Applicability of Wright’s Correction to Fuchs’ Boundary Sphere Method for TiRe-LII Calculations

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When LII experiments are performed on high pressure aerosols, transition-regime heat conduction from the laser-energized particles is usually calculated using Fuchs’ boundary sphere method. In this technique the Knudsen layer is represented by a collisionless boundary sphere enveloping the particle, which in turn is surrounded by a continuum gas. The analysis proceeds by equating the heat transfer through the two domains and then solving for the unknown boundary sphere temperature, $T_{\Delta}$.

This calculation requires specification of the spherical shell thickness, $\Delta$, which is usually chosen as the mean free path at $T_{\Delta}$, $\lambda_{\Delta} = \lambda(T_{\Delta})$. Filippov and Rosner [1] instead advocate a more complex equation that accounts for particle curvature and the directional distribution of incident molecules, originally proposed by Fuchs [2] and derived by Wright [3] to model evaporating droplets. If a colliding molecule has travelled a distance $l$ from its most recent collision at an angle $\theta$ relative to the surface normal, the corresponding radial distance is $\delta(l, \theta) = \left( l^2 + a^2 + 2la\cos \theta \right)^{1/2} - a$.

By integrating over all incident angles, the expected value of $\delta(l, \theta)$ for a given $l$ is

$$\delta(l) = \frac{\pi^2}{5} \int_0^\pi \delta(l, \theta) P_\theta(\theta) d\theta = \frac{a^3}{l^2} \left[ \frac{(1 + l^2/a^2)^3}{5} - \frac{(1 + l^2/a^2)(1 + l^2/a^2)^3}{3} + \frac{2}{15} \left( 1 + l^2/a^2 \right)^{1/2} - \frac{l^2}{a^2} \right]$$

where $P_\theta(\theta) = 2\cos \theta \sin \theta$. Filippov and Rosner [1] set $l = \lambda_{\Delta}$ in Eq. (1) to find $\Delta$, while Wright [2] also considers the distribution of incident paths, $P_\theta(l) = 1/\lambda_{\Delta} \exp(-l/\lambda_{\Delta})$,

$$\Delta = \int_0^\infty \delta(l) P_\theta(l) dl = \int_0^\infty \delta(l) \frac{1}{\lambda_{\Delta}} \exp(-l/\lambda_{\Delta}) dl$$

which can be solved numerically.

We use Direct Simulation Monte Carlo to investigate this phenomenon under typical LII conditions. The Knudsen layer thickness is found by sampling the radial distance that incident gas molecules travel before they collide with the surface. The DSMC results reveal that particle curvature increases the Knudsen layer thickness compared to a flat surface ($\Delta/\lambda_{\Delta} = 2/3$), an effect captured by both Wright’s equation [3] and Filippov and Rosner’s [1] approximation. This correction has a negligible influence on transition regime heat transfer rates, however, especially considering other uncertainties involved in the calculation, so it can be safely excluded when analysing TiRe-LII data.