interactions are the base of unions of state. Till now the process of world changing is realized without actual understanding of its laws and since that with hard errors within the States and their alliances. The general theory of new world construction can be build on the base of Aed theory.

Aed theory (A^{λ} mathematics) now has two own main hierarchical symbol images ${}^{x}\alpha^{\lambda}$ and ${}^{+}\alpha^{\lambda}$ which answer to acts of multiplying (learning) and uniting (design). They contain the new means of control and connect the strata (directions) of A^{λ}. Aed strata are: Λ, λ - level (time), Γ, γ - statute (law, connection), P, ρ - act (process), Ω, ω - unit (state) Σ, σ - construction (contents), B, β - new time (arising level), A, α - sway (coordinator).

As a object of Aed theory ((Novikava et all. 1990, 1991, 1995a, 1995b) the acts of original unit A^{λ} is multiplying and its symbol images uniting leads to the arising of the new time unit A^{β}.

$$A^{\beta}\overset{\gamma}{\underset{\rho}{\leftrightarrow}}\left\{ \begin{matrix} ? & \gamma & \beta \\ \omega & A & \sigma \\ & \rho & \end{matrix} \right\}^{\overset{A^{\beta}}{\beta \to ?}}_{\rho} \tag{1}$$

In this way all aed strata can renovate its original unit A^{λ}, they have all their signs and abilities, ↔ is the correspondence relation. (1) contains the «?» hazy symbol in its image (this is signed by the symbol «?» and they will be defined in time β) .

Unit of the arising times A^{β} can not be defined in the present time λ without the mathematical expression of higher level in lower level.

The second aspect is connected with meaning that the system development in our space is equivalent the motion of the point in the hierarchical space according to the general principles of point dynamics in the generalised space (Bremermann 1972, Miklashevich 2000). Trajectory of motion can be defined (Shapovalov, Evdokimov 1998) as a geodesic in the generalised N – dimension space.

$$\frac{d^2 p^j}{dt^2} + \Gamma^j_{kl}(p)\dot{p}^k \dot{p}^l = 0 \tag{2}$$

where Γ^j_{kl} are the coefficients of connection. All time we have summation by dummy index.

$$\Gamma^j_{kl}(p) = \frac{1}{2} g^{js}\left(\frac{\partial g_{ks}}{\partial p^l} + \frac{\partial g_{ls}}{\partial p^k} - \frac{\partial g_{kl}}{\partial p^s} \right) \tag{3}$$

g^{js} are the contravariants components of the metric tensor in the coordinates p_i. According to the standard procedure of the geometry the metric tensor determined all geometrical parameters in generalised space.

Active and passive approximation

According to our breakdown of the problem to the two aspects the problem of dynamics of hazy has two equivalent expressions:

- To determine the law, acting in the system (it is equivalence to the sway determination).
- To determine the geometry of the space of state in which the system is moving.

The first problem is equivalent to the active point of view. The system interacts with the media, and includes interaction between different hierarchical strata.

The second problem is equivalent to the passive point of view. We interpret the system as a unit mass in the space of state and the trajectory of evolution is formatted by the geometry of external space.

From active point of view the lower strata interprets the higher strata as a hazy systems. Because the sway (coordinator) situated on the higher level has his own tasks. The system situated on the lower strata can not understand the full meaning of these tasks and sway activity without the full mathematical image of the sway. For the reduction of hazy the system needs on the full expression of uniting operator ${}^{+}\rho^{\lambda}$

$$ {}^{+}\rho^{\lambda} \oplus A^{\lambda} \leftrightarrow A^{\beta} \tag{4}$$

$$ {}^{+}\rho^{\lambda} \oplus \Lambda^{\lambda} \leftrightarrow \Lambda^{\beta} \tag{5}$$

The operation of ${}^{+}\rho^{\lambda}$ is equal to the new level transition or ordinary unit connection. Thus the rate of hazy is tighty connected to the fullness of the operator expression.

From the passive point of view trajectory of evolution (3) is described by the nonlinear differential equation of second order. We can study this equation on stability or stochastic regime according to the standard procedure (Lichtenderg, Liberman 1983). Appearance of stochastic regime in the geodesic (3) (it is the same as the trajectory of evolution) is controlled by the value Γ^{i}_{kl} and defined from the metric g^{js}

GEOMETRY OF THE SPACE OF STATE

Because we investigate social systems as a open physical system the thermodynamics of open system must be a basis for the introducing of system variables. In this way we obtain

$$ g^{ij} = \frac{\partial p^{i}}{\partial z^{k}} \frac{\partial p^{j}}{\partial z^{k}} \tag{6}$$

where z^{k} is the basic vector in information representation. Consequently quantity of information for each variables is the important value. From point of view of mathematics the metric tensor determines the optimal trajectory of development. The optimal trajectory coincides with the geodesic line.

INFORMATION OF THE SYSTEM

As an example of social systems let's take an ethnos. There is a problem to select the major parameters to describe the ethnos's vital activity and viability. Regrettably, it is impossible to introduce the features from motivated physical models. We shall characterise the ethnos by number of ethnos's members *N(t)*, The value of this magnitude (thousands of members) allows us to consider the ethnos as statistical system. $L_i(t)$ is length of life *i*-th ethnos's member, $Q_i(t,t)$ is intellectual coefficient of members. Below the index *i* a number of current individual (member ethnos), *t* is the time. Time *t* has a global sense. It is time of existence of ethnos, $0 < t < 2000 \div 3000$ years. The parameter τ has sense of local time or time of existence of i - *th* member of ethnos $0 < \tau < 80 \div 90$.

We will introduce some definition for the next exposition. The factor of reproduction of information ζ is

$$ \zeta = \frac{I_e}{I_a} , \tag{7}$$

where I_e is input information, I_a is output information. Then we have three cases:

$$ \zeta < 1, \tag{8}$$

$$ \zeta = 1, \tag{9}$$

$$ \zeta > 1. \tag{10}$$

The case (8) is corresponding to destruction of information in the process of the ethnos activity. In our conception this corresponds to die down of ethnos. The case (10) is the case of conservation of information, stage homeostasis of ethnos, the equation (9) giving birth of ethnos, stage of development.

The ethnos and the society are strongly non-linear when considered as statistical systems, and the obtained result can not be a simple sum of external influence. Regrettably, we can't build the correct non-linear theory of ethnos by the first step and therefore research this system in linear approximation. Then we present total ethnos information quantity as two component sum

$$ I(t) = S_e(t) + S_i(t), $$

where S_e is information, defined by external environments for each ethnos, S_i is information, defined by internal characteristics of ethnos. It is possible to consider $S_e = const$ for all ethnoses when considering comparative ethnogeny without the restriction of generality. This is equivalent to propose that possible evolution is defined solely by internal features of ethnos.

Function of intellectual development

Function of intellectual development, connected with the ethnos information quantity, can be presented as

$$\Lambda_i = \Phi(\pi)A(\tau,t) = \Lambda_i(\pi,\tau,t) \tag{11}$$

where π is a value IQ, $A(\tau,t)$ is the age function, describing changes of the intellectual possibilities of person on stretch of local time and dependency of average length of life from a global time.

Value IQ is measured by methods, sufficiently good developed and motivated in psychology. It is shown that for the representative selection the density of distribution of IQ is given by normal (Gauss) distribution

$$\varphi(\pi) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\pi-\xi}{\sigma}\right)^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\pi-70}{\sigma}\right)^2} \tag{12}$$

In (12) ξ=70 is the mean of the value IQ, the dispersion of distribution σ is defined from standard reference conditions, that in the range $\xi \pm \sigma$ must get nearly half of population. The studies show that $\sigma \approx 25$. From the modern conditions it is known that flow of the distribution and the values ξ, σ are independent from that, where the test is conducted - or amongst educated inhabitants of Europe, or amongst uneducated Africans. So consider $\pi \neq \pi(t)$ For the age functions at condition $t = 0$ we can suppose that

$$A(\tau, t = const) = A(\tau) = \frac{25\ln(0{,}1x)}{x} \tag{13}$$

System information quantity

We can define a volume quantity information $I(t)$, which possesses ethnos. In our approximation

$$I(t) = I\left(S_i^c(t,\tau), S_i^n(t)\right) \tag{14}$$

is the internal information of the ethnos, where $S_i^c(t,\tau)$ is constructive information, i. e information necessary separate representative of ethnos for to function as part of the ethnos. $S_i^n(t)$ is nonconstructive information, required by function of separate object as biological essence (level of the instinct ensuring vital activity.). Regrettably, it is not always possible to conduct clear difference between types of information. For example, experience of collective processing of a land in the arid regions expects a survival of separate individual, but is realised by community of society.

Probably, the most universal possible relationship can be installed in the following type in the case of isolated approximation

$$I(t,\tau) = \sum_{k,l=1}^{N} V_{kl}(t) S_k^c(t,\tau) S_l^n(t) \tag{15}$$

where V_{kl} is matrix of interaction coefficients (in general case that factor present to functions a time), describing internal structure of the society and interaction between members of the society (ethnos). We consider the ideal case of an isolated ethnos, i.e. when its evolution is almost independent of other ethnoses. The summation draw in (15) obviously occurs on all members of the ethnos.

Structure matrix of interaction coefficients is going to be researched of the next paper.

If constructive and nonconstructive parts of information becomes weacker bound, then it is possible to expect that matrix of coefficients V_{kl} to come coupled. In first approximation this case corresponds to belief about independence of individual evolution from the social background. In this case we obtain

$$I_i(t,\tau) = \sum_{i=1}^{N} \left(\xi_{ik} S_k^c(t,\tau) + \zeta_{il} S_l^n(t) \right) \tag{16}$$

$I_i(t)$ is the partial information, i.e. quantity information, corresponding to i-th person. We accept that dependence on τ is enclosed in the dependence on the global time. $\xi_{ik}(\tau)$ is a function having sense of interaction of person and society. For stable societies (normal sociogenesis) this function is obviously periodic with slowly changing amplitude. The value $\xi_{ii}(\tau)$ will make sense selfinteraction (own evaluation, complacency or noncomplacency and etc.). ζ_{ij} is efficiency coefficient to social organisations.

The constructive part of the ethnos's information presents for us the main interest. Regrettably, we can not define the dynamics of changing this parameter from the experiment, because it is required too much time for the experiment. Constructive part of ethnos information, connected with individual characteristics of ethnos's members, must depend not only on individual characteristics of person but also on direction of its main interests (mentality) too

$$S_k^c(t,\tau) = \Lambda_i(\pi,\tau,t)\mathbf{e}_{ik} \qquad (17)$$

$\mathbf{e}_{ik}$ is the direction interests operator.

As far as number of components of functions (16) is big (practically value $I_i(t)$ possible considers as a two parametrical continuous functions), it can be quantified on the index i. It is introduced vector of state of ethnos

$$\vec{q}(t) = \frac{\nu_i}{N} I_i(t) \qquad (18)$$

where ν_i is partial share of people of certain age. Step of quantum can be selected in accordance to the necessary degree of discrete.

Share ν_i changes in the course of time according to historical and demographic data. Total sum by all share in the local time section is 1. It is proportional to the full number of members of ethnos. If we suppose that ethnogeny is isomorphous, i.e. development of social generalities look like one to one, then instead of undertaking a global permanent experiment on the determination of dynamics constructive information in the framework of one ethnos it is possibly to find several ethnoses, in which distribution of sharing ν_i complies with chosen as a test systems. Fitting together simultaneous data in one vector of condition, one possibly receipts a rule of evolution of condition vector for one ethnos.

CONCLUSION

The offered method allows to describe such complex and open objects as social system by means of the description of interaction of individual elements. For the study of received regularities it is possible to use hereinafter standard mathematical methods of analysis system dynamics. Each of elements should have function of distribution of characteristic parameters. The full set of variables must be determined from complex of other sociological, biological and other data. Thus the concept n-dimensional of space of condition is entered. In this space the system is represented as a point, and its (her) evolution is a line (trajectory) in this space. Depending on a choice of a choice of the basic characteristics we can formulate various criteria of stability. Depending on the strategic purposes we can require(demand) stability in small or global stability. As the example is considered introduction of partial system of co-ordinates for ethnoses and its evolution (process of ethnogeny) is considered. Thus the hierarchical hazy for this systems can be vanished.

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REFERENCES

Bertalanffy, von L., Beier W. and Lane R. (1977) *Biophysik des Fließgleichgewichts,* Akademie Verlag, Berlin.

Bremermann, H.J. (1972) *On the Dynamics and the Trajectories of Evolution Processes*. Wien- Heidelberg-N.Y.

Buka, P, Konash A., Siargeichyk A., et al. (1997) The Constructing of New Electronic Device with Aed Processor. *Preprints of Eleventh International Conference on Mathematical and Computer Modelling and Scientific Computing.* Washington, DC, USA. 1997. p.47.

Chestnut, H. (1995) Improving International Stability and Maintaining Peace. *Preprints of the IFAC Conference on Supplementary Ways for Improving International Stability,* SWIIS'95, Vienna, Austria, 1995, p.11-14.

Groumpos, P., et all. (1997). Design & Creating of New Engeneering Units in Reconstructing Regions,- *Preprints of Advanced Summer Institute'97, ICIMS - NOE*, Budapest. Hungary.

Jackson, M.C. (1985). Social systems theory and practice: the need for a critical approach. *Int. Jurn. General Systems* **10**, pp.135-151.

Kile, F. (1995) Desired peace. *Preprints of the IFAC Conference on Supplementary Ways for Improving International Stability*, SWIIS'95, p. 147-152.

Lichtenberg, A.J. and Liberman M.A.(1984) *Regular and stochastic motion.* Springer, New York, 1983.

Mesarovic, M.D., Masko D. and Takahara Y. (1970) *Theory of Hierarchical Multilevel Systems.* New York and London: Academic Press.

Mesarovic, M.D. and Takahara, Y. (1975) *General Systems Theory: Mathematical Foundation.* New York and London: Academic Press.

Miklachevich, I.A. (1998) *Mathematical description of ethnogenesis. 1. About isomorphism of evolution.* Trudy BGTU, Issue VI, Ser IV, p.64-69 (in Russian)

Miklachevich, I. (1999a) National idea and strategical plannyng of ethnogenesis. *1 International conference "Belarusian national idea"*, Grodno, Collection of abstracts pp. 44-45.

Miklachevich, I. (1999b) The scale of rank of evolutionary structure in the social processes and its mathematical description . *VII Int. Conference "Mathematics, Economics, Ecology. Education."*, Rostov, Collection of abstracts p. 175-176.

Miklachevich, I. (2000) Modeling and optimal control in ethnosocial systems. *18Th Int. System Dynamics Conference "Sustanaibility in the Third Millenium"* Bergen, Norway, August 6-10.

Novikava, S., Ananich, G. and Miatliuk, K. (1990) The Structure and the Dynamics of Information in Design Systems. *Proceedings of 7th Conference on Engineering Design*, ICED'90, Dubrovnik, **2**, pp.946-953,.

Novikava, S., Miatliuk, K. et all (1991) Aed Technology for Ecological, Social and Engineering Systems Coordination. *Proceedings of 8th International Symposium on Modular Information Computer Systems and Networks*, ICS-NET'91, Dubna, Russia, pp.145-152.

Novikava, S., et al. (1993) Hierarchical Multilevel Systems in Aed Realization. *Proceedings of 9th International Conference on Mathematical and Computer Modelling,* ICMCM'93, p.71, Berkeley, USA.

Novikava, S., Miatliuk, K., Gancharova, S., Kaliada V. (1995a) Aed Construction and Technology in Design. *Proceedings of the 7th IFAC Symposium on Large Scale Systems: Theory and Applications* LSS'95, London, UK, pp.379-381.

Novikava, S., Gancharova, S., Daronin, S., et al. (1995b) State Design: New Way in Exact Sciences. *Preprints of the IFAC Conference on Supplementary Ways for Improving International Stability* SWIIS'95, Vienna, Austria, pp.175-181.

Novikava, S., Gancharova S., Zhybul A., et al. (1997) The Statute of Hierarchical Mathematics and Its Cybernatical Maintenance. *Preprints of Eleventh International Conference on Mathematical and Computer Modelling and Scientific Computing.* Washington, DC, USA. 1997. p.149.

Shapovalov, A.V. and Evdokimov, E.V. (1998) Hamiltonian dynamics of Darwin Systems. *Physica D*, **112**, pp.441-450.