Fuzzy Logic-based Robust Control of a Flexible two-mass System (1990 ACC Benchmark Problem)

Adekunle C. Adediji and John Essegbey

School of Electronics and Computing Systems
University of Cincinnati
497 Rhodes Hall
Cincinnati, Ohio 45221-0030

Abstract

In intuitive design steps, a fuzzy logic-based robust controller is designed to address the first 1990-1992 American Control Conference benchmark problem. Using a conceptual transformation of the original flexible body into a perpetual rigid body mode, a final design which succeeds in stabilizing the system after a unit impulse disturbance is developed. The simulation results are shown to achieve and exceed the required design specifications of the benchmark problem, as well as those of other fuzzy logic-based solutions.

Introduction

As the complexity of engineered systems increased, it became imperative that the American Controls Conference (ACC) adopt a set of control design problems as robust control benchmark problems. This has led to several attempts by authorities in the field to come up with the best possible solutions, serving as a good basis for comparing the various heuristics and methodologies in designing for robust control. One of these problems, referred to by (Wie and Bernstein 1992) as ACC benchmark problem 1, was concerned with vibration control of a two-mass system with an uncertain spring constant (Figure 1). The flexible tow-mass system addresses, primarily, a disturbance rejection control problem in the presence of parametric uncertainty. This problem has been addressed in over 30 papers, including papers in special issues of the Journal of Guidance, Control and Dynamics and the International Journal of Robust and Nonlinear Control (Linder and Shafai 1999).

Probably due to the linearity of this problem, most published solutions have appropriated linear controllers of some sort, from H-infinity to game theory. (Niemann et al 1997) applied the μ-synthesis method for mixed perturbation sets using a modified D-K iteration approach, while (Wie and Liu 1992) proposed a solution using the H_∞ controller design methodology. In addition, (Farag and Werner 2002) compared the performance of his robust H_2 design with a collection of existing controllers such as Pole Placement, and Minmax Linear Quadratic Gaussian (LQG). (Hughes and Wu 1996) also presented an observer-based extension of a passive controller design, due to the fact that strictly passive feedback could no longer guarantee stability for the given problem.

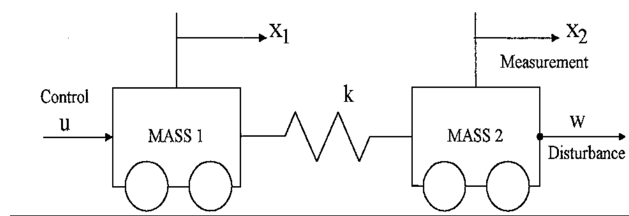


Figure 1: The ACC benchmark problem consisting of a dual mass, single spring system.

Some recent solutions, however, make use of qualitative approaches capitalizing on fuzzy reasoning, which have been shown to perform just as good as or even better than the existing quantitative methods (Cohen and Ben Asher 2001). It is worth noting that the presence of design constraints, and plant, as well as parameter uncertainties, drastically increases the complexity of modeling plant behavior, and makes the application of non-linear solutions worthwhile.

In this paper, we build on a solution using fuzzy logic. We start by generating a detailed model of the system and highlight the required design objectives for the controller. Next we obtain a reduced or simplified model of the system in the rigid-body mode, where spring oscillations have been effectively damped out using fuzzy logic heuristics (Linder and Shafai 1999). Finally, an additional fuzzy controller produces a superimposition of stability and tracking behaviors to ensure the achievement of stated design objectives.

Problem Description and Modeling

The benchmark plant shown in Figure 1 consists of two masses connected via a spring, with the following characteristics.

1) The system has a non-collocated sensor and actuator; the sensor senses the position of m_2 while the actuator accelerates m_1. This introduces extra phase lag into the system, making control of the plant difficult (Cohen and Ben Asher 2001).
2) The system is characterized by uncertainties in the temporal plant (spring constant that varies within a very wide range)
3) The system exists in both the flexible body mode (due to the spring) and rigid-body mode (when relative movements due to the spring are damped out).

For the above system, consider a simplification, where $m_1 = m_2 = 1$ and $k = 1$ with the appropriate units. A control force acts on body 1 (m_1), and x_2, which is the position of body 2 (m_2), is instead measured thus resulting in a non-collocated control problem. The state space representation of the system is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/m_1 & k/m_1 & 0 & 0 \\ k/m_2 & -k/m_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} w$$

$$y = x_2$$

where x_1 and x_2 are the positions of body 1 and body 2, respectively; $x_3$and x_4 are the velocities of body 1 and body 2, respectively; u is the control input acting on body 1; y is the sensor measurement, w is the disturbance acting on body 2, and k is the spring constant. The transfer function representation is

$$T_{uy} = \frac{(k/m_1 m_2)}{s^2[s^2 + k(m_1 + m_2)/m_1 m_2]}$$

and the corresponding transfer function between a disturbance to $m_2$and plant output is

$$T_{wy} = \frac{(1/m_2)(s^2 + k/m_1)}{s^2[s^2 + k(m_1 + m_2)/m_1 m_2]}$$

This paper considers only problems 1 and 2 as described by (Wie and Bernstein 1992) and ignores the effect of sensor noise (full state feedback) and disturbance acting on body 1. The constant-gain linear feedback controller design requirements are stated as

1. The closed-loop system is stable for $m_1 = m_2 = 1$ and $0.5 < k < 2.0$.
2. The disturbance w(t)=unit impulse at t=0 and y has a settling time of 15sec for the nominal plant parameters $m_1 = m_2 = 1$ and $k = 1$.
3. Reasonable performance/stability robustness and reasonable gain/phase margins are achieved with reasonable bandwidth.
4. Reasonable control effort is used.
5. Reasonable controller complexity is needed.
6. Settling is achieved when y is bounded by ± 0.1 units.

This problem addresses, primarily, a disturbance rejection control problem in the presence of parametric uncertainty. The plant has eigenvalues at $(\pm j\sqrt{k(m_1 + m_2)/(m_1 m_2)}, 0, 0)$, and a single-input/single-output (SISO) controller must close its loop around T_{uy}, which has a pole-zero surplus of four (Stengel and Marrison 1992).

Robust Design Solution using Fuzzy Logic

Fuzzy logic controller design was first started by (King and Mamdani 1977) on the basis of the fuzzy logic system generalized from the fuzzy set theory of (Zadeh 1965). It has gained wide practical acceptance providing a simple, intuitive, and qualitative methodology for control (Jamshidi, Vadiee, and Ross 1993), (Yen, Langari, and Zadeh 1992), (Zadeh 1994). In a typical implementation, a fuzzy controller consists of a set of if-then rules, where the controller output is the combined output of all the rules evaluated in parallel from the antecedents of the inputs. The inference engine, of a fuzzy logic controller, plays the role of a kernel that explores the fuzzy rules pre-constructed by experts to accomplish inferences.

Since the rules specify the implication relationships between the input variables and output variables characterized by their corresponding membership functions, the choice of the rules along with the membership functions makes significant impacts on the final performance of the controller and therefore becomes the major control strategy in Fuzzy Logic Controller design.

Common classifications of fuzzy controllers include fuzzy Proportional Integral Differential (PID) controllers, fuzzy sliding-mode controllers and fuzzy gain scheduling controllers (Driankov, Hellendoom, and Reinfrank 1996), (Jang and Sun 1995). Even though all three categories realize closed-loop control action and are based on quantitative control techniques, the first and second are implementations of the linear quantitative PID controller and a nonlinear, quantitative sliding-mode controller. The last category, however, utilizes Sugeno fuzzy rules to interpolate between several control strategies, and are suitable for plants with time varying or piecewise linear parameters (Jang and Sun).

A. Fuzzy Logic for Benchmark

For the robust control problem described above, plant stabilization is required first before performance objectives. Ensuring stability, however, entails the dampening of vibrations after an external disturbance is applied. (Linder and Shafai 1999) described an approach using Qualitative Robust Control (QRC) methodology, where stability and tracking behaviors are separately developed, and the superimposition of these behaviors achieves the final control objective. These behaviors exploit the rigid body mode of the plant, where the plant behaves as if the masses are rigidly connected. The stability behavior is derived from the heuristic that a control action is more effective in suppressing plant vibration if it is applied when the spring is neutral, and the control action opposes the motion of the spring.

Using fuzzy logic, a process model of the spring, needed to provide the qualitative state information that dampens plant vibrations and achieve stability, is achieved by abstracting the system to a state that indicates whether the spring is at its neutral length and whether the spring is in the process of compressing or elongating. In modeling the spring, the length of the spring and its rate of stretching or contraction are used as input parameters and the output, its state. The process utilizes a qualitative spring state that is specified by a qualitative partition of the spring length $L = x_2 - x_1$ and the spring length velocity $\dot{L} = \dot{x}_2 - \dot{x}_1$. These parameters are partitioned using five membership functions as shown in Figure 2. A Mamdani Fuzzy Inference System (FIS) applies 25 rules, shown in the Fuzzy Association Memory (FAM) of Figure 3, to infer the qualitative spring state from inputs L and $\dot{L}$. The fuzzy controller is developed using the minimum operator to represent the "and" in the premise, and the Center of Gravity (COG) defuzzification as the implication.

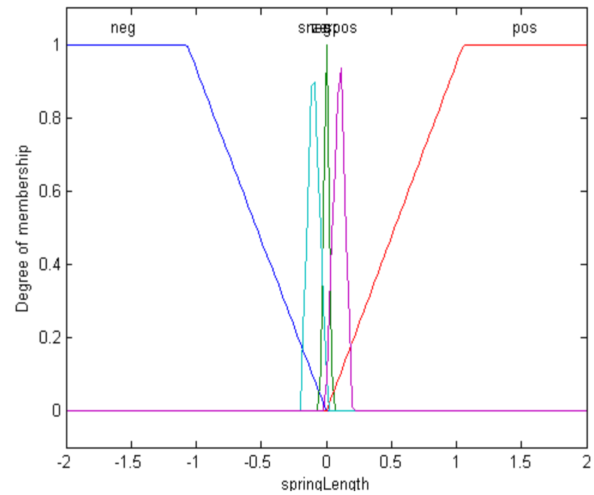
The qualitative behavior of the spring is based on a sense of direction and rate. Thus the parameters are defined on a bivalent range or universe of [-1, 1], and the outputs are described as follows;

NSCN: Not Stretching or Compressing with Neutral spring
CFN: Compressing Fast with Neutral spring
SFN: Stretching Fast with Neutral spring

The decision surface of Figure 4 is such that a vibration is observed when L is Small_positive or Small_negative, and $\dot{L}$ is Negative_large or Positive_large. A similar situation occurs when L is Zero and $\dot{L}$ is Small_negative or Small_positive.

B. State Observers

The above model is possible only if the states of the masses can be observed or correctly estimated. Due to the



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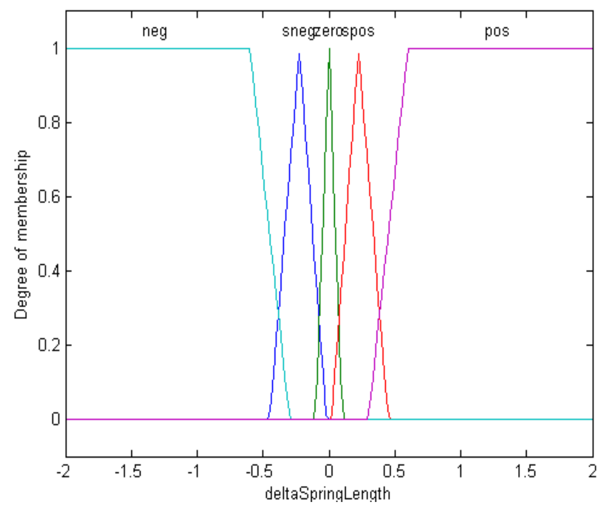

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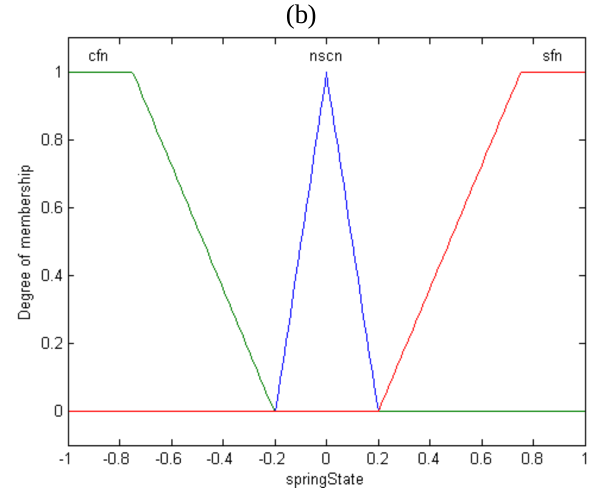

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Figure 2: Fig. 2(a) Membership functions of Spring Length $L = x_2 - x_1$ Fig. 2(b) Membership functions of the velocity of spring contraction or stretching $\dot{L} = \dot{x}_2 - \dot{x}_1$ Fig. 2(c) Membership functions of the output, spring state.

springLength \ deltaSpringLength	neg	sneg	zero	spos	pos
neg	nscn	nscn	nscn	nscn	nscn
sneg	cfn	nscn	nscn	nscn	sfn
zero	cfn	cfn	nscn	sfn	sfn
spos	cfn	nscn	nscn	nscn	sfn
pos	nscn	nscn	nscn	nscn	nscn

Figure 3: Fuzzy association memory of the spring model.

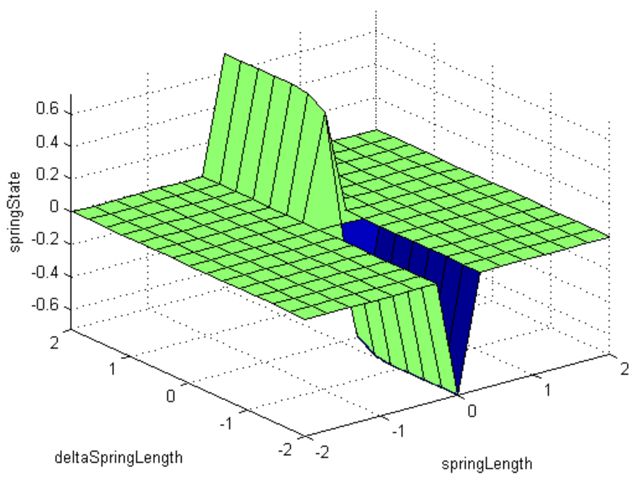


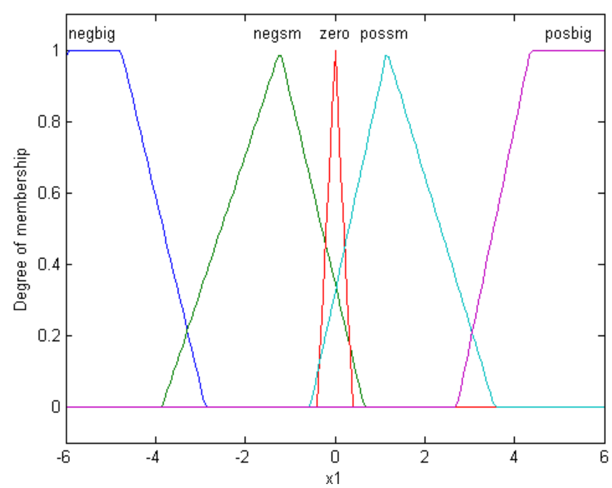
Figure 4: Output surface of the spring fuzzy process model.

non-collocated nature of this problem, designing for robust disturbance rejection requires the use of state observers to model disturbances and other uncertainties, such as position of the masses. In the deterministic case, when no random noise is present, the Luenberger observer and its extension may be used for time-invariant systems with known parameters. When parameters of the system are unknown or time varying, an adaptive observer is preferred. The corresponding optimum observer for a stochastic system with additive white noise processes, with known parameters, is the Kalman filter. As indicated earlier, this project assumes full state feedback of masses 1& 2.

C. Robust Tracking and Stability

With the system in a rigid-body mode, due to the damping effects on the interconnecting spring, it is evident that the position and velocity of body 2, x_2 and $\dot{x}_2$, are fixed relative to body 1. Hence, measuring $\dot{x}_1$ gives us $\dot{x}_2$, while the displacement of x_1 from its initial position at rest is equivalent to the displacement of x_2 from its own initial position. Essentially, the problem has been reduced to one that can be solved with a collocated controller on body 1. In robust control, collocation guarantees the asymptotic stability of a wide range of SISO control systems, even if the system parameters are subject to large perturbations, while also enabling the achievement of desired performance objectives.

We also use an additional Mamdani fuzzy controller which receives the position and velocity of body 1, x_1 and $\dot{x}_1$, as inputs and outputs an appropriate control action. This output is superimposed directly on the output of the spring controller to obtain the final control action on the system. The controller utilizes a qualitative partitioning of x_1 and $\dot{x}_1$ using five membership functions as shown in Figure 5. The input partitions of negbig (Negative), negsm (Negative_small), Zero, possm (Positive_small) and posbig (Positive) produce output partitions of nb (Negative), ns (Negative_small), Zero, ps (Positive_small) and pb (Positive), which represent the control force on body 1.



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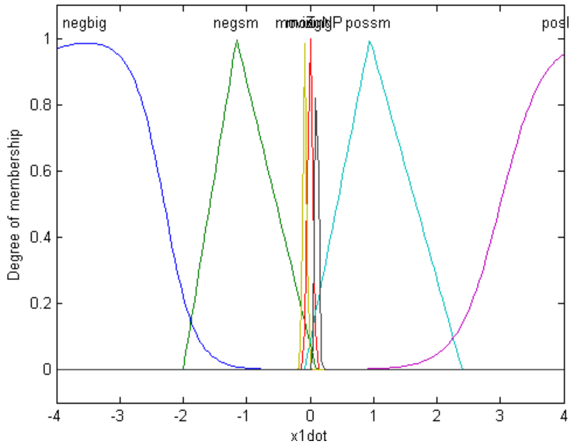

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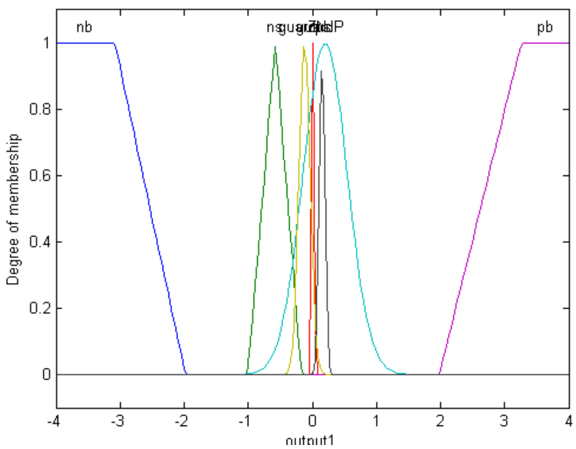

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Figure 5: Fig. 5 (a) Membership functions of position of body 1 x_1 Fig. 5 (b) Membership functions of the velocity of body 1 $\dot{x}_1$ Fig. 5 (c) Membership functions of output.

The observed decision surface of Figure 6 shows that the corresponding output produced, for a given set of inputs, has a somewhat inverse linear relationship to those inputs. Two special membership functions, movingN and movingP, with output membership functions of guardP and guardN respectively, were also added to $\dot{x}_1$ (velocity of body 1) to ensure full stability.

Simulation Results

The performance of our fuzzy controller was investigated using computer simulations in Simulink® and MATLAB®. Figure 7 shows the response to a unit impulse disturbance to m_2, w(t) at t=0, for the nominal plant parameters $m_1 = m_2 = 1$ and $k = 1$. The controller shows excellent vibration suppression properties as the position initially increases from 0 to 1.068 units before returning and staying bounded within the required ± 0.1 units of the final value in 4.8s. System stability was obtained as required in the design specifications, and a reasonable maximum value of u was obtained to be 1.262 units as shown in Figure 8.

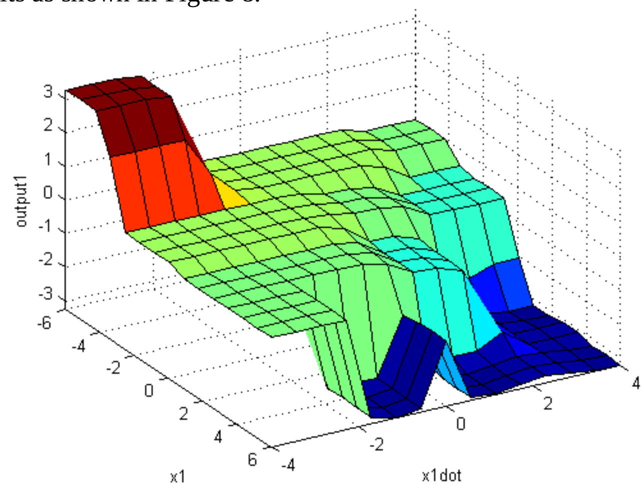


Figure 6: Output surface of the controller.

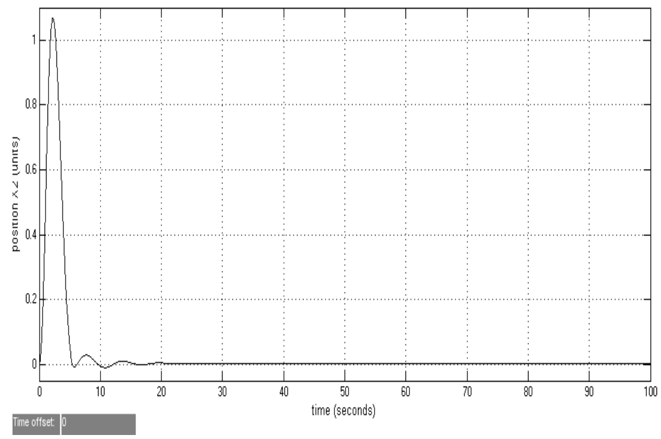


Figure 7: Time series of position of body 2, x_2, after a unit impulse disturbance on m_2 for nominal plant parameters $m_1 = m_2 = 1$ and $k = 1$. Settling time (Ts) = 4.8 seconds, Peak time (Tp) = 2.2 seconds and Peak Value (Pv) = 1.068 units.

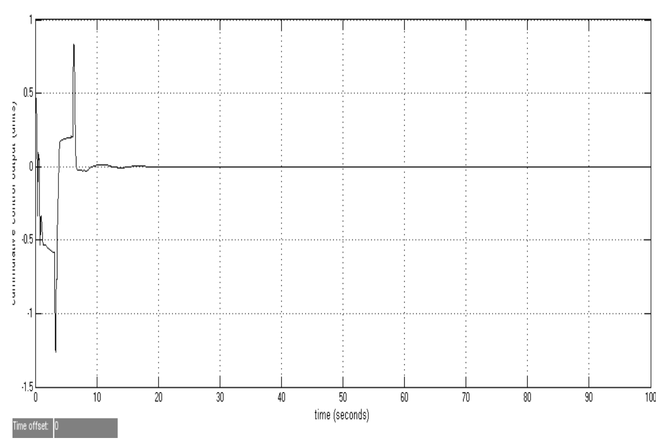
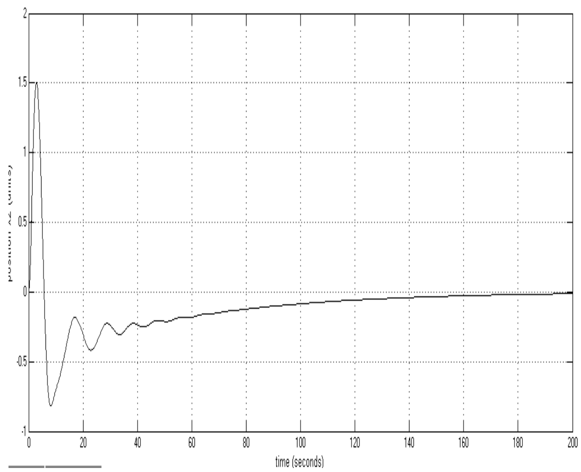


Figure 8: Time series of cumulative controller output u after a unit impulse disturbance on m_2 for nominal plant parameters $m_1 = m_2 = 1$ and $k = 1$. Maximum value of $u = 1.262$ units.

Figure 9 shows the stability of the system to varying spring constants in the range 0.5<k<2.0, while Table 1 summarizes the performances of the controller as compared to design objectives.



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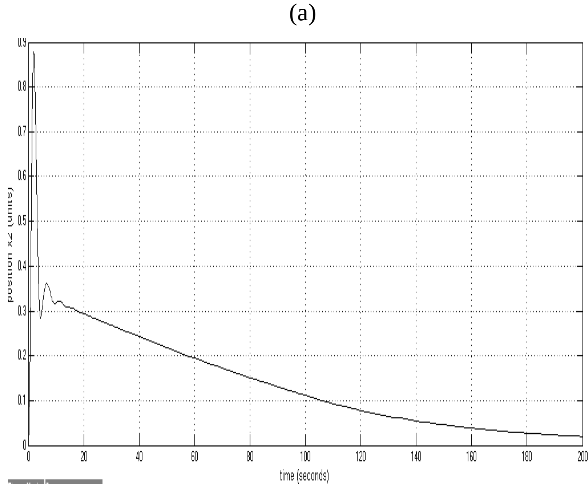

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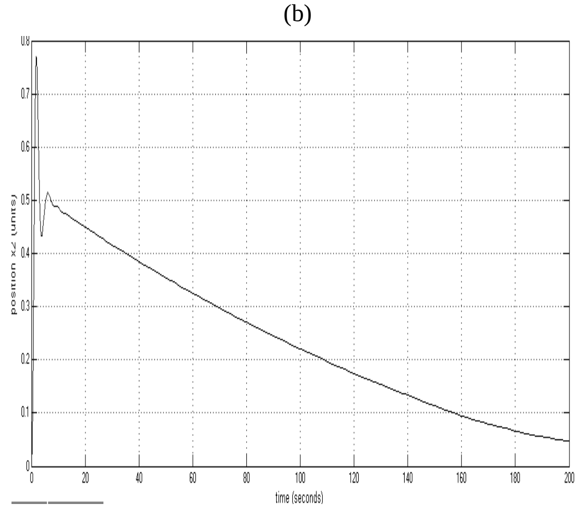

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Figure 9: Time series of position of body 2, x_2 after a unit impulse disturbance on m_2 for nominal plant parameters $m_1 = m_2 = 1$. Fig. 9(a) k=0.5 Fig. 9(b) k=1.5 Fig. 9(c) k=2.0

Table 1. Controller performances for nominal plant parameters $m_1 = m_2 = 1$ and , in comparison with other fuzzy-logic based solutions to the benchmark problem (base on Linder and Shafai 1999, and Cohen and Ben Asher 2001)

	Our Controller	Linder and Shafai A	Linder and Shafai B	Cohen and Ben Asher	Design objectives
Settling time (Ts)	4.8	15.0	8.0	8.8	15.0
Max controller output *u*	1.262	-	-	0.53	--

It is evident that the fuzzy logic-based controller solves the first two of the benchmark problems. It, however, achieves better settling time performance over other fuzzy logic solutions, while staying within the requirements of reasonable controller output. This is due to unique fuzzy membership function placements and tunings, especially for stability and robust tracking.

Also, as the value of the spring constant k is increased the peak time and peak value decreases simultaneously. This is due to the fact that an increase in the spring constant allows the system to exhibit more inherent natural dampness that ensures less oscillations or more rigidity. This, however, increases the settling time significantly as the controller has less "control" over the system. The designed controller has been optimized for the case where k=1. This can be repeated for other values of spring constants in order to achieve better performances.

Conclusion

This paper uses a superimposition of qualitative stability and tracking behaviors instantiated with fuzzy rules which have clear linguistic interpretations. The impressive performance of the fuzzy logic controller on the ACC robust control benchmark shows its suitability for designing and developing controllers for stability and performance robustness in view of plant uncertainties, and sensitivity to actuator/sensor noncollocation. Of significant interest is the fact that the developed control strategy leads to robust near time-optimal control while requiring a relatively small amount of control effort.

Further studies can be pursued to test and improve the controller presented herein for the vibration suppression of structures, such as beams, plates, shells, and those possessing very high modal densities at lower frequencies. Also, the effects of high frequency sensor noise can be modeled in to the system, and a stochastic robustness analysis, using Monte Carlo simulations, can be used to obtain performance metrics, as estimated probabilities of stability/performance.

References

Wie, B., and Bernstein, D. S. 1992. Benchmark Problems for Robust Control Design. *Journal of Guidance, Control, and Dynamics*. Vol. 15, No.5: 1057-1059.

Linder, S. P., and Shafai, B. 1999. Qualitative Robust Fuzzy Control with Applications to 1992 ACC Benchmark. *IEEE Transactions on Fuzzy Systems.* Vol. 7, No. 4: 409-421.

Niemann, H. H., Stoustrup, J., Tofnner-Clausen, S., and Andersen, P. 1997. Synthesis for the Coupled Mass Benchmark Problem. In Proceedings of the American Control Conference, 2611-2615. Technical University of Denmark, Denmark.

Wie, B., and Liu, Q. 1992. Robust Control Design for Benchmark Problem #2. Arizona State University, Tempe.

Farag, A., and Werner, H. 2002. Robust Controller Design and Tuning for the ACC Benchmark Problem and a Real-time Application. IFAC 15th Triennial World Congress, Barcelona, Spain.

Hughes, D., and Wu, J. T. 1996. Passivity Motivated Controller Design for Flexible Structures. *Journal of Guidance, Control, and Dynamics.* Vol. 19, No. 3: 726-729.

Cohen, K., and Ben Asher, J. Z. 2001. Control of Linear Second-Order Systems by Fuzzy Logic-Based Algorithm. *Journal of Guidance, Control, and Dynamics*. Vol. 24, No. 3: 494-501.

Stengel, R. F., and Marrison, C. I. 1992. Robustness of Solutions to a Benchmark Control Problem. *Journal of Guidance, Control, and Dynamics*. Vol. 15, No. 5: 1060-1067.

King, P. J., and Mamdani, E. H. 1977. The Application of Fuzzy Control Systems to Industrial Processes. *Automatica*. Vol. 13, No. 3: 235-242.

Zadeh, L. A. 1965. Fuzzy Sets. Inform. Contr., Vol. 8: 338-353.

Jamshidi, M., Vadiee, N., and Ross, T. 1993. Fuzzy Logic and Control: Software and Hardware Applications. Englewood Cliffs, NJ: Prentice-Hall.

Yen, J., Langari, R., and Zadeh, L. 1992. In Proceedings of 2nd Int. Workshop Indust. Fuzzy Contr. Intell. Syst. College Station, Texas.

Zadeh, L. A. 1994. Fuzzy Logic, Neural Networks, and Soft Computing. Communications Association of Computing Machinery. Vol. 37: 77-84.

Driankov, D., Hellendoorn, H., and Reinfrank, M. 1996. An Introduction to Fuzzy Control, 2nd ed. New York: Springer-Verlag.

Jang, J.-S. R., and Sun, C.-T. 1995. Neuro-Fuzzy Modeling and Control. In Proceedings of IEEE. Vol. 83: 378-406.