Visualizing Syllogisms: Category Pattern Diagrams versus Venn Diagrams

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Abstract. A new diagrammatic notation for Syllogisms is presented: *Category Pattern Diagrams, CPDs*. A CPD configures different styles of line segments to simultaneously assign quantification values to categorical variables and relations among them. The design of CPDs attempts to coherently visualize the structure of syllogisms at various conceptual levels. In comparison to Venn Diagrams and conventional verbal expressions of syllogisms, the potential benefits of CPD may include: a relatively straightforward inference method; simple rules for evaluating validity; applicability to multiple (>2) premise syllogisms.

1 Introduction: the project and programme

This paper is part of a project that is attempting to develop a novel set of related diagrammatic notations for various systems logic. The project's objective is to design a family of diagrammatic notations that share a common representational scheme for encoding logical states of affairs and a common inference method. The focus here is syllogisms, with the introduction of *Category Pattern Diagrams*, CPDs. The design of CPDs builds directly upon our previous work on *Truth Diagrams* for propositional logic [1,2]. In turn, CPDs are being used as intermediate stage to develop a related notational system for full predicate calculus. The overarching aim of the project is to show how re-codifying systems of logic in closely related notational systems may reveal the similarities and differences in the conceptual structures of those logics.

The project is, in turn, part of a larger *Representational Epistemic* research programme that is studying how notational systems encode knowledge and the potential cognitive benefits that novel codifications of knowledge may confer on higher forms of thinking [3-6]. The core principles of the Representational Epistemic approach address how to design representational systems for knowledge rich topics. They claim that directly encode the fundamental conceptual structure of a topic in coherent notational schemes will provide semantically transparency and thus enhance problem solving and conceptual learning in multiple ways [3-5]. Previous knowledge domains that have been re-codified as part of the programme include electricity, probability

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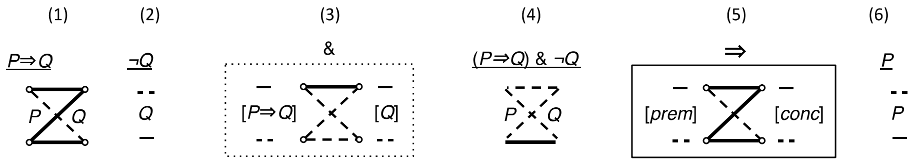

Fig. 1. Truth Diagram demonstration of the validity of Modus Tollens

theory and (school) algebra [3-5]. The project on logic is extending the scope of the programme by providing further stringent test cases for the Representational Epistemic claims.

The previous work in the logic notation design project developed *Truth Diagrams*, TDs, to re-codify propositional logic (and Boolean Algebra) [1,2]. Fig. 1 shows an example in which the validity of Modus Tollens is demonstrated. The details of TDs are not essential to consider here, rather it is the overall form of the notation that is of concern, because the design of CPDs aims to adopt a similar representational scheme and inference method. In TDs letters are labels for variables and configurations of line segments assign truth-values to propositional variables and relations among those variables. Solid lines stand for True and dashed for False. Fig. 1.1, 1.2, 1.4 and 1.6 are unary or binary relations of variables involving *P* and *Q*. The inference method creates a composite diagram, Fig. 1.4, by combining the premise diagrams, Fig. 1.1 and 1.2, using a diagrammatic operator, Fig. 1.3, which specifies the types of the lines to draw in the composite diagram given the permutations of line types in the premise diagrams. The validity of the inference is determined by comparing the structure of the composite diagram with the diagram for the given conclusion, Fig. 1.6, using a simple set of diagrammatic validity rules, Fig. 1.5, which specifies correct correspondences between the types of lines in the two diagrams. TDs constitute an efficient method to reveal how the propagation of patterns of truth-values determines the structure and validity of interferences. Taken together, the structure of the diagrams, the composition operators and the validity rules provide a novel, complete and sound, system that reveals conceptual structures (symmetries and regularities) on multiple levels that are typically hidden by standard formula notation [1,2].

The specific aims of this paper are: (a) to introduce *Category Pattern Diagrams*, *CPDs*, as a notation for syllogistic reasoning, that adopts a similar representational scheme and inference method to TDs; (b) to compare CPDs with syllogistic inferences using Venn diagrams and the traditional verbal approach; (c) to examine how codifications of syllogisms in these alternative notational systems provides quite different perspectives on the underlying conceptual structure of syllogisms, with varying degrees of coherence, and the impact this has on the ease of making inferences. Thus, the paper has the following sections: 2 is a brief reminder about syllogisms; 3 describes the graphical structure of CPDs; 4 gives the procedures of composing premise diagrams in to a result diagram; 5 provides the method to determine whether the result diagram correctly implies the given conclusion diagram; 6 extends the approach to multi-premise syllogisms, sorites; and, 7 discusses the overall efficacy of the CPD encoding of syllogisms and considers implications for the design of notations to encode logic.

2 Syllogisms: a brief reminder

See [7] and [9], for example, for full introductions to syllogisms; but as a reminder, consider syllogisms S1 and S2.

S1. No diagrams are sentential notations
All Venn Diagrams are diagrams
No Venn Diagrams are sentential notations

S2. All Category Pattern Diagrams (CPDs) are diagrams
All diagrams are effective representations
No effective representations are poor systems for learning
Some poor systems for learning are sentential notations
No CPDs are sentential notations

S1 is a classical two-premise syllogism, consisting of a *major premise*, a *minor premise* and a *conclusion*. The *middle* term, *M*, occurs in both premises and the *predicate* and *subject*, *P* and *S*, are the *major* and *minor* terms of the major or the minor premises, respectively. (M is a subject in the major premise and a predicate in the minor premise.) The *quantity* and *quality* of S1's *major* premise happens to be *universal* and *negative* (*No M are P*); such propositions are labelled 'E'. S1's *minor* premise is universal and affirmative (*All S* are *M*); labelled 'A'. The conclusion is also an E proposition (*No S are P*). *Particular affirmative* propositions are labelled 'I' and *particular negative* propositions labelled 'O'. The *mood* of a syllogism is its particular permutation of proposition types for the two premises and the conclusion: S1's mood is EAE. *S* always precedes *P* in syllogism conclusions. The four possible permutations of the order of the premise variables are called *Figures*; S1 is of Figure type 1: *M-P*, *S-M*. The Mood and Figure type of S1 may be summarised as 'EAE-1' and like all valid syllogisms has been given a name, "Celarent" [1,2].

To determine the validity of syllogisms in verbal form, one may apply five rules concerning the quality and quantity of the propositions. The quality rules state: (QL1) no conclusion may follow when both premises are negative; (QL2) a conclusion is negative when either premise is negative; (QL3) a negative conclusion cannot follow from two affirmative premises. The quantity rules rely upon the notation of *distribution*, which is the extent to which all the members of a category are affected in a proposition [7]; e.g., *S* is distributed in *All S are P*, but variable *P* is not. The quantity rules state: (QN1) the middle term must be distributed in one or both premises; (QN2) if a term in the premises is not distributed, then it must not be distributed in the conclusion. These rules are challenging to understand and apply, and explanations of why they govern the validity of inferences are not straightforward to give.

Venn Diagrams, e.g. Fig. 2, provide a more comprehensible means to assess the validity of syllogisms. First, a diagram is drawn with three fully intersecting circles to represent all the possible combinations of sub-sets, Fig. 2.3. Then, beginning with any universal premises, Fig. 2.1, corresponding regions in Fig. 2.3 are shaded for empty sets. Subsequently, for any particular premises, Fig. 2.2, a cross is drawn in any corresponding non-shaded region of Fig. 2.3. Care is needed to correctly locate

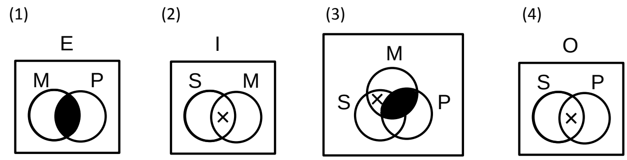

Fig. 2. Venn Diagram for the Ferio syllogism

the shading and crosses to take into account the term not mentioned in each premise. The inference is valid if the conclusion, Fig. 2.4, can be read directly from the pattern of shading and crosses in the three-circle diagram.

S2 is a sorites, a multiple premise syllogism. Their general form is P_1–P_2, P_2–P_3, …, P_{n-1}–P_n ⇒ P_1–P_n. The particular form of S2 is: *All C are D, All D are E, No E are P, Some P are S* ⇒ *No C are S*. Although Venn Diagrams can be systematically drawn for four and more sets [8], the diagrammatic benefits of that approach appear to be reduced for larger numbers of premises.

From this brief overview, it is clear that to re-codify syllogisms CPDs must do many things: identify the categorical variables; denote whether things belong to each category or not; specify relations among the variables, i.e., the mode and Figure; signify the quantification of the multiple subsets defined by those relations; have the potential to represent multiple premises (>2); provide a method to infer the quantity values of variables in the combined relations; establish a procedure to determine whether inferences correctly imply the given conclusion.

3 Graphical structure of CPDs

Fig. 3 shows examples of CPDs for unary, binary and ternary relations of categorical variables. Each diagram represents a state of affairs relating the categories identified by the letters. The letter is a label for a category and the lines run between the specific positions relative to the letters. In a unary variable diagram, Fig. 3.1, horizontal line segments are positioned above and below the letter. For the binary relation, Fig. 3.2, four lines run between the letters with their ends located at the four possible combinations of positions above or below each letter. We will call such line segments *connectors*. In the ternary relation diagram, Fig. 3.3, the eight connectors are composed of two binary connectors joined near the middle letter and with free ends associated with the other two letters. The eight connectors are arranged as four pairs: (1) an inverted triangular pattern; (2) an upright triangular pattern; (3) a descending parallelogram pattern, which slopes downwards from left to right; (4) an ascending parallelogram pattern. As will be seen below, this design of the ternary CPDs is intended

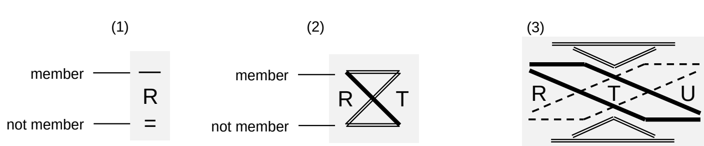

Fig. 3. Unary, binary and ternary Category Pattern Diagrams

to support judgments about the validity syllogisms.

The position of the ends of a connector, top or bottom, refers to possible membership or possible non-membership of the category, respectively. Fig. 3.1 and 3.2 include labels making this explicit. Each connector is a particular *case*, a combination of inclusion or exclusion of things in the subsets of the variables of the relation. The unary relation has two cases and the binary relation has four cases.

Consider examples of some cases in the ternary CPD of Fig. 3.3. In the top inverted triangle the upper straight connector refers to the case of the membership of all variables, where as the \/ shape connector is the case of subsets *R* and *U* membership but *T* not. In general, the local altitude of the middle point or free ends of a connector within a triangle or parallelogram indicates the membership status of a subset, with a high position in the shape for membership and low position for absence. In the descending parallelogram the ¯\ connector refers to the membership of *R* and *T* and the absence of *U*, whereas the _ connector refers the membership just of *R*.

The line style of the connector assigns a quantity to a case. There are three styles for three quantifiers: (1) a single solid connector is *some* – at least one instance of the case; (2) a dashed connector is *none* – no instance of the case; (3) a solid double-line connector means *no information* (*no-info*) – the quantification of the case is not known; it may either be *some* or *none*. In Fig. 3.1 the top some *connector* specifies that something is a member of *R* and the bottom *no-info* connector means it is not known if things are excluded from *R* or not. In Fig. 3.2 the three double-line connectors means the only specific information provided relates *R* and *not T*, and its solid single-line connector says that at least one thing is a member of R is not a member of T: in other words, the diagram reads *Some R are not T*. Consider three cases in Fig. 3.3. The double-line top connector of the upper triangle pair says that there is nothing known about the assignment of members to the intersection of *R*, *T* and *U*. In the descending parallelogram the solid line of the lower _ shape connector indicates that at least one thing is a member of *R* but it is absent from *T* and *U*. The dashed line of the /¯ connector says there is nothing that is *T* and *U* and not *R*.

Each connector in a CPD is equivalent to a region in a Venn diagram.

Fig. 4 shows CPDs diagrams for the four syllogistic propositions, A, E, I and O (and gives their verbal expressions). Notice that all the CPDs have three *unknown* connectors (double-lines) and either a single *some* or a *none* connector to constitute the particular and universal propositions. The intersection of the two sets is the top connector; the two exclusive subsets of the variables are the ascending and descending diagonals; the exclusion of both sets is the bottom connector. When the order of the terms in a proposition is swapped, the order of the letters in the CPD is simply reversed. Equivalently, the pattern of the lines may be reflected with the letter positions fixed. (If both the letters and lines are reversed, the proposition is unchanged.)
Notice that the patterns of lines in E and I are symmetrical, which has interesting implications for the validity of certain syllogisms; as will be seen below.

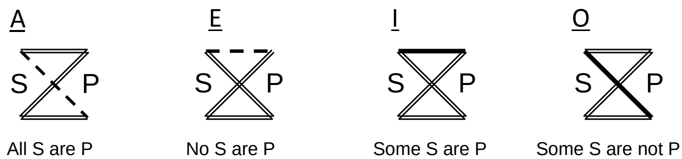

Fig. 4. The four types of syllogistic propositions as binary CPDs

Turning to ternary CPDs, Fig. 5.2 shows a generic ternary relation CPD with numbered connectors that show the corresponding regions of the Venn diagram in Fig. 5.1. Fig. 5.3 and 4 are two examples of specific ternary relations. In Fig. 5.3 the descending parallelogram says there is *nothing* that is *S and not P*, whatever the case with *M*. The ascending parallelogram says that *no-info* holds for *not S and P*, for both values of *M*. The top and bottom triangles both possess cases in which either there are no instances present or that *no-info* occurs, for different values of *M*. Fig. 5.4 shows other patterns of connectors including the assignment of *some* to one case. With a little experience, identifying individual cases in CPDs appears to be as easy as finding sub-sets in a Venn diagram. Similarly, selecting pairs of cases for the same values of *S* and *P*, as is required to judge the validity of a syllogism, also appears to be comparable in both notations. However, we will see below that the CPDs and Venn diagrams diverge when more than three propositions are considered.

That completes the overview of the syntax and semantics of relational CPDs. The next section considers how to make inferences with CPDs.

4 Composition of Binary CPDs

Fig. 6 shows the CPD for the *Celarent* syllogism (EAE-1: *No M are P*, *All S are M*, therefore *No S are P*; Figure type 1). In outline, the overall procedure for syllogistic inferences with CPDs has two stages. First, given the two premises (Fig. 6.1 & 6.3), the conjunction operator (6.2, see below for a full explanation) is applied to generate the ternary *result* diagram (6.4). (The term *result* refers to the set of implications derived from the premises as distinct from the given conclusion.) In the second stage, the pairs of connectors of the result diagram are compared to the conclusion diagram (6.5.E) to check that the result diagram fully and correctly implies the conclusion diagram. (In Fig. 6 the desired conclusion (6.5.E) is highlighted but three others are included for the discussion of invalid inferences below.) This stage compares the types of connectors in the result diagram with the corresponding conclusion connectors using a table of validity rules (6.6). This section describes the construction of the ternary result diagram and the next section gives the procedure for testing validity.

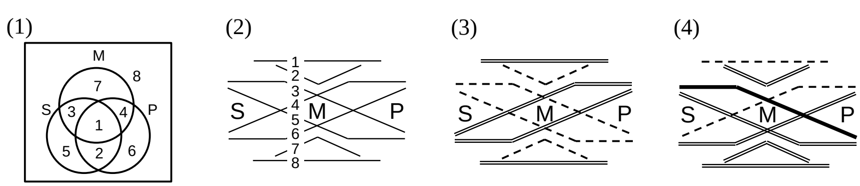

Fig. 5. Ternary CPDs

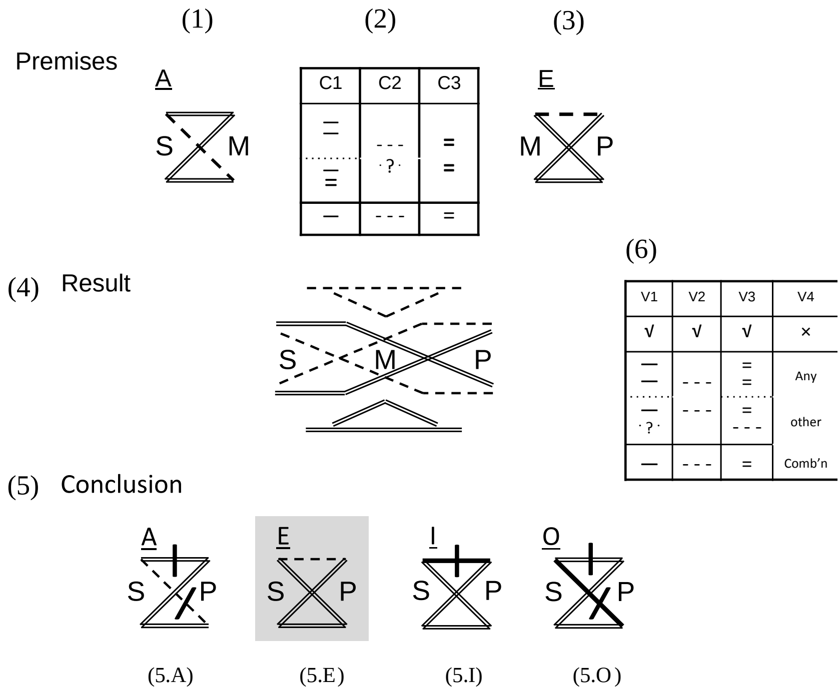

Fig. 6. EAE-1 Syllogism (and some alternative conclusions)

To construct a ternary result diagram we simply consider each of the eight connectors in the CPD in turn. Fig. 7 shows two examples of the construction of two connectors in the result CPD of Fig. 6.4. Three steps are required for each connector. Step 1 – Fig. 7, arrows 1: determine the shape and position of the new connector from the relevant connectors in the two premise CPDs. Step 2 – arrows 2: from the styles of the pair of premise connectors find the relevant composition rule. Step 3 – arrows 3: the style of the new result connector is given by the output of the selected rule.

In preparation for step 1, the two premise diagrams are drawn so that the *middle* term (*M*) will be in the centre of the new diagram and the subject term (*S*) on the left and predicate term (*P*) on the right, see Fig. 6 and 8. *M* is in the middle because it is common to both binary premise diagrams. The *S* and *P* arrangement will facilitate the comparison of the result diagram with the conclusion diagram later (see below). Fig 8.1 illustrates this process for a ternary CPD of no particular mood (faint lines for arbitrary connector types). If *S* is to the right and *P* to the left of *M* in the premise diagrams, as in Figure type 1 syllogisms such as Fig. 6, they can simply be put together without further ado. If the premises are different syllogism Figures, then one or both of the premise diagrams is reflected before they are combined; for example, in a type 2 *Figure* syllogism, the *M* term occurs on the right of both the binary premise diagrams, so just the *P-M* diagram needs to be reversed. Thus, all the possible moods and Figures of syllogism handled.

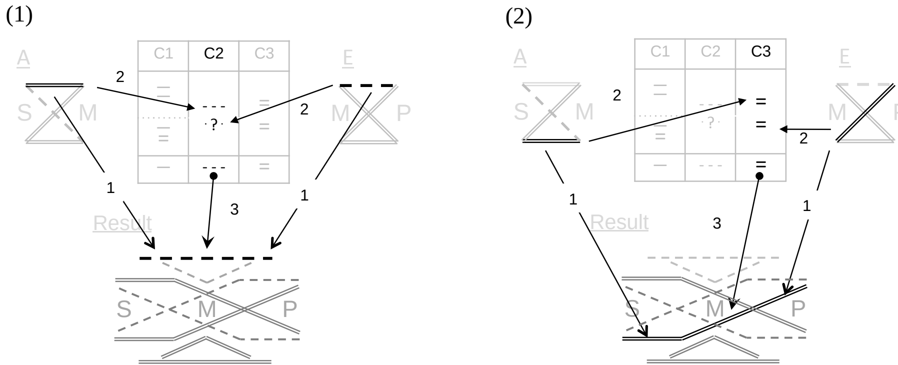

Fig. 7. Composition of ternary CPDs – connector shape and style

Now, step 1 builds each ternary result connector from possible pairs of the premise connectors. The top connector of result CPD, Fig. 7.1 (arrows 1), combines the top connectors of the premises. The _/ shape result connector, Fig. 7.2 (arrows 1), is assembled from the bottom and ascending diagonal connectors of the premises. In general, for each connector in one binary diagram there are two possible associated connectors in the other diagram. Fig. 8.2 shows how the four pairs of connectors in Fig. 3.3 and 6.4 are obtained from the binary diagrams in Fig. 8.1. Each of the four patterns in Fig. 8.2 corresponds to a particular case of *S* and *P* values, but the values of *M* differ.

In Step 2, we find the quantification value for the new connector by looking up the values of the premise binary connectors in the composition operator look up table in Fig. 9. A copy of this table is reproduced between the two binary diagrams in Fig. 6 and 7 for convenience. Given the three possible types of each of the two premise connectors, 3^2=9 permutations are possible. The table determines mappings from pairs of premise connector types at the top of each column to the result connector type at the bottom. The ‘?’ symbol in Fig. 9 means any type of connector. (C1) The result of the operator will obviously be a *some* connector when both premises are *some* connectors. Whenever one premise connector is a *some* connector and the other a *no-info* connector, the result is also a *some* connector, because just one premise possessing a member will ensure that the new case contains a member. (C2) When both of the connectors are *no-info* types, combining them provides no new information; therefore, the result is also a *no-info* connector. (C3) Given a single *none* connector, or pair of them, the result must be a *none* connector, because the presence of any members of the new combined category is forbidden. For example, in Fig. 7.1 rule C2 applies to the left *no-info* and the right *none* connectors, so the new connector will be have *none* style.

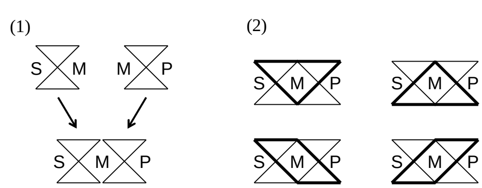

Fig. 8. Composing ternary CPDs

In step 3, we simply draw the result connector in the style given by the output of the rule selected in step 2, in the position determined in step 1; in Fig. 7.1 (arrow 3) this is a dashed top connector. In Fig. 7.2 the bottom and the ascending diagonal premise connectors will give a _/ shaped result connector (step 1), rule C3 applies because both premise connectors are both *no-info* (step 2), so the result connector is drawn in position as a *no-info* connector (step 3). Repeating the steps for the other six connectors completes the result CPD. As the two premise CPDs in Fig. 6 both possess just *no-info* or *none* connectors the resulting ternary CPD contains only connectors of these types.

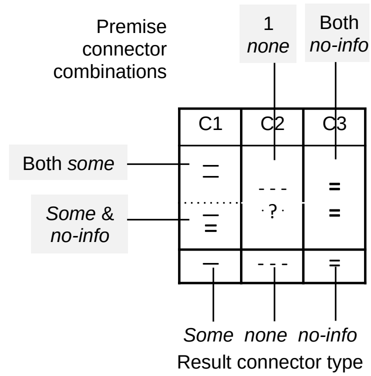

Fig. 9. Composition operator rules

5 Determining Validity

The second stage of the CPD approach compares the result diagram with the given conclusion diagram to establish whether, or not, each case of possible assignments of values of *S* and *P* in the conclusion is validly implied by the two possible cases for the same assignment of *S* and *P* in the result. As noted, the pairwise design of the connectors in the ternary CPDs supports these comparisons. Fig. 10 shows the correspondence between the pairs of result connectors and the conclusion connectors: the upper result triangle maps to top conclusion connector; the descending parallelogram to the descending connector; the ascending parallelogram to the ascending connector; the bottom triangle to the bottom connector. The subsets of *S* and *P* are the same in the result and conclusion for each matching case.

For each of these matches, we now determined whether the types of the two result connectors correctly imply the type of the conclusion connector. Fig. 11 provides a look up table for valid matches, where each column is a validity rule. (V1) If either or both of the result connector types are *some*, then the conclusion connector is *some*, because the presence of any member in the result implies the conclusion will have a member. (V2) Two *none* result connectors imply a *none* conclusion, because the total absence of any category members in the result implies an absence of members in the conclusion. (V3) Two *no-info* connecters, or one with a *none* connector, implies a *no-info* conclusion connector, because these combinations provide no information about whether there are category members or not. (V4) No other permutations of result and conclusion connectors are valid. Given that each of the three connectors may be one of three types, a total 3^3 different permutations exist, so the four rules of Fig. 11 constitute

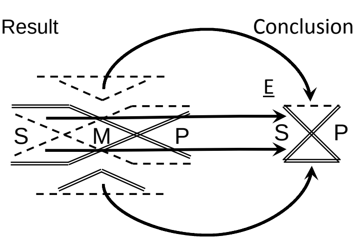

Fig. 10. Matching result connectors to the conclusion connectors

a concise encoding of the 27 possible ways result connectors may, or may not, validly imply the conclusion. Again, this conciseness may be attributed to the representation of the possible quantification values as three styles of lines.

Now applying the validity rules to Fig. 6, the *none* top connector of the target conclusion (Fig. 6.5.E) is correctly implied by the result, because the upper triangle has two *none* connectors (Fig. 6.4) – Rule V2. The descending parallelogram correctly implies the respective *no-info* descending conclusion connector, because the result parallelogram has one *no-info* connector and one *none* connector – V3. This rule applies to the ascending parallelogram in the same fashion. It also applies to the bottom triangle but in respect to the two *no-info* connectors. Therefore, as all four of the result connector pairs correctly imply their conclusion connectors, the overall inference is valid. Had just any one of these matches been invalid, the overall implication would have been invalid.

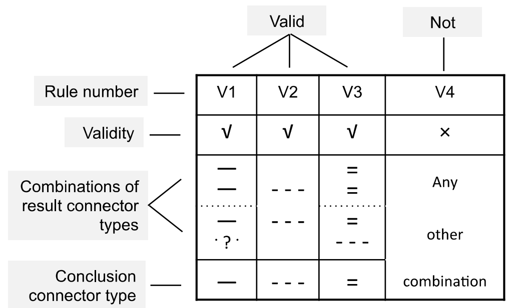

Fig. 11. Validity rules

The other types of proposition, A, I and O, are shown in Fig. 6 as alternative conclusions, which we now demonstrate are not implied by the conjunction of premises E and A; i.e., EAA-1, EAI-1 and EAO-1 are not valid syllogisms. The bars on the conclusion connectors in Fig. 6.5.A/I/O identify those that are not satisfied in the result diagram. In the case of the A conclusion, the top *no-info* connector is not implied by the pair of *none* connectors in the upper triangle (V4 true, V3 violated), and the descending *none* diagonal is not implied by a single *none* connector in the parallelogram (V4 true, V2 violated). For the I proposition the top *some* connector is not implied, because there is no *some* connector among the pair of in the upper triangle of the result (V1 violated), and similarly for the *some* descending connector in the O proposition (Fig. 6.5.O).

Fig. 12 derives the valid *Ferio*, *Festino* and *Ferison* and *Fresison* syllogisms (EIO-1, 2, 3, 4), and has three points of interest. (1) The presence of the *some* connector yields a *some* connector or a *none* connector in the result CPD when it is combined with a *no-info* or a *none* connector, respectively, from the other premise. (2) The match of the *some* connector in the result diagram and the O conclusion satisfies V3, but the A, E and I conclusions neatly show how different forms of mismatch are easily spotted. (3) Both premise diagrams are symmetric, because their only non *no-info* connectors are the top lines, which means that the overall configuration of the result ternary CPD is invariant: the orders *S*, *P* and *M* does not matter, which is why the EIO mood is the only one that is valid for all four *Figures*. By the same reasoning, this explains why valid syllogisms often occur in pairs; they have an E or I as a premise.

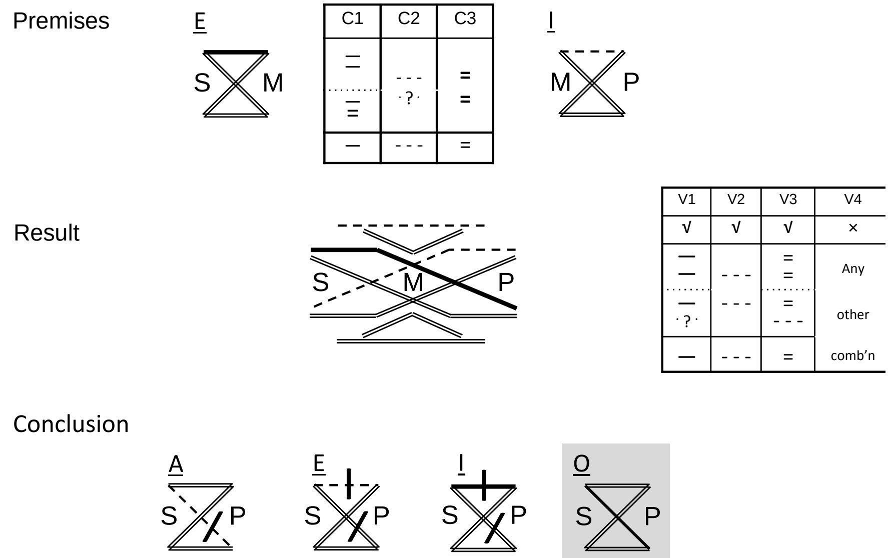

Fig. 12. EIO-1/2/3/4 syllogism (and some alternative conclusions)

Is the CPD approach for classical syllogisms complete and sound? All the 256 possible combinations of mood and figures have been examined. CPDs are complete because all 15 valid syllogisms [7] are found to be valid in the approach. It is sound because none of the 241 invalid syllogisms [7] are found to be valid. (As the composition and validity rules are few in number and simple, a spreadsheet was setup to test all 256 syllogisms en masse.)

6 Sorites

The CPD approach extends beyond classical syllogisms. Fig. 13 shows two examples of sorites, or polysyllogims. In each, the sequence of premises is on the left and the conclusion on the right. A ternary CPD has eight connectors, and as each additional proposition doubles the number of cases, quaternary and quinary CPDs will have sixteen and thirty-two connectors, respectively, so would consequently be cumbersome to draw. However, given the relative simplicity of the composition rules and validation rules it is not essential to expand the row of premise CPDs, but rather we may consider possible paths along connectors from the first variable through to the last. The composition rules in Fig. 9 may be applied iteratively to a sequence of connectors. The top row of Fig. 13.1 are all *no-info* connectors, so Rule C3 (Fig. 9) yields an overall *no-info* path. The \ _ _ shaped path has one *none* connector and two *no-info* connectors, so its overall path is *none*. Is this sorite valid? In an equivalent fashion to Fig. 10, all the paths through the premises from a specific start point to a specific end point are compared to the conclusion connector that has corresponding points; for example, paths from the top-left to bottom-right through of the sequence of

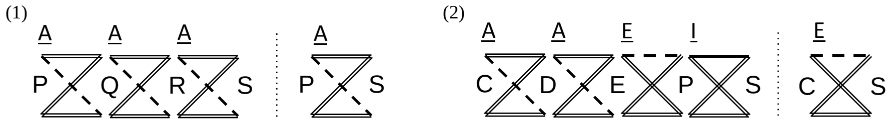

Fig. 13. Two sorites as CPDs

premises (top *P* to bottom *S*) corresponds to the descending diagonal connector of the conclusion (top *P* to bottom *S*). The rules in Fig. 11 are used to judge whether all the types of these paths correctly imply the conclusion connector type. The top connector of the conclusion of Fig. 13.1 is a *no-info* connector and by inspect we can see that all paths (‾‾‾‾‾, \/‾‾, _/, ‾‾\/) from the top left to the top-right of the premises are either *no-info* or *none* paths by rules C2 and C3. Thus, all four paths satisfy V3. Similarly, the ascending connector in the conclusion is correctly implied by the four paths from the bottom-left to top-right, because there is at least one *no-info* path and the rest are *none* paths, applying C3, C2 and V3. The same is true of bottom conclusion connector and the bottom-left to bottom-right premise paths. The descending line in the conclusion is a *none* connect. Again by inspection, we see that all four paths from the top-left to the bottom-right contain one or two *none* connectors, so by rule C2 all the paths are *none* paths, which means that the conclusion is correctly implied (V2 satisfied). As all the conclusion connectors are correctly implied the overall inference is valid.

Our second syllogism above, S2, is a four-premise inference with a mixture proposition types (A, E and I). The letters of the variables in Fig. 13.2 have been chosen to match the terms in the S2. Although this example is more complex than Fig. 13.1, testing its validity is relatively straightforward. Consider the top *none* connector of the conclusion. Rule V2 say that all premise paths must be the *none* type for this to be correct, however we immediately see that there is a path consisting only of *no-info* connectors, ‾‾\/, so this case is not valid, and in turn the overall inference is invalid: QED. (Testing the other cases is not arduous. All the other premises paths correspond to three *no-info* connectors in the conclusion. By inspection all the cases include at least one *no-info* path (C3) and *none* paths as the only other type (C2), so all have a mixture of *no-info* and *none* paths, therefore all three conclusion connectors are correctly implied, because the conditions for V3 are met. Nevertheless, the sorite is invalid, because the validation of top conclusion connector failed.)

This inspection method may, of course, be applied to two-premise syllogisms, and is simpler than constructing of the ternary result diagram (Fig. 6 and 12). However, the ternary result diagrams are nevertheless worth considering, because they provide an explicit introduction to the analysis of the structure for binary CPDs sequences that is needed to familiarize learners on the composition of connectors and about the matching of multiple connectors to test validity. Quaternary CPDs can be drawn with four groups of distinct patterns of four connectors that serve the same role as the four pairs of connectors in ternary CPDs. However, they are cumbersome, because they include 16 distinct lines. Clearly, higher order CPDs will be impractical to draw. Fortunately, this limitation of CPDs is mitigated by the potential to use the inspection method on linear sequences of binary CPDs for many purposes when dealing with multi-premise syllogisms.

7 Discussion

Two of the aims of developing Category Pattern Diagrams were (1) to investigate whether a new notation for syllogisms could be designed using a similar representational scheme and inference method to that devised for propositional logic Truth Diagrams (c.f., Fig. 1), and (2) to examine whether the possible benefits of the CPD notation were similar to those of the TD notation.

CPDs have been successfully developed using a scheme in which assignments of values to variables, and values to relations among variables, is based on the position, shape and style of line segments running among letters for categories. CPDs used three styles of lines for *some*, *none* and *no-info* connectors, whereas TDs have two styles for truth-values. Unlike the many diagrammatic composition operators of TDs, there is just a single composition operator for CPDs, as syllogisms merely concern conjunctions of propositions. Although there are many possible permutations of values for a pair of connectors, the CPD composition operator includes just three simple rules. Similarly, the method for testing the validity of an inference consists of just four simple rules to compare connectors in the conclusion and combined diagram of the premises. When one is new to CPDs, an explicit result diagram may be drawn in order to work methodically through all the permutations of connectors (e.g., Fig. 6 and 12). However, when one is familiar with the system, the validity of an inference may be determined by inspecting paths running through the sequence of premises (e.g., Fig. 13). This approach is feasible because (i) the small number of simple composition rules enables one to mentally compute the overall type of a path traced along successive connectors and (ii) the small number of simple validity rules means that the implications of a group paths can be readily judged in relation to a conclusion connector.

The simple rules of the CPD approach stands in marked contrast to the conventional verbal approach to the evaluation of syllogisms that relies on the three quantitative and two qualitative rules given in section 2. Because the quality rules, QL1-3, are stated in terms of negatives or even double negatives, this inevitably makes them somewhat tricky to apply. (They may be restated in positive terms, but at the cost of introducing awkward disjunctions to work through.) The same comment holds for quantity rule QN2. Further, both quantity rules are also challenging to apply, because they not only concern the distribution of terms among the premises and conclusions, but very notion of distribution is conceptually demanding to apply to all the terms in all four types of syllogistic proposition. Inferences with CPDs works at a more elemental level, with judgments about the overall validity of an inference depending upon simple comparisons of whether the assignment of values to conjunctions of variables are compatible, which is done by visually matching the styles of simple patterns of line segments.

A similar claim holds for Venn Diagrams, as the assessment of the validity of an inference revolves around whether the presence of a cross or the shading of particular region in the three circle diagram are consistent with the conclusion. Whether CPDs or Venn diagrams, in themselves, are better visualization for classical two premise syllogisms will depend on particular representation design issues. One such is the explicit representation of the absence of information in CPDs (i.e., the *no-info* connector) versus the implicit encoding in Venn Diagrams (i.e., no × and no shading). Another issue is the efficacy of representing sub-sets using spatially contained regions versus distinct line segments. Such design issues will require empirical tests with users. However, an advantage of CPDs over Venn diagrams is in relation to multi-premise syllogisms. Venn himself show how to draw his diagrams for four and more sets, but even with more simpler modern designs (e.g., [8]), the difficulty of dealing with large numbers of premises increases more rapidly for Venn Diagrams than with CPDs. The complexity of constructing the diagram and interpreting its relations appears to grow with the power of the number of sets. In CPDs the difficult arises with the growing number of paths, but this is mitigated by the multiple constraints that the construction rules and validity rules usefully offer. For example, composition rule C2 means all combinations of paths up or down stream of a *none* connector in a sequence of binary premise diagrams will be *none* paths. Finding just a single *no-info* or *some* path corresponding to a *none* connector in the conclusion invalidates the whole inference.

The comparisons of CPDs to the verbal and Venn diagrams approaches allow some observations to be made about the general nature of how notations systems might effectively codify logic. First, although both CPDs and Venn Diagrams are graphical representations they use quite different schemes to encode the same concepts, which again supports the theoretical claim that it is the nature of the relation between the conceptual structure of the ideas being encoded and the characteristics of a notational that largely determines the efficacy of a representational system [3-6]. It is not merely that a graphical representation is spatial or geometric in nature that provides potential benefits to reasoning, but how particular diagrammatic properties encode and interrelate the concepts. Although the spatial containment on the plane provides an initially compelling device to encode a small number of set memberships, the scheme becomes rather less efficacious with larger numbers of sets.

The second observation is that the composition and validity rules of CPDs operate at the "elemental" level of the assignment of fundamental quantity values (*some*, *no-info* and *none*) to the "atomic states" of member and non-membership of the subsets of variables and relations. As a consequence the basic rules of the system are simple and relatively few in number. It is therefore possible to hypothesize that the conceptual difficulties we face in order to understand syllogisms does not arise from the intrinsic nature of the topic, but rather is due to the complexity generated by combinatorics of these fundamental elements in situations with multiple terms. The design of CPDs appears to demonstrate that directly encoding the fundamental concepts of the syllogism domain in the primary representational schemes of a notational system creates an effective codification of the topic (potentially). As such, this would be a further example of the core Representational Epistemic principle, which was previously demonstrated in a range of other knowledge rich topics [3-6].

The third observation concerns how the direct encoding of the fundamental concepts supports reasoning with the new notation. It has previously been theorized such codifications of knowledge produce a semantically transparent system, in which many of the concepts at different levels of granularity, levels of abstraction and in alternative perspectives are readily accessible in the same of expressions of the notation [3-6]. It appears that this claim is also true for CPDs, as they provide explicit access to multiple types of information that are variously used to make inferences with and to explain syllogisms. These include: the identification of categories (labels); distinguishing the subsets of variables (high and low position); specification of relations among variables (connector shapes); assignment of values to the variables (positioning of connectors relative to letters); the type and order of propositions, or moods (arrangement of binary CPDs); the ordering of the variables within a proposition, or Figures (arrangement of unary CPDs in each binary CPD). As the composition operator and validity rules apply directly to patterns of connectors their effect on the categorical state of affairs tends to be plain. Further, by examining overall patterns of connectors for different combinations of mood and Figures one can gain a sense of regularities that follow from the underlying categorical constraints (e.g., the impact of the symmetry of the E and I) and also the implications of concepts such as distribution (e.g., by adding symbols to explicitly show the distributional status of terms).

The next challenge for the project is to extend CPDs beyond syllogisms to cover predicate logic in full.

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