

# An Argument for an Analogical Perspective on Rationality & Decision-Making

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**Abstract.** Humans are without any doubts the prototypical example of agents that can hold rational beliefs and can show rational behaviour. When modeling human decision-making, it seems reasonable to take the remarkable abilities of humans into account with respect to rational behaviour, but also the apparent deficiencies of humans shining up in certain rationality tasks. Based on well-known challenges for human rationality, together with results from psychological studies on decision-making and from the field of computational modeling of analogy-making, we argue that analysis and modeling of rational belief and behaviour should also consider cognitive mechanisms like analogy-making and coherence maximization of the background theory.

## 1 Introduction

At times, human behaviour seems erratic and irrational. Still, it is widely undoubted that humans can act rational and, in fact, appear to act rational most of the time. In explaining behaviour, we use terms like beliefs and desires. If an agent's behaviour makes the most sense to us, then we interpret it as a reasonable way to achieve the agent's goals given his beliefs. We take this as indication that some concept of rationality does play a crucial role when describing and explaining humans' behaviour in a large variety of situations.

Based on ideas from vernacular psychology, in many cases rational beliefs are interpreted as a foundation of rational behavior. In this extended position paper, we will be mostly concerned with beliefs and knowledge, i.e. the epistemic aspects of rationality.

In the following, we want to shed light on some aspects of rationality from a mostly computational cognitive science point of view. Although, even in psychology or economics there is no generally accepted formal framework for rationality, we will argue for a model that links rationality to the ability of humans to establish analogical relations. This is an attempt for proposing a new perspective and framework for rationality. Furthermore, in the course of a mostly overview-like presentation, we want to give some hints at how already existing frameworks for computational analogy-making integrate some aspects considered characteristic for human decision making.

## 2 Rationality Concepts and Challenges

### 2.1 Rationality

Many quite distinct frameworks for modeling rationality have been proposed, and an attempt at clustering these frameworks to the best of our knowledge results in at least four classes: logic-based models (cf. e.g. [1]), probability-based models (cf. e.g. [2]), heuristic-based models (cf. e.g. [3]), and game-theoretically based models (cf. e.g. [4]).

Several of these models have been considered for establishing a normative theory of rationality, not only trying to model “rational behaviour”, but also to offer predictive power for determining whether a certain belief, action, or behaviour may be considered rational or not. Also, every of these theories specifies some sort of *definition* of rationality. Unfortunately, when comparing the distinct frameworks, it shows that these definitions are in many cases almost orthogonal to each other (as are the frameworks). Therefore, in this paper, we will propose certain cognitive mechanisms for explaining and specifying rationality in an integrated, more homogeneous way.

### 2.2 Well-Known Challenges

Although the aforementioned frameworks have gained merit in modeling certain aspects of human intelligence, the generality of each such class of frameworks has at the same time been challenged by psychological experiments. For example, in the famous Wason-selection task [5] human subjects fail at a seemingly simple logical task (cf. Table 1). Also, experiments by Byrne on human reasoning with conditionals [6] indicated severe deviations from classical logic (cf. Table 1). Similarly, Tversky and Kahneman’s Linda problem [7] illustrates a striking violation of the rules of probability theory (cf. Table 1). Heuristic approaches to judgment and reasoning [8] are often seen as approximations to a rational ideal and in some cases could work in practice, but often lack formal transparency and explanatory power. Game-based frameworks are questioned due to the lack of a unique concept of optimality in game-theory that can support different “rational behaviors” for one and the same situations (e.g. Pareto optimality vs. Nash equilibrium vs. Hick’s optimality etc., [9]).

**Wason Selection Task:** This task shows that a large majority of subjects are seemingly unable to verify or to falsify a simple logical implication: “If on one side of the card there is a D, then on the other there is the number 3”. In order to check this rule, subjects need to turn D and 7, i.e. subjects need to check the direct rule application and the contrapositive implication. After a slight modification of the content of the rule (content-change), while keeping the structure of the problem isomorphic, subjects perform significantly better: In [11], the authors show that a change of the abstract rule “ $p \rightarrow q$ ” to a well-known problem significantly increases correct answers of subjects. The authors use the rule “If a person is drinking beer, then he must be over 20 years old.” The cards used in the task were “drinking beer”, “drinking coke”, “25 years old”, and “16 years old”. Solving this task according to the rules of classical logic comes down to turning “drinking beer” and “16 years old”.

**Inferences and Conditionals:** Also Byrne’s observations question whether human reasoning can be covered by a classical logic-based framework. Presented with the information given in Table 1, from 1.46% of subjects conclude that Marian will not study

**Wason-Selection Task [10]:**

Subjects are given the rule “Every card which has a D on one side has a 3 on the other side.” and are told that each card has a letter on one side and a number on the other side. Then they are presented with four cards showing respectively D, K, 3, 7, and asked to turn the minimal number of cards to determine the truth of the sentence.

**Inferences and Conditionals [6]:**

1. If Marian has an essay to write, she will study late in the library. She does not have an essay to write.
2. If Marian has an essay to write, she will study late in the library. She has an essay to write.
3. If Marian has an essay to write, she will study late in the library. She has an essay to write. If the library stays open, she will study late in the library.

**Linda-Problem [7]:**

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Linda is a teacher in elementary school.

Linda works in a bookstore and takes Yoga classes.

Linda is active in the feminist movement. (F)

Linda is a psychiatric social worker.

Linda is a member of the League of Women Voters.

Linda is a bank teller. (T)

Linda is an insurance salesperson.

Linda is a bank teller and is active in the feminist movement. (T&F)

**Table 1.** The Wason-selection task questions whether humans reason in such situations according to the laws of classical logic. Byrne’s experiments on how humans handle conditionals also shed doubt on a logic-based model. Tversky and Kahneman’s Linda problem questions the ability of humans to reason according to the laws of probability theory.

late in the library, erring with respect to classical logic (as denial of the antecedent does not validate a negation of the consequent). Also, from 2. 96% of subjects conclude that Marian will study late in the library, whilst only 38% of subjects reach the same conclusion from 3.. Thus an introduction of another antecedent (without any indication that the antecedent should not hold) dramatically reduced the number of subjects applying a simple modus ponens in their process of forming a conclusion.

**Linda Problem:** With respect to the Linda problem it seems to be the case that subjects are amenable to the so-called conjunction fallacy: subjects are told a story specifying a particular profile about the bank teller Linda. Then, eight statements about Linda are shown and subjects are asked to order them according to their probability (cf. Table 1). 85% of subjects decide to rank the eighth statements “Linda is a bank teller and active in the feminist movement” (T & F) as more probable than the sixth statement “Linda is a bank teller” (T). This ranking contradicts basic laws of probability theory, as the joint probability of two events (T & F) is less or at most equal to the probability of each individual event.

**Classical Resolution Strategies:** Strategies that have been proposed to address the mentioned challenges include non-classical logics for modeling subjects' behavior in the Wason-Selection task [12], or a switch from (syntactic) deductions to reasoning in semantic models [13]. Still, these are only individual case-based solutions, which do not (or only hardly) generalize, and thus don't provide a basis for a unified theory or the genesis of a generally accepted broad concept of rationality.

### 3 Non-Standard Interpretations of Challenges for Rationality

An immediate reaction to the challenges for rationality depicted above may be to deny that humans are always able to correctly reason according to the laws of classical logic or the laws of probability theory. Still, concluding that human behaviour therefore is irrational in general does not seem convincing. The most that can be concluded from the experiments is that human agents are neither deduction machines nor probability estimators, but perform their undisputable reasoning capabilities with other means. From our point of view, subjects' behavior in the described tasks is connected to certain cognitive mechanisms that are used by humans in such reasoning tasks, giving rise to the emergence of behavior commonly described as rational.

#### 3.1 Interlude: Analogy and Analogical Reasoning

Analogies can basically be described as claims of similarity, which are often used in argumentation or when explaining complex situations. Putting it more formally, analogy-making refers to the human ability of perceiving dissimilar domains as similar with respect to certain aspects based on shared commonalities in relational structure or appearance. Analogy and analogy-making research has received growing attention during the last decades, changing the perception of analogy from interpreting it as a special and rarely applied case of reasoning to placing it in the center of human cognition itself [14]. The literature on analogies knows a distinction between two subcategories of analogical mapping: attribute mappings (surface mappings) and relational mappings [15]. Whilst both mapping types are standardly assumed to be one-to-one, attribute mappings are based on attributes or surface properties, such as shape or color (i.e., two objects can be said to be similar with respect to a particular attribute or set of attributes), whilst relational mappings are based on relations between objects, such as having the same role or the same effect (i.e., two objects can then be said to be similar with respect to some relation to one or more other objects). Once such an analogical bridge has been established between two domains, analogical reasoning now allows for carrying over inferences from the base to the target domain in order to extend knowledge about the latter, i.e., an inference which holds between elements in the base domain is also assumed to analogically hold between the corresponding elements of the target domain.

#### 3.2 How Analogy-Making Enters the Picture

In a short reply to Colman's article "*Cooperation, psychological game theory, and limitations of rationality in social interaction*" [16], Kokinov challenges traditional views

on rationality [17]. Taking an initial stance similar to Colman's, agreeing on that rationality fails as both, descriptive theory of human-decision making and normative theory for good decision-making, Kokinov reaches a different, more radical conclusion as Colman did before. Instead of trying to fix the concept of rationality by redefining it, adding formerly unconsidered criteria for optimization of some kind, he proposes to replace the concept of rationality as a theory in its own right by a multilevel theory based on cognitive processes involved in decision-making. Where Colman proposes a collection of ad-hoc strategies for explaining the deviations from rationality which people exhibit in their behaviour, Kokinov proposes analogy as means of unifying the different, formerly unconnected parts of Colman's attempt at describing the mechanisms of decision-making. In Kokinov's view, the classical concept of utility making has to be rendered as an emergent property, which will emerge in most, but not all, cases, converting rationality itself in an emergent phenomenon, assigning rational rules the status of approximate explanations of human behavior.

But evidence for a crucial role of analogy in decision-making cannot only be found in conceptual cognitive science, but also in psychological studies on decision-making and choice processes. An overview by Markman and Moreau [18], based on experiments and observations from psychological studies, amongst others on consumer behaviour and political decision-making, reaches the conclusion that there are at least two central ways how analogy-making influences choice processes. Analogies to other domains can provide means of representation for a choice situation, as generally speaking the making of a decision relies on a certain degree of familiarity with the choice setting. In many cases of this kind, analogy plays a crucial role in structuring the representation of the choice situation, and thus may strongly influence the outcome of a decision. Also, structural alignment (a key process of analogy-making) plays a role when comparing the different possible options offered by a decision situation, with new options being learned by comparison to already known ones. An experimental study by Kokinov [19] demonstrated that people actually do use analogies in the process of decision-making, with significant benefit already if only one case is found to be analogous to the choice situation under consideration. Furthermore, evidence has been found that there is no significant difference between close and remote analogies in this process, and that people are not limited to rely only on analogous cases from their own experience, but that also cases which were only witnessed passively (e.g., by being a bystander, or learning about a situation from reports in the media) may have beneficial influence.

Taking all this together, we strongly argue in favor of taking into account cognitive mechanisms centered around the concept of analogy when analysing and modeling rational belief and behaviour in humans. In the following, we want to provide an analogy-inspired point of view on the aforementioned well-known challenges for rationality.

### **3.3 Resolving the Wason-Selection Task by Cognitive Mechanisms**

As mentioned above, according to [11] subjects perform better (in the sense of more according to the laws of classical logic) in the Wason-Selection task, if content-change makes the task easier to access for subjects. In our reading, subjects' performance is tightly connected to establishing appropriate analogies. Subjects perform badly in the

classical version of the Wason-Selection task, simply because they fail to establish a fitting analogy with an already known situation. In the “beer drinking” version mentioned above, i.e. the content-change version of the task, the situation changes substantially, because subjects can do what they would do in an everyday analogous situation: they need to check whether someone younger than 20 years is drinking beer in a bar. This is to check the age of someone who is drinking beer and conversely to check someone who is younger than 20 years whether he is drinking beer or not. In short, the success or failure of managing the task is crucially dependent on the possibility to establish a meaningful analogy.

### **3.4 Resolving the Inferences and Conditionals Problem by Cognitive Mechanisms**

The results concerning conclusions drawn by the subjects in Byrne’s experiments can also be explained through analogy-making. People faced with the information given in 1. will recall similar conversations they had before, using these known situations as basis for their decision on what to conclude. According to Grice [20], in conversations speakers are supposed to provide the hearer with as much information as is needed for exchanging the necessary information, a rule which goes in accordance with our everyday observation. Thus, when being given the additional information that “Marian does not have to write an essay.”, the set of candidate situations for establishing an analogy will be biased towards situations in which this information had an impact on the outcome, resulting in the conclusion that Marian would not study late in the library either. Regarding 2. and 3., a similar conjecture seems likely to hold: By additionally mentioning the library, similar situations in which the library might actually have played a crucial role (e.g., by being closed) will be taken into account as possible base domains of the analogy, causing the change in conclusions made.

### **3.5 Resolving the Linda Problem by Cognitive Mechanisms**

In case of Tversky and Kahneman’s Linda problem, a natural explanation of subjects’ behavior is that people find a lower degree of coherence between Linda’s profile and the mere statement “Linda is a bank teller”, than they do with the expanded statement “Linda is a bank teller and is active in the feminist movement”. In the latter one, at least one conjunct of the statement fits quite well to Linda’s profile. In short, subjects prefer situations that seem to have a stronger inner coherence. Coherence is important for the successful establishment of an analogical relation, as it facilitates the finding of a source domain for an analogy. We conjecture that in order to make sense of the task, humans rate statements with a higher probability where facts are arranged in a theory with a higher degree of coherence. Thus, seeing coherence in the first place as a means for facilitating analogy-making, and taking into account that analogy has been identified as a core element of human cognition, the decision for the coherence-maximizing option is not surprising anymore, but fits neatly into the conceptual analogy-based framework, and could even have been predicted (providing inductive support for our general claim).

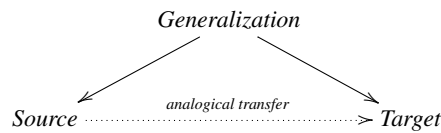


Fig. 1. HDTP's overall approach to creating analogies

## 4 Rationality, Decision-Making and Analogy-Making Systems

In this section we want to give an overview-like sketch of how computational analogy-making systems can be related to some of the discussed challenges for rationality, as well as to decision-making and choice in general, demonstrating their value as models also in this domain.

### 4.1 Heuristic-Driven Theory Projection

Heuristic-Driven Theory Projection is a symbolic framework for computing analogical relations between two domains that are axiomatized in first order logic [21]. HDTP, after being given the logic representations of the two domains, by means of anti-unification [22] computes a common generalization of both, and uses this resulting theory as basis for establishing an analogy, also involving analogical transfer of knowledge between the domains (i.e., the system provides an explicit generalization of the two domains as a by-product of the analogy-making process). Thus, conceptually, HDTP proceeds in two phases: in the *mapping phase*, the formal representations of source and target domain are compared to find structural commonalities, and a generalized description is created, which subsumes the matching parts of both domains. In the *transfer phase*, unmatched knowledge in the source domain can be transferred to the target domain to establish new hypotheses in an analogical way, cf. Figure 1.

Think about Rutherford's model of the atom [23] in analogy to a model of the solar system: HDTP, after finding commonalities in the logical representation of the solar system as base domain, and the atom model as target domain (for example, that in both cases less massive objects are somehow related to a more massive central object, or that always a positive distance and a positive force between these lighter objects and the heavier core can be found), a generalization is computed, via which known laws from the base can be reinstated in the target (e.g., that a lighter object revolves around a heavier one when there is negative centrifugal force between the lighter and the heavier one, yielding the revolution of the electrons around the nucleus, or that the centrifugal force between two spatially separated objects with positive gravitational force between both is equal to the negative value of that gravity, resulting in stable orbits of the electrons in the model).

HDTP implements a principle (by using heuristics) that maximizes the coverage of the involved domains [21]. Intuitively, this means that the sub-theory of the source (or the target) that can be generated by re-instantiating the generalization is maximized. Putting it the other way round, the original domain-specific information and structure

shall implicitly be preserved as far as possible. The higher the coverage the better, because more support for the analogy is provided by the generalization (in a way, the higher the achieved degree of coverage, the more firmly the analogy is rooted in the underlying domains, used for creating the generalization). A further heuristics in HDTP is the minimization of substitution lengths in the analogical relation, i.e. the simpler the analogy the better [24]. The motivation for this heuristics is to prevent arbitrary associations. Clearly there is a trade-off between high coverage and simplicity of substitutions: An appropriate analogy should intuitively be as simple as possible, but also as general and broad as necessary in order to be non-trivial. Unfortunately, high coverage normally comes with higher complexity of substitutions (as a more complex generalization allows for a higher degree of re-representation of domain-specific structures and information), whilst the simplicity constraint is trying to steer the analogy-making process in exactly the opposite direction. This kind of trade-off is similar to the kind of trade-off that is usually the topic of model selection in machine learning and statistics.

#### **4.2 The Wason-Selection Task Revisited**

A modeling of the Wason-Selection task with HDTP is quite simple as long as appropriate background knowledge is available, in case an analogy should be established, or the lack of appropriate background knowledge prevents analogy-making, in case no analogy should be established: On the one hand, if background knowledge for an analogous case is missing (i.e., in the case of HDTP, no domain representation which offers sufficient structural commonalities to the target domain as to serve as a base for the analogy process can be retrieved from memory), then there is no chance to establish an analogical relation. Hence, subjects have to apply other auxiliary strategies, possibly deviating from the expected “right” answer. If there is a source theory with sufficient structural commonalities on the other hand, then the establishment of an analogical relation is straightforward, resulting in a smooth solution process of the task.

#### **4.3 Analogy in Choice**

Coming back to Markman and Moreau’s meta-study of the role analogy and analogical comparison play in the process of human choice, presented in [18], we want to show some connections of their findings to computational systems for analogy-making.

It is without doubt that the choice of options taken into account when making a decision is of crucial importance for the entire process of decision-making. Markman and Moreau present the formation of consideration sets (i.e., the set of options taken into account by a decision maker) as one of the places at which the influence of analogy on decision-making clearly shines up. An analogical reasoning process is involved when deciding on which scenarios are likely to happen, and thus have to be considered (see, e.g., also [25] for related results). According to their findings, there are different factors influencing which analogies will be used in a choice situation, resulting in a set of analogies which are considered similar or familiar to the current situation. Close analogs have the advantage of probably allowing the transfer of more lower-order relations than distant analogs would, i.e., closer concepts are more likely to be considered as an option due to an easier and more fruitful analogy-making process. This goes in accordance with



characteristics exhibited by many computational models of analogy-making, where we just want to mention HDTP. As pointed out in [21], although HDTP basically aligns any entity, function or predicate, it clearly prefers literally-matching alignments over non-literally ones, and equivalent structures to structural mismatches, thus reconstructing a preference and behaviour also shown by humans.

Also, experiments indicate that commonly shared surface elements of domains are more useful as retrieval cues than are connected relational systems. Also this carries over to the principles underlying HDTP, with HDTP trying to minimize the complexity of analogical relations whilst maximizing the degree of coverage: Connected relational systems have the strong tendency of reaching higher-order stages, whilst direct surface correspondences stay on a low level, allowing for a direct matching of features. Thus, handling common surface elements allows for a certain degree of coverage without having to escalate complexity, probably also making HDTP prefer surface elements for supporting an analogy over relational ones (if both types are equally available).

Finally, it shows that elements related to a person's individual experience do influence the way decisions are taken. These elements have the advantage of being (mostly) highly accessible, with base domains which form part of someone's past being more likely to have richly connected relational structures, providing good ground for eventual analogical inference. When searching for a way of computationally modeling this phenomenon, it comes to mind that a similar effect can already be found in AMBR, Kokinov's well-documented hybrid analogy-making system [26]. This system exhibits signs of priming effects in the retrieval process of a fitting base domain for an analogy's given target domain, together with a general influence of earlier memory states on later ones.

#### **4.4 Modeling Judgement and Choice**

In [27], Petkov and Kokinov present JUDGEMAP, a computational model of judgement and choice based on the general-purpose cognitive architecture DUAL [28], and the aforementioned corresponding AMBR analogy-making system. JUDGEMAP is capable of performing both tasks, giving a judgement on a scale and deciding a choice situation, by means of a process of making forced analogies, exclusively using mapping principles inherited from the underlying AMBR system. JUDGEMAP has been demonstrated to replicate phenomena known from observations of human judgement as, for example, range and frequency effects, or sequential assimilation effects.

Furthermore, several simulations are described, in which it is demonstrated that mechanisms designed for modeling analogy can have influence on judgement and choice, possibly reproducing contextual effects in tasks which don't seem to be related to analogy-making. Among others, it is shown that the pressure for one-to-one mapping, which has been introduced to AMBR for the purpose of analogy-making, can in the model cause phenomena similar to the frequency effect in judgement (i.e., people using all available ratings almost equally often in their judgements), and for the concave form of the functional relation between subjective value (i.e., utility) and money. Also, the effect that humans when judging tend to use middle ratings more often than extreme ones can be explained in terms of a dynamic mechanism used for hypothesis creation in AMBR. Also, the occurrence of the preference reversal effect in choice can be explained

by a feature originating from the analogy-making system. The most remarkable part of all this is, that not a single one of the mechanisms used in the JUDGEMAP model had been created for this purpose, but were all obtained from the AMBR model, which supports our claim that structural mapping and analogy play a fundamental role also in judgement and choice, and therefore ultimately also in decision-making.

## 5 Concluding Remarks

The evidence for a crucial role of analogy-making presented over the last pages falls far from being complete. Yet another example can be given in form of well-known studies on human decision-making under time pressure, which show a change in the applied inference procedure. In [29], the authors report that, whilst the best predicting model of human inference for decision making in an unstressed conditions was a weighted linear model integrating all available information, when time pressure was induced, best predictions were obtained by using a simple lexicographic heuristic [30]. This presumed change from a more complex strategy using complex relational structures to a simple single-attribute-based procedure also can be found in research on analogy-making: In [31], it is reported that anxiety made participants of an analogical-reasoning experiment switch from a preference for complex relational mappings to simple attribute-based mappings.

Still, whilst not claiming completeness of our overview of evidence, we are convinced that even the already given examples and indications are sufficient as not to allow for leaving analogy and cognitive processes out of consideration.

A criticism with respect to the analogy-making approach might be a seeming lack of normativity as a theory. Although work on this topic is still in a very early stage, we are confident that this objection grasps at nothing: Normativity can be introduced in a very natural way by considering the reasonableness (or unreasonableness) of made analogies. Roughly speaking, it is obvious that different analogies may have different degrees of reasonableness, e.g., based on the level to which they result in coherent beliefs and to which they encompass both, the source and the target domain of the analogy.

In this paper, we argue in favor of an introduction of the concept of analogy into conceptual research on rationality and decision-making on a foundational level. Based on a review of some basic concepts and existing work within the fields of analogy research and research on decision-making and choice, together with an exemplifying proposal of new resolution strategies for classical rationality puzzles, we think that the usage of frameworks for establishing analogical relations and the usage of frameworks that can maximize the coherence of a theory necessarily have to be taken into account when modeling (and possibly implementing) what is commonly considered rational belief in a not overly simplified manner.

Of course, this paper is just a very first conceptual step in constructing and establishing the promoted new view, still a great amount of substantial fundamental work has to be done, and numerous open questions have to be answered. Nevertheless, considering the evidence indicating a connection between decision making and analogy originating from psychology, together with characteristics shown by already existing models

of analogy-making (which were designed without any consideration of rationality or an application in decision making), we are strongly confident that an undertaking as argued for in this paper merits the effort, and can lead to important results and insights.

## References

1. Evans, J.: Logic and human reasoning: An assessment of the deduction paradigm. *Psychological Bulletin* **128** (2002) 978–996
2. Griffiths, T., Kemp, C., Tenenbaum, J.: Bayesian Models of Cognition. In: *The Cambridge Handbook of Computational Cognitive Modeling*. Cambridge University Press (2008)
3. Gigerenzer, G., Hertwig, R., Pachur, T., eds.: *Heuristics: The Foundation of Adaptive Behavior*. Oxford University Press (2011)
4. Osborne, M., Rubinstein, A.: *A Course in Game Theory*. MIT Press (1994)
5. Wason, P.C.: Reasoning. In: *New Horizons in psychology*. Penguin (1966)
6. Byrne, R.: Suppressing valid inferences with conditionals. *Cognition* **31**(1) (1989) 61–83
7. Tversky, A., Kahneman, D.: Extensional versus intuitive reasoning: The conjunction fallacy in probability judgement. *Psychological Review* **90**(4) (1983) 293–315
8. Gigerenzer, G.: *Rationality for Mortals: How People Cope with Uncertainty*. Oxford University Press (2008)
9. Chinchuluun, A., Pardalos, P., Migdalas, A., Pitsoulis, L., eds.: *Pareto Optimality, Game Theory and Equilibria*. Springer (2008)
10. Wason, P.C., Shapiro, D.: Natural and contrived experience in a reasoning problem. *The Quarterly Journal of Experimental Psychology* **23**(1) (1971) 63–71
11. Cosmides, L., Tooby, J. In: *Cognitive Adaptions for Social Exchange*. Oxford University Press (1992)
12. Stenning, K., van Lambalgen, M.: *Human Reasoning and Cognitive Science*. MIT Press (2008)
13. Johnson-Laird, P.: *Mental Models*. Harvard University Press (1983)
14. Holyoak, K., Gentner, D., Kokinov, B.: Introduction: The Place of Analogy in Cognition. In: *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press (2001) 1–19
15. Gentner, D.: Structure mapping: A theoretical framework for analogy. *Cognitive Science* **7** (1983) 155–170
16. Colman, A.M.: Cooperation, psychological game theory, and limitations of rationality in social interaction. *Behavioral and Brain Sciences* **26**(2) (2003) 139–198
17. Kokinov, B.: Analogy in decision-making, social interaction, and emergent rationality. *Behavioral and Brain Sciences* **26**(2) (2003) 167–169
18. Markman, A., Moreau, C.: Analogy and analogical comparison in choice. In: *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press (2001) 363–399
19. Kokinov, B.: Can a Single Episode or a Single Story Change our Willingness to Risk? The Role of Analogies in Decision-Making. In: *Advances in Cognitive Economics*. NBU Press (2005)
20. Grice, H.P.: Logic and Conversations. In: *Syntax and Semantics, Vol. 3: Speech Acts*. Academic Press (1975) 41–58
21. Schwering, A., Krumnack, U., Kühnberger, K., Gust, H.: Syntactic principles of heuristic-driven theory projection. *Cognitive Systems Research* **10**(3) (2009) 251–269
22. Plotkin, G.D.: A note on inductive generalization. *Machine Intelligence* **5** (1970) 153–163
23. Rutherford, E.: The scattering of  $\alpha$  and  $\beta$  particles by matter and the structure of the atom. *Philosophical Magazine* **21** (1911) 669–688

24. Gust, H., Kühnberger, K., Schmid, U.: Metaphors and heuristic-driven theory projection (hdt). *Theoretical Computer Science* **354** (2006) 98–117
25. Schwenk, C.: Cognitive simplification processes in strategic decision-making. *Strategic Management Journal* **5**(2) (1984) 111–128
26. Kokinov, B., Petrov, A.: Integrating Memory and Reasoning in Analogy-Making: The AMBR Model. In: *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press (2001) 59–124
27. Petkov, G., Kokinov, B.: JUDGEMAP - integration of analogy-making, judgement, and choice. In: *Proceedings of the 28th Annual Conference of the Cognitive Science Society (CogSci)*. (2006) 1950–1955
28. Kokinov, B.: The DUAL cognitive architecture: A hybrid multi-agent approach. In Cohn, A., ed.: *Proceedings of the Eleventh European Conference of Artificial Intelligence*, John Wiley & Sons, Ltd. (1994) 203–207
29. Rieskamp, J., Hoffrage, U.: Inferences under time pressure: How opportunity costs affect strategy selection. *Acta Psychologica* **127** (2008) 258–276
30. Fishburn, P.: Lexicographics orders, utilities and decision rules: A survey. *Management Science* **20** (1974) 1442–1471
31. Tohill, J., Holyoak, K.: The impact of anxiety on analogical reasoning. *Thinking & Reasoning* **6**(1) (2000) 27–40