P systems: State of the Art with Respect to Representation of Geographical Space

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Abstract. Membrane computing is an emergent branch of natural computing, taking inspiration from the structure and functioning of a living cell. P systems, computing devices of this paradigm, are parallel, distributed and non-deterministic computing models which aim to capture processes taking place in a living cell and represent them as a computation. In last decade, a great variety of extensions of model, introduced by Paun in 1998, were presented. In this paper we present a comprehensive review of current progress in the field of membrane computing, focusing on representation of geographical space in P systems. Two approaches are commonly used in Geographic Information Science (GIS) for representation of entities: field-based and object-based. Both approaches are discussed from the point of P systems and possibilities of using inherent hierarchical structure of P systems in spatial modeling are mentioned.

Keywords: membrane computing, P systems, Geographical Information Systems, representation of space

1 Introduction

Membrane computing represents new and rapidly growing branch of natural computing, which starts from observation that the processes taking place in a living cell can be understood as a computation. Membrane computing and its computational device – *P system* – were introduced by Păun [32] and gained a lot of interest in last decade. P systems start from observation, that membrane plays a fundamental role in the functioning of a living cell. Membranes act as three-dimensional compartments which delimit various regions of a living cell. They are essentially involved in a number of reactions taking place inside cell and moreover act as selective channels of communication between different compartments of a cell [5].

P systems take inspiration from cell on two levels – the structure and the functioning. Structure of cell is represented by its membranes and functioning is governed by biochemical reactions. Every P system therefore has three main elements: a *membrane structure*, where *object* evolve according to given *evolution rules* [35]. Some authors add fourth basic element of membrane systems – *communication* [5, 34]. Communication is always encoded in rules (they are called

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communication rules instead of *evolution rules*) and will be dealt with later in the text. From the point of view of *Geographical Information Systems* (GIS), communication (e.g. topology) is essential feature of most real world phenomena.

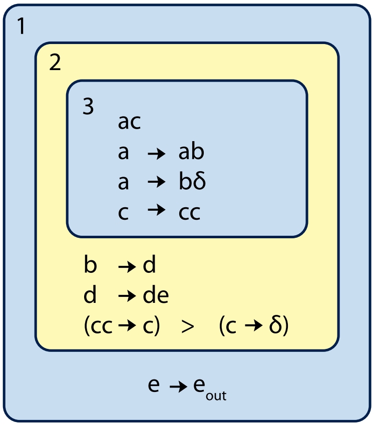

Fig. 1. Graphical representation of P system, [46]

Simple example of P system is depicted in Fig. 1. Membrane structure is hierarchically arranged set of membranes, contained in a distinguished outer membrane, called *skin* membrane. System is surrounded by the *environment*, which may collect objects leaving the system, or in some variants of P systems, the environments can actively support system with objects [4, 11]. Membrane structure can be represented in many ways – as Venn diagram, as rooted tree, or by linear notation. Membrane structure of P system depicted in Fig. 1 in linear notation is written as $[_1\ [_2\ \ [_3\ ]_3\ ]_2\ ]_1$. Membranes delimit *regions*, with which they are in one-to-one relation. Therefore the terms *membrane* and *region* are mostly interchangeable. Each membrane is identified by its *label*, which can be with membranes in one-to-many relation. The position of inner membranes does not matter; we assume, that in membrane *there is no ordering, everything is close to everything else* [35]. Please note the difference with the first law of geography: *everything is related to everything else, but near things are more related than distant things* [39]. This applies not only to membranes, but also to objects in them. From the biological point of view, inner membranes are considered floating free in their parental membranes and therefore definition of topological or metric relations between them makes no sense. This is of course not valid for geographical space and we will discuss it later.

Second basic element of P systems are *objects*. By objects in biological sense are meant chemicals, ions, molecules etc. Those substances are present in a cell in enormous amount, but the ordering again does not matter. What matters is the concentration, the population, the number of copies of each molecule [35]. Abstracting from biological reality, we represent each substance by a symbol from given alphabet and since the multiplicity matters, instead of objects we use *multisets* of objects. Common notation of multisets in P systems is following: if, for example, objects a, b, c are present in 7,2 and 5 copies, they will be represented by multiset $a^7b^2c^5$.

In basic variant of P systems, multisets of objects are considered to be floating in inner regions of membrane systems. They evolve by the means of *evolution rules*, which are localised with the regions of the membrane structure. There are three main types of rules [35]: (1) multiset-rewriting rules, (2) communication rules and (3) rules for handling membranes. In this section only first type of rules will be described.

Multiset-rewriting rules take form $u \to v$, where u and v are multisets of objects. For example, rule $ab \to cd^2$ says, that one copy of a and one copy of b are consumed and one copy of c and two copies of d are produced. A number of possible extensions of rules will be discussed later.

Two crucial features of P systems have to be mentioned at this point. As mentioned earlier, in membranes everything is close to everything else. Therefore, if one instance of an object can be processed by two or more rules, the rule to be applied is chosen *non-deterministically*. All rules have the same probability to be chosen. The rules also have to be used *in maximally parallel manner*.

More specifically, the objects are assigned to rules, non-deterministically choosing the objects and the rules, until no further assignment is possible. An evolution step in a given region of membrane system consists of finding the maximal applicable multiset of rules, removing from region all objects specified in the left hand of the chosen rules and producing the objects on the right hand side of the rules.

After giving short introduction to basic notions of P systems, let us continue to more detailed survey on spatial properties of P systems. In Sect. 2 we will give a formal definition of *transition P system* and in Sect. 3 some possible extensions of P systems are presented. In Sect. 4 we will discuss how geographical space is represented in Geographical Information Systems, in Sects. 5 and 6 we will describe object-based and field-based representation of geographical space in P systems and in Sect. 7 we will discuss how hierarchical structure of P systems can be used to represent geographical phenomena. We will conclude with some final remarks in Sect. 8.

2 Transition P system

P systems based on application of *multiset-rewriting rules* are called *transition P system*. Formally, transition P system is a construct of the form:

$$\Pi = (O, C, \mu, w_1, w_2, \ldots, w_m, R_1, R_2, \ldots, R_m, i_o), \tag{1}$$

where:

- O is the finite and non-empty alphabet of objects,
- $C \subset O$ is the set of catalysts,
- μ is a membrane structure, consisting of m membranes, labeled $1, 2, \ldots, m$; one says, that the membrane structure, and hence the system, is of degree m,
- $w_1, w_2, \ldots, w_m$ are strings over O representing multisets of objects present in regions $1, 2, \ldots, m$ of membrane structure,
- $R_1, R_2, \ldots, R_m$ is finite set of evolution rules associated with regions $1, 2, \ldots, m$ of membrane structure,

- i_o is either one of the labels $1, 2, \ldots, m$ and then the respective region is the *output region* of the system, or it os 0 and then the result of the computation is collected in the environment of the system.

A sequence of transitions of P system constitutes a *computation.* A computation is successful if it halts, it reaches a configuration where no rule can be applied to the existing objects, and output region i_o still exists [35].

The rules are of form $u \to v$, where $u \in O$ and $v \in (O \times Tar)$, where $Tar = \{here, in, out\}$. Target indications Tar extend transition P system in following way: rule $ab \to c_{here}d_{in}e_{out}$ consumes one instance of each a and b and produces one copy of c in current membrane, one copy of d in a child of current membrane and one copy of e in the parent of current membrane. If current membrane is skin membrane, object e is send to environment of the system. If current membrane does not have a child, rule can not be applied.

Another extension comes from the existence of *catalysts.* Catalysts are objects, which participate in a chemical reaction, but are not consumed or produced by it. They just enable the application of rule. Rule with catalysts takes following form: $ac \to bc$, with object c being the catalyst.

One more extension must be mentioned at this place, and that is *dissolution* of membranes. During dissolution, membrane disappears and its content (both objects and inner membranes) are left free in the surrounding membrane. Dissolution rule takes form $u \to v\delta$, where δ denotes the action of dissolution.

3 Possible Extensions of P systems

In this section we will mention some elementary extensions of P system, which however constitute only a fracture of possibilities. We refer reader to The P systems Webpage [45] for complete list of publications and further information. Already in the text, three types of rules were mentioned. Evolution rules were briefly covered in previous sections.

Communication rules were introduces in [34]. Basic idea of communicating P systems is, that computation is achieved only by transporting object between membranes. Direct inspiration from biology are *symport* and *antiport.* When two chemical pass through membrane only together, in the same direction, the process is called *symport.* When the two chemicals pass only with help of each other, but in opposite directions, the process is called *antiport* [34]. Symport rules take a form (ab, in) or (ab, out), and antiport rules take a form $(a, out; b, in)$, where a, b are object from alphabet of all possible objects. Meaning of rules is following: for symport rule (ab, in) or (ab, out), if objects a, b are present in current membrane, they are sent together into child (or parent, in second case) of the membrane. For antiport rule $(a, out; b, in)$, if a is present in current membrane and b is present in the parent of a membrane, than a exits current membrane and b enters it. Universality of P systems with symport and antiport have been proven [34] and simplified version of communication, *conditional uniport* have been studied [40]. Comprehensive review of communication strategies in P systems can be found in [41].

Third type of rules are *rules for handling membranes*. Dissolution of membranes has already been mentioned, but other ways to obtain dynamical membrane structure, evolving during the course of computation, have been presented. Most simple of those is assigning *electrical polarization* $+, -, 0$ to each membrane. Polarization replaces target indicators $in, out, here$. Polarization of objects is introduced by the rules and polarized objects can enter only membranes with opposite polarization. For example, $ab \rightarrow c^+d^-$ means, that one instance of c enters inner membrane with negative polarization and one instance of d enters inner positive membrane. Rules can also be used to change the polarization of membranes during the computation.

Another possibilities to alter membrane structure have been proposed. Division of membranes can be used to obtain exponential working space in linear time [33] and have been used to solve **NP**-problems [31]. An optimalization algorithms based on membrane computing were proposed [29, 24]. Also biological processes of *exocytosis, endocytosis* and *gemmation* were translated into the language of P systems and examined in detail [25].

Two issues seem essential, when P systems are used to simulate biological phenomena rather than for computation. Non-determinism is first of the issues. Some chemical reactions are more likely to occur than others. First attempt to solve this is by assigning priorities to rules. Firstly, set of rules with the highest priority is chosen and according to the principle of maximal parallelism, all rules which can be applied, are applied. Then, the rules with second highest priority are selected and the procedure repeats, etc.

Second approach is to assign probabilities to all rules. Probability can be introduced to P system on different levels [30], but here we will mention only probability on the level of rule selection. Different approaches have been proposed [4, 13, 36]. Basic idea is to associate each rule with a constant k, so the rule takes a form $u \xrightarrow{k} v$, where u, v are multisets of objects and k can be interpreted either as a probability, or as a "stoichiometric coefficient", using which the true probability is calculated.

Last extension, which we will mention at this point, is representation of time of P systems. In real world, every biochemical reaction takes some time. Representation of time in P systems is similar to representation of probability. A constant t is assigned to each rule, so the rule takes the form $u \xrightarrow{t} v$, where u, v are multisets of objects and t is number of time units, which must pass to complete the application of the rule [12]. In the first time step, multiset u is consumed and removed from the current membrane. After $t-1$ more time steps, multiset v is introduced into the system. Time can also be introduced into P systems as a lifetime of objects or even membranes [1].

For the sake of brevity, we will not discuss more extensions of P systems, although many possibilities were explored within this framework. Every real-world application of P systems requires careful and accurate definition of system to be modeled and once the real-world system is defined, P system as a mathematical model for simulation can be developed. It is very unlikely, that any of presented variants of P systems would accurately describe the complex real-world phenomena, however when considered as a modelling paradigm, P systems offer great variety of extensions, and arbitrary P system for concrete application can be developed.

4 Geographical Representation of Space

Object-based and field-based models of space are accepted as two alternative approaches for conceptualization and geographical modelling [20]. Object-based conceptualization understands geographical entities as sharply bounded and therefore represented by mostly polygonal boundaries. Objects are located in space, i.e. location is attribute. Fields are continuous phenomena and characterize space by properties – functions and values – related to locations [42]. Raster and vector represetations are dual to field-based and object-based conceptualization of space [37].

In field-based model, every location in a spatial framework is associated with a set of attributes. Fields are spatially continuous by definition. Field can be viewed as a mapping between spatial location and an attribute domain [43]. The most common field-types are scalar, vector or tensor; with scalar fields being the most commonly used in GIS modelling. Representation of field must be always approximate and rectangular, triangular or hexagonal tessellations are used [16]. Fields are usually stored as a georeferenced raster.

In an object-based perspective, space is viewed as a container populated by objects. Location is an attribute of each object. Object's spatial projection is mostly represented in GIS environment by points, lines (polylines, networks) or polygons [16].

Possible merge of field-based and object-based approaches have been discussed in literature [16, 42], but GIS applications include only two basic representations of space.

5 Object-based Representation of Geographical Space in P systems

Two basic terms in representation of geographical space are *distance* and *topology*. The mathematical theory of metric spaces is well-known to be inadequate as a formal foundation for distance measures in geographic spaces [28]. Contextual knowledge is a key feature of human apprehension of geographic space [44]. For example distance between cities A and B is different from the point of view of cyclist and pilot of an airplane. There is an important distinction between global view (top-table space) and geographical view (geographical space).

Top-table space can be viewed from one single point, whether geographical space is context-based [44]. Therefore classic definition of metric in mathematical space does not apply (geographical space is asymmetric and triangle inequality does not apply).

In geographical space, neighborhood relations are commonly treated as prior to metrics. For example we know that Austria and Germany share the border, but we are unable to estimate the length. Regarding the fractal nature of geographical boundaries [26], measuring the length may not even make sense.

Definition of topology between geographical entities is therefore essential for any geographical analysis.

As mentioned earlier in the text, object-based representation of geographical space understands space as a container with entities, which are defined by their locations. The relation of entities is described by their topology. Nearness of two neighboring entities can be quantified as a distance between them. Distance can be context-based (i.e. time necessary to overcome distance between two points can depend on the mean of transportation) and also dependent on direction. Those relations can be formalized using graph theory. Entities are represented as nodes of the graph and links represent topological relation between them. Cost of links represents distance between geographical entities.

Special variant of P systems with membranes arranged in the net have been proposed as *tissue-like P system* [27]. In this variant, membranes are arranged in an arbitrary graph instead of in a hierarchy. The computation is achieved using symport/antiport rules, but generalization using evolution rules can be considered. Links between membranes are represented using *synapses*. Formal definitions are here omitted and can be found in [19, 27, 35]. This formalization is suitable for representing topological relations between entities in geographical space. Entities can be represented as membranes (nodes of the graph) and their topology could be stored in links. Adding cost to synapses will achieve further representation of distances between entities. This approach has been adapted by [6, 7] to simulate interaction between spatially separate regions (so called metapopulations). This research was however only theoretical.

Cardona et al. in [8] started research on modelling population of Beraded Vulture in the Pyrenees using model with several spatially separated regions, which could however interact with each other by sending objects to the environment and retrieving them. This model was later expanded to model population of 12 animals [9, 10], modules for modelling biomass of plants were added [14] and model was also used for management of the area [15]. Also, [10] used similar model to simulate population growth of invasive species of zebra mussel in water reservoir. The water body was represented by 17 regions with different regime of water temperature fluctuations with their topology represented by oriented graph.

Object-based representation of space can be coherently represented using P systems. Different geographical entities can be represented by membranes, topology can be stored in a graph and distances between entities could be represented using costs of links of graph. Moreover, each node – membrane – can have inner hierarchical structure. Take agglomeration of larger city as an example. This agglomeration is connected with other cities by roads and rails, therefore represented as a membrane with synapses to other membranes – cities. In the same time the agglomeration can have inner structure, represented either by smaller set of interconnected membranes (public transportation network with nodes representing stations) or a hierarchy of membranes, representing for example administrative zoning with several levels. Moreover, both representations can be stored within this membrane separately.

6 Field-based Representation of Geographical Space in P systems

Field-based representation of space understands geographical space as a continuous field, where every location has a set of attributes (temperature, air pressure or elevation, for example). For analytical purposes, continuous fields must be approximated by grid, mostly rectangular. This representation of space is similar to cellular automata.

Recently, [3] introduced *spatial P systems*, which embody concept of space and position inside membranes in similar manner that cellular automata do. Rules, as usual, specify the objects which are consumed and which are produced, moreover, the position of produced objects can be specified. Although P systems were used before to model processes in geographical space (see previous chapter), [3] was first to inherently include space into P systems. In classical view of P systems, position does not matter. Also [7, 9] and others, who worked with space, did consider position of membranes only as an attribute of membranes, and position of object inside membrane was never considered before.

We will not give formal definition of spatial P system and will focus on possible applications instead, because up to date, there are none. Barbuti et al. [3] defines spatial P systems in two-dimensional space, but living cells are three-dimensional compartments, therefore extension to 3D would be suitable to enhance expressiveness of the model.

Cellular automata are used in GIS for various applications, including modelling of forest fires, urban growth and dispersion of pollution. Among these application, modelling of spread of pollution seems as most promising ground for P systems. Pollutants are mostly chemicals and their consumption, creation and alteration can be naturally described using P systems. Their dispersion can be described using spatial P systems.

Another example would be simulating growth of collonies of bacteria (both in microscale and macroscale), where P systems can accurately describe behavior of such simple organisms. In many other application, like modelling of deforestation, urban growth and qualitative changes in landscape, spatial P systems could achieve similar results like cellular automata.

7 Using Hierarchy to Represent Space

Geographical space is without doubts structured in a hierarchical manner. Administrative division of Czech republic is an example. Hierarchical data structures such as quadtree or octtree [38] are widely used in GIS and spatial databases. Even methodology for adapting such data structures to globe was developed [21].

Also hierarchical spatial reasoning gained interest in the community of geographers in last two decades [23]. This hierarchical approach mimics human reasoning when performing spatial operations: an appropriate scale is selected and results are computed. The quality of results obtained is assessed, and if it is satisfactory, the computation stops. If not, more detailed level of spatial data is consulted.

Our literature search showed, however, that spatial modelling of hierarchically structured phenomena is rare. Eckhardt and Thomas [17] used multilevel regression models for inspection of patterns of road accident occurrences. Other publications dealing with hierarchical modelling also exist, but mostly use hierarchical Bayesian models, where the structure of model is hierarchical, but not in geographical sense [2]. Some research has been dedicated to possibilities of visualization of geographical hierarchies [22].

P systems offer different approach, where modelling can be taking place on multiple levels of the model simultaneously. In Fig. 2, a simple system of forest is depicted (rules are omitted). This model has two levels. On upper level, the population of animals (deers) is modelled. On lower level, the competition between two parts of forests coniferous and hardwood is modelled. Because population of deers is not dependent on the inner structure of the forest, it can be modelled separately, on upper level, and inherent hierarchical structure of P systems can be exploited.

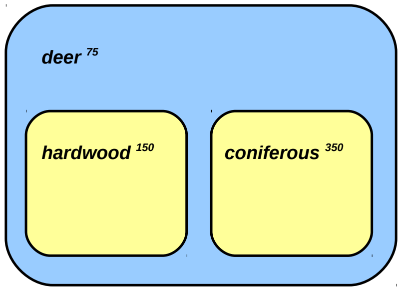

Fig. 2. Hierarchical structure of geographical space in P systems

In natural systems, ecosystems are hierarchically structured and this structure plays prominent role [18]. P systems for ecological application therefore offer expressiveness, which most other computational models do not posses.

To our knowledge, no application, where multiple levels of a single system were simultaneously modelled, were presented and also no theoretical research of this topic was conducted. Hence, further research on application of P systems should focus on this aspect of modelling.

8 Concluding Remarks and Future Work

We have presented brief introduction to membrane computing with emphasis on description of handling of geographical space in P systems. Both representations of geographical space – object-based and field-based were already discussed in the literature and some application are available for object-based representation.

Only one publication so far was dedicated explicitly to computing with space in P systems.

Currently, many extensions of classical transitional P system exist, and expressiveness of this model can be significantly enhanced. For real-world application, however, unique models must be defined. We proposed some possible application of P systems in geography, from which modelling the spread of pollution seems the most promising, given the nature of the prenomenon.

Also perspectives of multi-level modelling were mentioned. This approach exploits inherent hierarchical structure of P systems to simulate the behavior of systems on multiple levels. Possible application can be seen in ecological studies, since ecosystems are deeply hierarchized structures.

Our current research is focused on modelling of transportation using P systems. Individual based modelling, paralellism and evolution of components of a system are key features needed to model complex behaviour of transport systems. However this research is in its initial stage and experimental results are not available at the moment.

References

1. Aman, B., and Ciobanu, G. Adding Lifetime to Objects and Membranes in P Systems. *International Journal of Computers Communications and Control 5 (3)*. 2010. Pages 268-279.
2. Banerjee, S. *Hierarchical Modeling and Analysis for Spatial Data*. Chapman and Hall, 2004, London, UK.
3. Barbuti, R., and Maggiolo-Schettini, A., and Milazzo, P., and Pardini, G., and Tesei, L. Spatial P systems. *Spatial P systems 10 (1)*. 2011. Pages 3-16.
4. Bernardini, F., and Manca, V. Dynamical aspects of P systems. *BioSystems 70 (2)*. 2003. Pages 85-93.
5. Bernardini, F., and Gheorghe, M., and Krasnogor, N., and Muniyandi R.C., and Perez Jimenez, M.J., and Romero-Campero, F.-J. On P Systems as a Modelling Tool for Biological Systems. *Pre-Proc. of the sixth Workshop on Membrane Computing*. Vienna, Austria, 2005. Pages 114-133.
6. Besozzi, D., and Cazzaniga, P., and Pescini, D., and Mauri, G. Seasonal variance in P system models for metapopulations. *Pre-proceedings of International Conference on Bio-Inspired Computing - Theory and Applications, BIC-TA 2006, Membrane Computing Section*. Wuhan, China, 2006. Pages 27-36.
7. Besozzi, D., and Cazzaniga, P., and Pescini, D., and Mauri, G. Modelling metapopulations with stochastic membrane systems. *Biosystems 91 (3)*. 2008. Pages 499-514.
8. Cardona, M., and Colomer, M.A., and Margalida, A., and Pérez-Hurtado I., and Pérez-Jiménez, M.J. and Sanuy, D. Modeling Ecosystems Using P Systems: The Bearded Vulture, a Case Study. *Membrane Computing*. Springer-Verlag, 2009, Berlin/Heidelberg. Pages 137-156.
9. Cardona, M., and Colomer, M.A., and Margalida, A., and Pérez-Hurtado I., and Pérez-Jiménez, M.J. and Sanuy, D. A P System Based Model of an Ecosystem of Some Scavenger Birds. *Workshop on Membrane Computing*. 2009. Pages 182-195.
10. Cardona, M., and Colomer, M. and Margalida, A., and Palau, A., and Pérez-Hurtado, I., and Pérez-Jiménez, M.j., and Sanuy, D. A computational modeling for real ecosystems based on P systems. *Natural Computing*. 2010. Pages 39-53.

11. Cavaliere, M. Evolution-Communication P Systems. Proceeding WMC-CdeA '02 Revised Papers from the International Workshop on Membrane Computing. 2003. Pages 134-145.
12. Cavaliere, M., and Sburlan, D. Time-independent P systems. *Membrane Computing. International Workshop WMC5*. Milano, Italy, 2005. Pages 239-258.
13. Cazzaniga, P., and Pescini, D., and Romero-Campero, F.-J., and Besozzi, D., and Mauri, M. Stochastic Approaches in P Systems for Simulating Biological Systems. *Fourth Brainstorming Week on Membrane Computing*. Seville, Spain, 2006. Pages 145-164.
14. Colomer-Cugat, M.A., and Fondevilla, Ch., and Valencia-Cabrera, A New P System to Model the Subalpine and Alpine Plant Communities. *Ninth Brainstorming Week on Membrane Computing*. Seville, Spain, 2011. Pages 91-112.
15. Colomer, M.A., and Lavín, S., and Marco, I., and Margalida, A., and Pérez-Hurtado, I., and Pérez-Jiménez, M.J., and Sanuy, D., and Serrano, E., and Valencia-Cabrera, L. Modeling population growth of Pyrenean Chamois (Rupicapra p. pyrenaica) by using P systems. *Lecture Notes in Computer Science*. 2011. Pages 144-159.
16. Cova, T.J., and Goodchild, M.F. Extending geographical representation to include fields of spatial objects. *Journal of geographical information science 16 (6)*. 2002. Pages 509-532.
17. Eckhardt, N., and Thomas, I. Spatial nested scales for road accidents in the periphery of Brussels. *IATSS Research 29 (1)*. 2005. Pages 66-78.
18. Forman, R.T.T. *Land Mosaics: The Ecology of Landscapes and Regions*. Cambridge University Press, 1995, Cambridge, Uk.
19. Freund, R., and Păun, Gh., and Pérez-Jiménez M.J. Tissue-like P systems with Channel-States. *Second Brainstorming Week on Membrane Computing*. Seville, Spain, 2004. Pages 101-116.
20. Goodchild, M.F. Geographical data modelling. *Computers and Geosciences 18*. 1992. Pages 401-408.
21. Goodchild, M.F., and Yang, S. A hierarchical spatial data structure for global geographic information systems. *CVGIP: Graphical Models Image Processing 54 (1)*. 1992. Pages 31-44.
22. Hadlak, S., and Tominski, Ch., and Schulz, H.-J., and Schumann, H. Visualization of Attributed Hierarchical Structures in a Spatio-Temporal Context. *International Journal of Geographical Information Science*. 2010. Pages 1497-1513.
23. Hirtle, S.C., and Frank, A.U. Spatial Information Theory: A Theoretical Basis for GIS. *Lecture Notes in Computer Science 1329*. 1997. Pages 1-15.
24. Huang, L., Suh, I.H., and Abraham, A. Dynamic multi-objective optimization based on membrane computing for control of time-varying unstable plants. *Information Sciences 181*. 2011. Pages 2370-2391.
25. Krishna, S.N., and Păun, Gh. P Systems with Mobile Membranes. *Natural Computing 4 (3)*. 2005. Pages 255-274.
26. Mandelbrot, B. How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension. *Science 156 (3775)*. 1967. Pages 636-638.
27. Martin-Vide, C., and Pazos, J. and Păun, Gh., and Rodríguez-Patón, A. A New Class of Symbolic Abstract Neural Nets: Tissue P Systems. *Proceedings of the 8th Annual International Conference on Computing and Combinatorics*. Springer-Verlag, 2002, London, UK. Pages 290-299.
28. Montello, D.R. The Geometry of Environmental Knowledge. *Spatio-Temporal Reasoning*. 1992. Pages 136-152.

29. Nishida, T.Y. An application of P systems: A new algorithm for NP-complete optimization problems. *Proceedings of the 8th World Multi-Conference on Systems, Cybernetics and Informatics*. 2004. Pages 109-112.
30. Obtulowicz, A., and Păun, Gh. (In search of) Probabilistic P systems. *Biosystems 70 (2)*. 2003. Pages 107-121.
31. Pan, J., Alhazov, A. Solving HPP and SAT by P Systems with Active Membranes and Separation Rules. *Acta Informatica 43 (2)*. 2005. Pages 131-145.
32. Păun, Gh. Computing with Membranes. *Technical report, Turku Center for Computer Science-TUCS*. Turku, Finland, 1998.
33. Păun, Gh. P Systems with Active Membranes: Attacking NP Complete Problems. *Journal of Automata, Languages and Combinatorics 6*. 1999. Pages (75-90)
34. Păun, A., and Păun, Gh. The Power of Communication: P Systems with Symport/Antiport. *New Generation Computation 20 (3)*. 2002. Pages 295-306.
35. Păun, Gh. Introduction to Membrane Computing. *First brainstorming Workshop on Uncertainty in Membrane Computing*. Palma de Mallorca, Spain, 2004. Pages 1-42.
36. Pescini, D., and Besozzi, D., and Mauri, G., and Zandron, C. Dynamical probabilistic P systems. *International Journal of Foundations of Computer Science 17 (1)*. 2006. Pages 440-447.
37. Peuquet, D.J. A Conceptual Framework and Comparison of Spatial Data Models. *Cartographica*. 1984. Pages 66-113.187-260
38. Samet, H. The Quadtree and Related Hierarchical Data Structures. *ACM Computing Surveys 16 (2)*. 1984. Pages 187-260.
39. Tobler, W. A computer movie simulating urban growth in the Detroit region. *Economic Geography 46 (2)*. 1970. Pages 234-240-
40. Verlan, S., and Bernardini, F., and Gheorghe, M., and Margenstern, M. Computational completeness of tissue p systems with conditional uniport. *Proceedings of the 7th international conference on Membrane Computing*. Springer-Verlag, 2006, Berlin/Heidelberg. Pages 521-535.
41. Verlan, S., and Bernardini, F., and Gheorghe, M., and Margenstern, M. Generalized communicating P systems. *Theoretical Computer Science 404 (1-2)*. 2008. Pages 170-184.
42. Winter, S. Bridging vector and raster representation in GIS. *Proceedings of the sixth ACM international symposium on Advances in geographic information systems GIS 98*. ACM Press. 1998. Pages 57-62.
43. Worboys, M.F. *GIS: a computing perspective*. Taylor and Francis, 1995, London, UK.
44. Worboys, M.F. Metrics and topologies for geographic space. *Advances in Geographic Information Systems Research II: Proceedings of the Symposium on Spatial Data Handling*. Delft, The Netherlands, 1995. Pages 170-184.
45. The P Systems Web Page. `http://ppage.psystems.eu/`. Last revision: 2012-03-18.
46. P system - Wikipedia, the free encyklopedia. `http://en.wikipedia.org/wiki/P_system`. Last revision: 2012-03-18.