An Evidential Approach for Modeling and Reasoning on Uncertainty in Semantic Applications

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Abstract. Standard semantic technologies propose powerful means for knowledge representation as well as enhanced reasoning capabilities to modern applications. However, the question of dealing with uncertainty, which is ubiquitous and inherent to real world domain, is still considered as a major deficiency. We need to adapt those technologies to the context of uncertain representation of the world. Here, this issue is examined through the evidential theory, in order to model and reason about uncertainty in the assertional knowledge of the ontology. The evidential theory, also known as the Dempster-Shafer theory, is an extension of probabilities and proposes to assign masses on specific sets of hypotheses. Further on, thanks to the semantics (hierarchical structure, constraint axioms and properties defined in the ontology) associated to hypotheses, a consistent frame of this theory is automatically created to apply the classical combinations of information and decision process offered by this mathematical theory.

Keywords: Ontologies, OWL, Uncertainty, Dempster-Shafer Theory, Belief Functions, Semantic Similarity.

1 Introduction

Uncertainty is an important characteristic of data and information handled by real-world applications. The term "uncertainty" refers to a variety of forms of imperfect knowledge, such as incompleteness, vagueness, randomness, inconsistency and ambiguity. In this approach, we consider only the epistemic uncertainty, due to lack of knowledge (incompleteness) and the inconsistency, due to conflicting testimonies or reports. This paper presents a proposal on a possible way to tackle the issue of representing and reasoning on this type of uncertainty in semantic applications, by using the Dempster–Shafer theory [1], also known as "evidential theory" or "belief function theory". The general objective of our applications is to form the most informative and consistent view of the situation, observed by multiple sources. These observations populate our domain ontology. Thus, we consider that the uncertainty has to be embodied in the instantiation rather than in the structural knowledge of ontology. One of our requirements is that a source can assign a belief on any instance without worrying of any level of granularity or disjointness of these instances. For example, one source could assign a belief on an instance of class *Vehicle* and, at the same time, another belief on an instance of type *Car*, which inherits from the class *Vehicle*.

The following section of this paper introduces the basic definitions and notations of the Dempster–Shafer theory. Section 3 presents our ontology modeling of the representation of uncertainty, using evidential theory. In the fourth section, we address how to reason with the evidential theory while benefiting from the semantics included in the domain ontology. Section 5 proposes to position our approach by comparing it with already existing works in the domain of uncertainty and the Semantic Web.

2 Basis of Dempster-Shafer Theory

The Dempster–Shafer theory [1] allows the combination of distinct evidence from different sources in order to calculate a global amount of belief for a given hypothesis. It is often presented as a generalization of the probability theory. It permits to manage uncertainties as well as inaccuracies and ignorance.

2.1 Frame of Discernment

Let Ω be the universal set, also called the discernment frame. It is the set of all the N states (hypothesis) under consideration: $\Omega = \{H_1, H_2, ..H_N\}$.

The universal set is supposed to be exhaustive and all hypotheses are exclusives. Exhaustivity refers to the closed-world principle. From this universal set, we can define a set, noted 2^Ω. It is called the power set and is the set of all possible sub-sets of Ω, including the empty set. It is defined as follows:

$$2^\Omega = \{A \mid A \subseteq \Omega\} = \{\varnothing, \{H_1\}, ..., \{H_N\}, \{H_1, H_2\}, ..., \Omega\}.$$

2.2 Basic Mass Assignment and Belief Measures

A source, who believes that one or more states in the power set of Ω might be true, can assign belief mass to these states. Formally, a mass function is defined by:

$$m : 2^\Omega \rightarrow [0,1] \ . \tag{1}$$

It is also called a basic belief assignment and it has two properties:

$$m(\varnothing) = 0 \quad \text{and} \quad \sum_{A \in 2^\Omega} m(A) = 1 \ . \tag{2}$$

This quantity differs from a probability since the total mass can be given either to singleton hypothesis H_n or to composite ones.

The main other belief measures are belief and plausibility. Belief *bel(A)* for a set *A* is defined as the sum of all the masses of the subsets of the set of interest:

$$bel(A) = \sum_{B | B \subseteq A} m(B) \qquad \forall A \subseteq \Omega . \tag{3}$$

It is the degree of evidence that directly supports the given hypothesis *A* at least in part, forming a lower bound. The plausibility *pl(A)* is the sum of all the masses of the sets *B* that intersect the set of interest *A*:

$$pl(A) = \sum_{B | B \cap A \neq \varnothing} m(B) \qquad \forall A \subseteq \Omega . \tag{4}$$

pl(A) can be interpreted as the part of belief which could be potentially allocated to *A*, taking into account the elements that do not contradict this hypothesis. It is seen as an upper bound.

2.4 Information Fusion

Modeling by masses through the evidential theory would be useless without an adequate combination enabling the fusion of a set of information sources. This is especially the role of the Dempster's rule of combination. Namely, it combines two independent sets of mass assignments (i.e. from difference sources). The combination (called the joint mass) is calculated from the two sets of masses m_1 and m_2 in the following manner:

$$(m_1 \oplus m_2)(A) = \begin{cases} \dfrac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K_{12}} & A \neq \varnothing \\ 0 & A = \varnothing \end{cases} . \tag{5}$$

$$\text{where} \qquad K_{12} = \sum_{B \cap C = \varnothing} m_1(B) m_2(C) . \tag{6}$$

K is a measure of the amount of conflict between the two mass sets. K is ranging from 0 to 1. Dempster's rule corresponds to the normalized conjunctive operator. Other combination rules exist, such as the disjunctive combination and other operators that reassign the amount of conflict differently [2].

3 DS-Ontology Modeling

The first step of our approach is to model and represent the uncertainty through ontologies. Modeling is proposed through a specific ontology that needs to be imported in the initial domain ontology. This initial domain ontology is the ontology we want to instantiate in an uncertain way. The imported ontology is called DS-Ontology. It is described in the following.

3.1 Structural Knowledge of the DS-Ontology

This ontology is a formal representation of the theory of Dempster-Shafer, as it proposes a shared understanding of the main concepts: mass, belief, plausibility, source, etc. It is non-domain specific, since one can use it in every area of knowledge. It has been coded in OWL2 language [3]. Hereafter is an informal schema of the terminology of DS-Ontology.

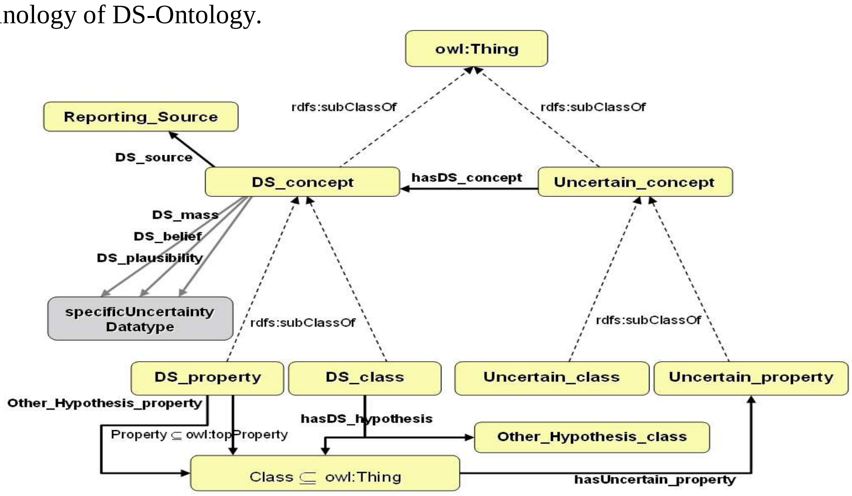

Fig. 1. Informal ontology structure schema. Yellow boxes represent OWL classes. Grey ones refer to datatypes (XML ones and user defined datatype). Arrows symbolize properties. Resources appearing without namespace prefix come from the DS-Ontology whose namespace is http://DS-Ontology.owl.

The main classes are *Uncertain_concept* and *DS_concept*. The *DS_concept* class links the hypothesis, with the source and the numerical amount of belief related to the hypothesis. The hypothesis consists either of a singleton or a union of hypotheses. Hypotheses are in fact instances of the domain ontology. Instances are either individuals of classes or instances of properties. The *Uncertain_concept* class links together all the *DS_concept* that are related to the same context. Indeed, the uncertainty is embodied by several candidate instances (with an assigned belief) and the uncertainty is concretely instantiated through one instance of *Uncertain_concept*. *Uncertain_concept* enables to retrieve the set of hypotheses under consideration, i.e. the power set 2^{Ω}.

In order to represent uncertainty both on individuals and on asserted properties, *DS_concept* and *Uncertain_concept* have been specialized. They are specified in subclasses *XX_class* and *XX_property* (XX prefix representing both DS and Uncertain). *Uncertain_concept* is now an equivalent class to the union of *Uncertain_property* and *Uncertain_class*, while the latter two are disjoint. Respectively, this holds for *DS-concept* and its subclasses.

The *hasDS_hypothesis* object property relates an instance of *DS_class* to a set of candidate individuals. Concerning candidate properties, things have been done differently. Indeed, OWL properties are not first-class citizens, contrary to OWL classes; as such OWL properties cannot be related to each others: a property cannot be the subject or object of another property. To get around this, an object property *hasUncertain_property* has been introduced. The original subject of the candidate property is the subject of *hasUncertain_property*. The domain of *hasUncertain_property* is intuitively the class *Uncertain_Property*. Then, *DS_Property* instances are directly the subject of the candidate properties while their object remains unchanged.

An illustration of the use of the DS-Ontology is given in the next section.

As with the Dempster-Shafer theory, the modeling of ignorance is made possible. It is realized through an instance of *DS_concept* linked to all hypothetical instances. Ontologies evolve within the open world assumption. However, the original evidential theory assumes a closed world and that is why the measure of the amount of conflict exists. Therefore, we should for instance opt for an Open Extended World extension of the Dempster-Shafer theory [4]. Applied to ontologies, it consists in modeling another concept, with prefix: "*Other_Hypothesis*". This element is included in the DS-Ontology (both as a class and a property) and is asserted if needed to embody hypothesis, which does not correspond to any already defined concept in the domain ontology.

We represent numerical evidential belief through a s*pecificUncertaintyDatatype* which is a user-defined datatype defined in our DS-Ontology to restrict its value to an *xsd:double* ranging from 0 to 1.

In our model, *Uncertain_concept* and *DS_concept* are classes that let grouping together collected pieces of information about an uncertain instance we want to model and reason about. It can be viewed as a reification process, where an addressable object is created as a proxy for non-addressable objects. Informally, reification is often referred to as "making something a first-class citizen" within the scope of a particular system. Reification is one of the most frequently used techniques of conceptual analysis and knowledge representation. Even if RDF language enables reification process [5], we choose to model explicitly in an ontology our full representation, instead of using annotations not defined in the ontology. As a consequence, the uncertainty extension of OWL through the DS-Ontology is completely compliant with the basic principle of OWL ontologies to structure knowledge in two levels: structural and assertional.

3.2 Instantiation Example

Our applications aim mainly at observing real world situations through different perspectives (sources) and give an understandable and fused analysis of what is going on in this situation to the final decision maker. This simplified scenario involves here two distinct sources. One is a human while the other is an automatic sensor, such as radar. They both want to express that something is going into a specific direction; the "something" entity is the same object for both sources; however, they are not sure about how to identify this object. Indeed, the radar source can only distinguish a land vehicle from an aircraft; it assigns here a more important belief on the fact that it is an instance of a land vehicle. The second source is a human, who has a slight and far away view of the situation is assigning different beliefs to an instance of car which looks like red, or a fire truck or a more imprecisely one to a land vehicle. In most cases, we do not have to assess the belief assigned to hypotheses by ourselves, it is directly given by the sources according to their condition of use (e.g. meteorology, proximity, etc.) and we apply possibly a weakening coefficient according to the source reliability. The structural knowledge of this domain is modeled through an ontology (http://ontology-uri.owl), whose hierarchical structure is captured in figure 2.

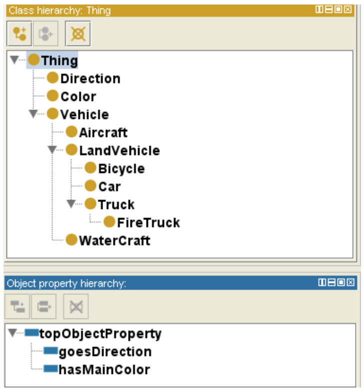

Fig. 2. Protégé snapshot of the structural knowledge of the ontology.

In addition to the hierarchical structure of the knowledge, domain and range of properties are also defined, as well as additional information concerning a priori information about the world. For instance, in this domain ontology, it is mentioned that a fire truck individual is always associated to the property *hasMainColor* with the value red. According to the sources and to the domain ontology, the assertional knowledge of this ontology involves:

- http://ontology-uri.owl#direction: an individual of class http://ontology-uri.owl#Direction
- respectively #landVehicle for class #LandVehicle
- respectively #aircraft for class #Aircraft
- respectively #fireTruck for class #FireTruck
- respectively #red for class #Color
- respectively #car for class #Car which is linked to the individual #red through the #hasMainColor property.

The set of candidate instances are: {#landVehicle, #aircraft, #fireTruck, #car}. We refer here to IRI instances only with their local name, omitting the namespace.
Regarding the Dempster-Shafer theory, the masses are assigned by the sources as:

- m_{radar}({#landVehicle}) = 0.6 ; m_{radar}({#aircraft}) = 0.1 ; m_{radar}({#landVehicle, #aircraft}) = 0.3
- m_{human}({#car}) = 0.2 ; m_{human}({#fireTruck}) = 0.4 ; m_{human}({#landVehicle}) = 0.4

This domain ontology imports the DS-Ontology, in order to represent all these pieces of knowledge within the domain ontology. Two more individuals are created to represent the sources:

- #human for class http://DS-Ontology.owl#Reporting_Source
- #radar for class http://DS-Ontology.owl#Reporting_Source

The following figure illustrates through a non-formal ontological schema, how the instances are linked together.

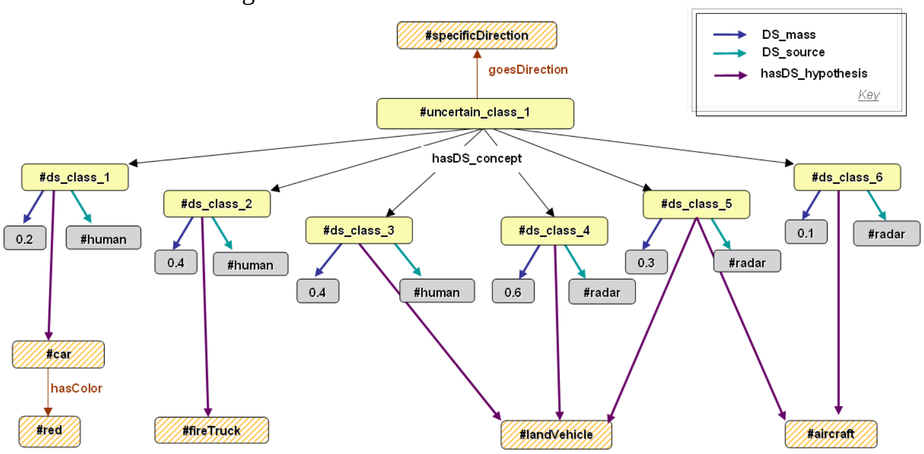

Fig. 3. Uncertain individuals scenario

4 Evidential Reasoning on DS-Ontology

Once the uncertainty contained in the information has been represented, reasoning processes have to be conducted to fuse the different observation and eventually decide of the instance with the most likelihood. This section has to be viewed as the chronological steps that are realized by the system in order to reason on the uncertain pieces of information represented through the DS-Ontology.

4.1 Generate automatically the Discernment Frame

One *Uncertain_concept* instance of the DS-Ontology groups a set of candidate instances together (either individuals or properties). From this set of instances, we want to determine automatically a consistent frame of discernment, according to the Dempster-Shafer theory. The underlying assumptions of the theory are: an exhaustive frame of discernment and the exclusivity of elements of Ω (see section 2.1). In this paper, we have already managed the first constraint within the Open World assumption of ontologies in the modelling of the DS-Ontology. The second constraint of the frame of discernment is the exclusivity of its elements. This implies that each singleton hypothesis (i.e. the elements of Ω) are disjoint. In other words, if H_1 and H_2 are two singletons, we cannot have $H_1 \subset H_2$ or even $H_1 \cap H_2 \neq \emptyset$. In the instantiation example, #fireTruck and #car individuals are semantically "included" in #landVehicle. As there is an inclusion, #fireTruck and #car individuals have also a non-null intersection with the #landVehicle individual. Moreover, #fireTruck has a non-null intersection with #car. Indeed, these two individuals are sharing many characteristics in common: they are both land vehicles and their main colors are in both cases red.

To deal with this second constraint, we take into account the explicit and inferred semantics of the domain ontology to generate the discernment frame. The granularity of the set of candidate instances affects the generation of the discernment frame. The semantic will help us determining the inclusion of hypotheses as well as the semantic similarity between instances. The whole set of candidate instances will help us fixing a threshold for semantic distances.

4.1.1 Semantic Inclusion/Intersection

The semantic inclusion is quite straightforward to determine. Indeed, in case the instances are property assertions, for example if a property P1 has for ancestor P2, then we say that P1 is included in P2. Otherwise, in case the instances are individuals and they have zero or the same properties (or some included property), then there is an inclusion. In all other cases, the inclusion does not hold.

Concerning semantic intersection, things go a little further. First of all, logically, if two instances have already a semantic inclusion, then they also have a non-null semantic intersection. In all other cases, we will consider that two instances have a non-null intersection when their semantic similarity is exceeding a certain threshold. More specifically, our similarity measure is a global function, which combines existing similarity measure defined in literature. As for individuals, it is a mixture of similarity measure of their respective types and similarity measures concerning their relations. Wu & Palmer similarity measure [6] is used to qualify the similarity between two instances based on their respective type. It takes into account the distance that separates two types in the hierarchy and their position with the root. Equation (7) depicts their formula. $C1$ and $C2$ are two classes. Class C is the immediate mother-class of $C1$ and $C2$ that subsumes both classes. $depth(C)$ function is the number of edges separating C from the root. $depth_C(Ci)$ is the number of edges which separate Ci from the root while passing by C.

$$conSim(C1, C2) = \frac{2 * depth(C)}{depth_C(C1) + depth_C(C2)}. \quad (7)$$

The other combined similarity measures count the number of identical properties versus the number of different properties related to the two individuals. This is calculated both for object properties and datatype properties. On equation (8), $I1$ and $I2$ are the two individuals for which the global semantic similarity measure is calculated. For object properties (respectively for datatype properties), $nbProp(I)$ is the number of object properties (resp. of datatype properties) of individual I. $nbPropComm(I1,I2)$ is the number of common properties - identical predicate and related individual or value - for the two individuals $I1$ and $I2$. These three similarity measures focusing on the similarity of the types of individuals and their characteristics (through the datatype and object properties) are combined through a weighted mean.

$$propSim(I1, I2) = \frac{2 * nbPropComm(I1, I2)}{nbProp(I1) + nbProp(I2)}. \quad (8)$$

Once the cross-similarity measure of the set of all candidate instances is calculated, the threshold is fixed through a clustering method. The threshold is thus varying according to all the computed semantic similarities. This process permits to adapt the granularity of the set of candidate instances. It translates our general impression that the concept of a compact car is closer to the concept of minivan than of a plane's; however the concept of a compact car is closer to the concept of plane than of a book's. In the first case, the intersection should be brought by the pair (compact car, minivan), whereas in the latter, it should be brought by the pair of (compact car, plane). It should be noted that, in both cases, the concepts of compact car and plane have the same semantic similarity. As a consequence, the semantic intersection is seen as a Boolean condition on the similarity measure exceeding the threshold.

Finally, we consider the evidential set inclusion (respectively intersection) as equivalent to the semantic inclusion (respectively intersection). In case of our scenario, the intersection and inclusion are graphically represented on the figure below.

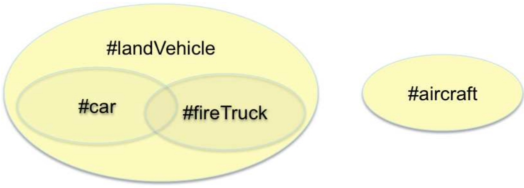

Fig. 4. Inclusion and intersection of candidate instances

4.1.2 From the Set of Candidates Instances to the Discernment Frame

Once the intersection and inclusion of candidate instances identified, we are able to set up a consistent frame of discernment. For this, we reframe the set of candidate instances into single or composite disjoint hypotheses.

In case of a discovered intersection between two candidate instances #inst1 and #inst2, #inst1 is reformulated as the union of two singletons $\{H_1, H_{inters}\}$ and #inst2 as $\{H_2, H_{inters}\}$. In case of discovered inclusions between two candidate instances #inst1 and #inst2, where #inst1 is included in #inst2, #inst1 is represented by a single hypothesis $\{H_1\}$ and #inst2 by the union of hypotheses $\{H_2, H_1\}$. Single hypotheses, grouped together, constitute the frame of discernment. In fact, each initial candidate instance belongs to the power-set of the frame of discernment. Taking our scenario, each candidate instance can now be decomposed as such:

- #aircraft = $\{H_1\}$
- #car = $\{H_2, H_3\}$
- #fireTruck = $\{ H_3, H_4\}$
- #landVehicle = $\{H_2, H_3, H_4, H_5\}$

Indeed, relying on Figure 4, #aircraft instance has no intersection nor inclusion; thus, it constitutes a single hypothesis within the frame of discernment. The non-null intersection, between #fireTruck and #car instances, has been modeled through a common and shared single hypothesis: H_3. Finally, the inclusion brought by #landVehicle results in the union of the set of single hypotheses of #fireTruck and #car, in addition to its own singleton H_5.

4.2 Use Dempster-Shafer Calculations on DS-Ontology

Once the discernment frame has been obtained, we can reformulate in the Dempster-Shafer formalism, the basic mass assignment of the scenario:

- $m_{radar}(\{H_2, H_3, H_4, H_5\}) = 0.6$; $m_{radar}(\{H_1\}) = 0.1$; $m_{radar}(\{H_1, H_2, H_3, H_4, H_5\}) = 0.3$
- $m_{human}(\{H_2, H_3\}) = 0.2$; $m_{human}(\{H_3, H_4\}) = 0.4$; $m_{human}(\{H_2, H_3, H_4, H_5\}) = 0.4$

We are now able to apply directly the classical combination rules found in the Dempster-Shafer theory, and then go through the decision process.

5 Related Work

During the last decade, approaches considering both uncertainty and the Semantic Web have been proposed. In this section, we mention some of them in order to position and compare our work. We consider their goal, underlying mathematical theory and processes.

Fuzzy and rough set theories aim to model vagueness and uncertainty. Regarding fuzzy sets, classes are considered to have unsharp definitions. fuzzyDL approach [7] aims to represent and reason about a membership function specifying the degree to which an instance belongs to a class. Even if it could be interesting to take into account fuzzy aspect of hypotheses especially those formulated by human sources, it is not the purpose of our approach to model more precisely our knowledge, but to decide among multi hypotheses and have a more coherent and reliable view of the situation. Approaches in [8, 9] are relying on rough set theory – which considers the indiscernability between objects. In that case, classes are not restricted to a crisp representation; they may be coarsely described with approximations. In [9], the author is using rough classes to generate new subclasses or relations by mining an important set of instances already existing. This can be part of the ontology engineering process. The goal is here also different to ours; however, some notions and process are similar. First, the design of a rough OWL ontology can be seen as the matching piece to our DS-Ontology for the Dempster-Shafer theory. Moreover, the use of p-indistinguishable properties notion for two individuals can be linked to our so-called common properties in Equation (8) when processing the similarity measure between two instances. Finally, descriptions for lower and upper approximation – through intersection and inclusion considerations - remind us the definition of the exclusivity of our frame of discernment; however, they consider here intersection and inclusion between two classes whereas we calculate it between two individuals.

Probabilistic adaptations or extensions (Pr-OWL [10], BayesOWL [11], Fire [12]) are more relevant to our objective of assessing the most likelihood instances that holds. However, probabilities suffer from the lack of ignorance and imprecision management in comparison to evidential theory.

Approaches in [13, 14 and 15] are more related to our chosen mathematical theory as they directly deal with evidential theory. [13] and [14] transform uncertain statements in belief networks. However, these network representations are themselves extensions of evidential theory. Moreover, they do not take into account the semantic attached to the hypotheses, in order to consider the most conflicting hypotheses or on the inverse the implied hypotheses. Looking this way, they can be considered complementary to ours. A recent published approach [15] is concentrating on uncertain reasoning on instances of an ontology using the evidential theory and some similarity measures. While we handle the same mentioned tools, our process and aspiration are quite different. Indeed, their main objective is to propose an alternative ABox inductive reasoning - by classifying individuals (determining their class- or role- memberships or value for datatype properties) through a prediction based on an evidential nearest neighbor procedure. Their reasoning addresses here another way to tackle automatic inference from a classical ontology. This automatic inference aims to derive new or implicit knowledge about the current representation of the world, on the basis of the asserted knowledge. Whereas, our current reasoning goal is to rely on the semantic description of candidate instances (hypothesis) describing a same and unique entity or phenomenon in order to decide which candidate instances should be chosen.

Other reports enlarging the state-of-the-art to all ontology languages can be found in [17, 18].

6 Conclusion and Future Work

This paper proposes a solution in order to handle uncertainty within ontologies. Our approach is relying on current W3C standards. Modeling of uncertainty is realized through an imported pre-defined ontology: the DS-Ontology. Uncertain instantiation of the domain ontology is performed through the use of this imported DS-Ontology. The DS-Ontology relies on the theory of Dempster-Shafer, which manages uncertainty, as well as imprecision and ignorance. This paper has underlined some key issues that have to be dealt when implementing such parallelism between a formal mathematical theory to manage uncertainty and semantic world. The assumption of Open World in ontologies is one of these issues. Reasoning on uncertainty is made possible through an automatic generation of the frame of discernment. For that purpose, Boolean semantic operators, such as the intersection and inclusion, have been developed based on the semantic expressivity of the domain ontology. As a consequence, this paper provides a double and mutual contribution in the domains of the Semantic Web and of uncertain theories, which benefits clearly from the semantics of the hypotheses.

Further researches are also in discussion to extend the reasoning over the Boolean inclusion and intersection of candidate instances. Indeed, it could be interesting to keep the semantic similarity degree (which is a real ranging from 0 to 1) and use it instead of Boolean notions within the theory of Dempster-Shafer. This could be made by rearranging the basic measures of belief and plausibility and of the rules of combination.

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