The Methods of Maximum Flow and Minimum Cost Flow Finding in Fuzzy Network

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Abstract. This article considers the problems of maximum flow and minimum cost flow determining in fuzzy network. Parameters of fuzzy network are fuzzy arc capacities and transmission costs of one flow unit represented as fuzzy triangular numbers. Conventional rules of operating with fuzzy triangular numbers lead to a strong "blurring" of their borders, resulting in loss of self-descriptiveness of calculations with them. The following technique of addition and subtraction of fuzzy triangular numbers is proposed in the presented paper: the centers are added (subtracted) by the conventional methods, and the borders of the deviations are calculated using linear combinations of the borders of adjacent values. The fact that the limits of uncertainty of fuzzy triangular numbers should increase with the increasing of central values is taken into account. To illustrate the proposed method numerical examples are presented.

Keywords: Fuzzy arc capacity, linear combination of borders, fuzzy triangular number, fuzzy flow.

1 Introduction

This article deals with flow problems arising in networks. The network corresponds to a directed graph $G = (X, A)$, where X – the set of nodes, A – the set of arcs with distinguished initial (source) and final (sink) nodes. Each arc $(x_i, x_j) \in A$ has capacity determining the maximum number of flow units, which can pass along the arc. The relevance of the tasks of maximum and minimum cost flow determining lies in the fact that the researcher can effectively manage the traffic, taking into account the loaded parts of roads, redirect the traffic, and choose the cheapest route. Suppose a network, which arcs have capacities (q_{ij}). Formulation of the problem of maximum flow finding in the network is reduced to maximum flow determining that can be passed along arcs of the network in view of their capacities [1]:

$$
\begin{aligned}
&\nu = \sum_{x_j \in \Gamma(s)} \xi_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt} \to \max, \\
&\sum_{x_j \in \Gamma(x_i)} \xi_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki} = \begin{cases} \nu, \ x_i = s, \\ -\nu, \ x_i = t, \\ 0, \ x_i \neq s, t, \end{cases} \\
&0 \le \xi_{ij} \le q_{ij}, \ \forall (x_i, x_j) \in A.
\end{aligned} \tag{1}
$$

In (1) ξ_{ij} – the amount of flow, passing along the arc (x_i, x_j); ν – the maximum flow value in the network; s – initial node (source); t – final node (sink); q_{ij} – arc capacity, $\Gamma(x_i)$ – the set of nodes, arcs from the node $x_i \in X$ go to, $\Gamma^{-1}(x_i)$ – the set of nodes, arcs to the node $x_i \in X$ go from. ξ_{ij} represents, for example, the amount of cars, going from the node $x_i \in X$ to the node $x_j \in X$. The first equation of (1) defines that we should maximize the total number of flow units emanating from the source (ν), which is equal to the total number of flow units entering the sink (ν). The second equation of (1) is a flow conservation constraint, which means that the total number of flow units emanating from the source (ν) must be equal to the total number of flow units entering the sink (ν) and the total number of flow units emanating from any node $x_i \neq s,t$ must be equal to the total number of flow units entering the node $x_i \neq s,t.$ The third inequality of (1) is a bound constraint, which indicates that the flow of value ξ_{ij}, passing along any arc (x_i, x_j) must not exceed its arc capacity.

The task of minimum cost flow determining in a network can be formulated as follows: suppose we have a network, which arcs have two associated numbers: the arc capacity (q_{ij}) and transmission cost (c_{ij}) of one flow unit passing from the node $x_i \in X$ to the node $x_j \in X$. The essence of this problem is to find a flow of the given value ω from the source to the sink, which doesn't exceed the maximum flow in the graph ν and has minimal transmission cost. In mathematical terms the problem of minimum cost flow determining [2] in the network can be represented as follows:

$$
\begin{aligned}
&\sum_{(x_i, x_j) \in A} c_{ij} \cdot \xi_{ij} \to \min, \\
&\sum_{x_j \in \Gamma(x_i)} \xi_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki} = \begin{cases} \omega, \ x_i = s, \\ -\omega, \ x_i = t, \\ 0, \ x_i \neq s, t, \end{cases} \\
&0 \le \xi_{ij} \le q_{ij}, \ \forall (x_i, x_j) \in A.
\end{aligned} \tag{2}
$$

In (2) c_{ij} – transmission cost of one flow unit along the arc (x_i, x_j), ω – given flow value, that doesn't exceed the maximum flow v in the network.

In practice, the arc capacities, transmission costs, the values of the flow entering the node and emanating from the node cannot be accurately measured according to their nature. Weather conditions, emergencies on the roads, traffic congestions, and repairs influence arc capacities. Variations in petrol prices, tolls can either influence transmission costs. Therefore, these parameters should be presented in a fuzzy form, such as fuzzy triangular numbers [3]. Thus, we obtain a problem statement of maximum and minimum cost flow problems in fuzzy conditions.

2 Literature Review of the Maximum and Minimum Cost Flow Determining Tasks

The problem of the maximum flow finding in a general form was formulated by T. Harris and F. Ross [4]. L. Ford and D. Fulkerson developed famous algorithm for solving this problem, called "augmented path" algorithm [5]. Maximum flow problem was considered in [1, 6].

There are different versions of the Ford-Fulkerson's algorithm. Among them there is the shortest path algorithm, proposed by J. Edmonds and R. Karp in 1972 [7], in which one can choose the shortest supplementary path from the source to the sink at each step in the residual network (assuming that each arc has unit length). The shortest path is found according to the breadth-first search.

Determining the maximum flow in the transportation network in terms of uncertainty has been studied less. In [8] a solution taking into account the interval capacities of arcs was proposed. S. Chanas [9] proposed to solve this problem by using so-called "fuzzy graphs". There are contemporary articles which solve the problem by the simplex method of linear programming [10].

Many researchers have examined the task of minimum cost flow finding in crisp conditions in the literature. Methods of its solution can be divided into graph techniques and the methods of linear programming. In particular, solutions by the graph methods are considered in [1, 6]. The advantages of this approach are great visualization and less cumbersome. The minimum cost flow is proposed to find by Busacker-Gowen and M. Klein's algorithms in [2]. In [2, 6] a task of minimum flow determining is considered as a task of linear programming. This approach is cumbersome.

The methods of minimum cost flow finding in networks in fuzzy conditions can be divided into two classes. The first class involves the use of conventional flow algorithms for determining the minimum cost flow, which operate with fuzzy data instead of crisp values and require cumbersome routines with fuzzy numbers. The second class of problems implies the use of "fuzzy linear programming", which was widely reported in the literature [11, 12].

Authors [13] consider the tasks of "fully fuzzy linear programming". These tasks are cumbersome and can not lead to optimal solutions in the minimum cost flow determination. The solution of fuzzy linear programming tasks by the comparison of fuzzy numbers based on ranking functions is examined in [14].

3 Presented Method of Operating with Fuzzy Triangular Numbers

Researcher is faced with the problem of fuzziness in the network, when considering the problems of maximum and minimum cost flow finding. Arc capacities, flow values, passing along the arcs, transmission costs per unit of goods cannot be accurately measured, so we will represent these parameters as fuzzy triangular numbers.

We will represent the triangular fuzzy numbers as follows: (a, γ, δ), where a – the center of the triangular number, γ – deviation to the left of the center, δ – deviation to the right of center, as shown in Fig. 1.

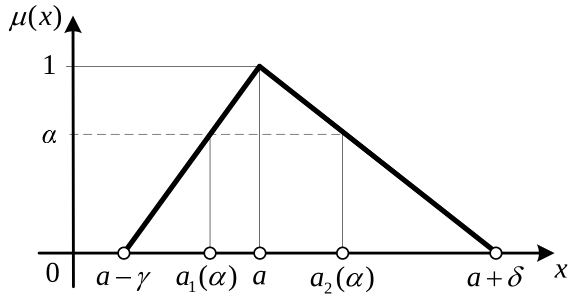

Fig. 1. Fuzzy triangular number.

Conventional operations of addition and subtraction of fuzzy triangular numbers are as follows: let $\tilde{A}_1$ and $\tilde{A}_2$ be fuzzy triangular numbers, such as $\tilde{A}_1 = (a_1, \gamma_1, \delta_1)$ and $\tilde{A}_2 = (a_2, \gamma_2, \delta_2)$. Therefore, the sum of triangular numbers can be written as: $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, \gamma_1 + \gamma_2, \delta_1 + \delta_2)$ and the difference represented as: $\tilde{A}_1 - \tilde{A}_2 = (a_1 - a_2, \gamma_1 + \delta_2, \delta_1 + \gamma_2)$ [3]. The disadvantage of the conventional methods of addition and subtraction of fuzzy triangular numbers is a strong "blurring" of the resulting number and, consequently, the loss of its self-descriptiveness. For example, when adding the same triangular number with itself, the borders of its uncertainty increase: (2,1,1) + (2,1,1) = (4, 2, 2) and (2,1,1) + (2,1,1) + (2,1,1) = (6, 3, 3). Generally, it is not true, because the center of the triangular number should increase, while its borders must remain constant. The fact that the degree of borders "blurring" of fuzzy number depends on the size of its center is not usually considered, when specifying the triangular fuzzy numbers. Therefore, the more the center, the more "blurred" the borders should be (while measuring 1 kg of material, we are talking "about 1 kg", implying the number "from 900 to 1100 g", but while measuring 1 t. of material, imply that "about 1 t." is the number "from 990 kg to 1110 kg").

Comparison of fuzzy triangular numbers according to various criteria is also very difficult and time-consuming. Consequently, following method is proposed to use when operating with triangular fuzzy numbers. Suppose there are the values of arc capacities, flows or transmission costs in a form of fuzzy triangular numbers on the number axis. Then when adding (subtracting) the two original triangular fuzzy numbers their centers will be added (subtracted), and to calculate the deviations it is necessary to define required value by adjacent values. Let the fuzzy arc capacity (flow or transmission cost) "near $\tilde{x}'$" is between two adjacent values "near $\tilde{x}_1$" and "near $\tilde{x}_2$", $(x_1 \le x' \le x_2)$ which membership functions $\mu_{\tilde{x}_1}(x_1)$ and $\mu_{\tilde{x}_2}(x_2)$ have a triangular form, as shown in Fig. 2.

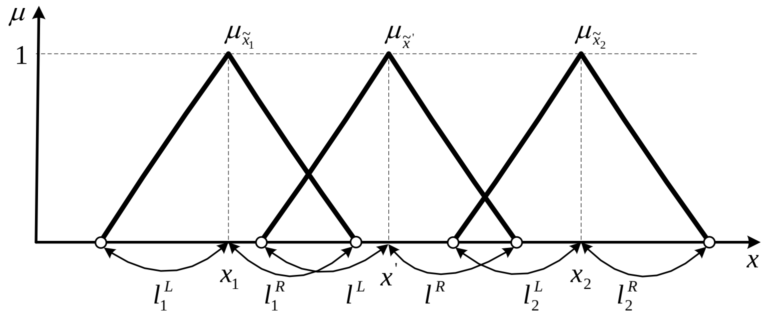

Fig. 2. Given values of arc capacities (flows or transmission costs).

Thus, one can set the borders of membership function of fuzzy arc capacity (flow or transmission cost) "near $\tilde{x}'$" by the linear combination of the left and right borders of adjacent values:

$$l^L = \frac{(x_2 - x)}{(x_2 - x_1)} \times l_1^L + \left(1 - \frac{(x_2 - x)}{(x_2 - x_1)}\right) \times l_2^L,$$
$$l^R = \frac{(x_2 - x)}{(x_2 - x_1)} \times l_1^R + \left(1 - \frac{(x_2 - x)}{(x_2 - x_1)}\right) \times l_2^R. \qquad \textbf{(3)}$$

In (3) l^L is the left deviation border of required fuzzy number, l^R is the right deviation border. In the case when the central value of triangular number resulting by adding (subtracting) repeats the already marked value on the number axis, its deviation borders coincide with the deviation borders of the number marked on the number axis. If required central value is not between two numbers, but precedes the first marked value on the number axis, its deviation borders coincide with those of the first marked on the axis. The same applies to the case when the required central value follows the last marked value on the axis.

4 Solving the Task of Maximum Flow Finding in Fuzzy Network

The task of maximum flow finding in fuzzy network can be formulated as follows:

$$\tilde{v} = \sum_{x_j \in \Gamma(s)} \tilde{\xi}_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \tilde{\xi}_{kt} \to \max,$$

$$\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ki} = \begin{cases} \tilde{v},\ x_i = s, \\ -\tilde{v},\ x_i = t, \\ \tilde{0},\ x_i \neq s, t, \end{cases} \tag{4}$$

$$\tilde{0} \le \tilde{\xi}_{ij} \le \tilde{q}_{ij},\ \forall (x_i, x_j) \in A.$$

In (4) $\tilde{v}$ is required maximum fuzzy flow value in the network; $\tilde{\xi}_{ij}$ – fuzzy amount of flow, passing along the arc (x_i, x_j); $\tilde{q}_{ij}$ – fuzzy capacity of the arc (x_i, x_j); $\tilde{0}$ is fuzzy number of the form (0, 0, 0), as it reflects the absence of the flow.

Let's consider an example, illustrating the solution of this problem, represented in Fig. 3. Let network, representing the part of the railway map, is given in a form of fuzzy directed graph, obtained from GIS "Object Land" [15, 16]. Let the node x_1 is a source, node x_{12} is a sink. The values of arc capacities in the form of fuzzy triangular numbers are defined above the arcs. It is necessary to calculate the maximum flow value between stations "Kemerovo" (x_1) and "Novosibirsk-Gl." (x_{12}) according to Edmonds-Karp's algorithm [7] and the method, described for operations with fuzzy triangular numbers. Determining of maximum flow is based on sending flows along the arcs of the network until one cannot send any additional unit of flow from the source to the sink. Edmonds-Karp's algorithm represents the choice of the shortest supplementary path from the source to the sink at each step in the residual network (assuming that each arc has unit length). Fuzzy residual network contains the arcs of the form (x_i, x_j) with the fuzzy residual arc capacity $\tilde{q}_{ij} - \tilde{\xi}_{ij}$, if the arcs (x_i, x_j) have the flow value $\tilde{\xi}_{ij} < \tilde{q}_{ij}$ in the initial network; and the arcs of the form (x_j, x_i) with the residual arc capacity $\tilde{\xi}_{ij}$, if the arcs (x_i, x_j) have the flow value $\tilde{\xi}_{ij} > \tilde{0}$. The arc (x_i, x_j) is called "saturated" when the flow, passing along it, equals to arc capacity $\tilde{q}_{ij}$. Other words, residual arc capacity defines how many flow units can be sent along the arc (x_i, x_j) to reach arc capacity. Residual arc capacity of arc saturated arc (x_i, x_j) is zero.

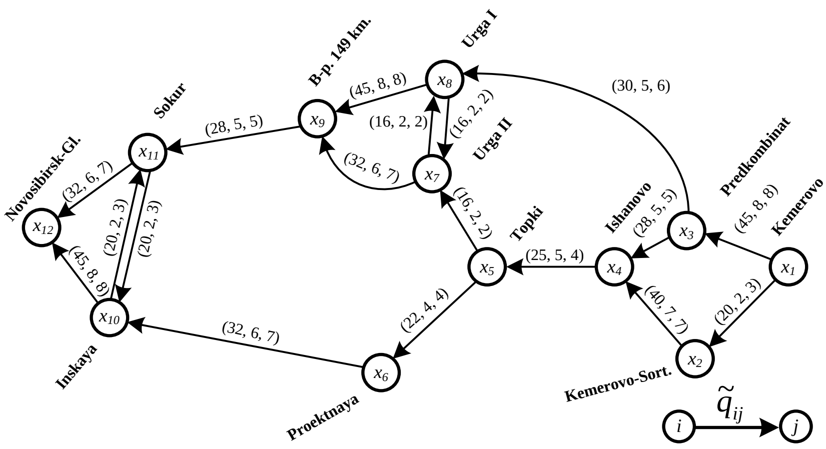

Fig. 3. Initial network.

Therefore, the algorithm proceeds as follows: the first iteration of the algorithm performs an augmenting chain $x_1x_3x_8x_9x_{11}x_{12}$. Push the flow, equals to (28, 5, 5) units along it. The arc (x_9, x_{11}) becomes saturated, consequently, fuzzy residual capacity of the arc (x_9, x_{11}) equals to (0, 0, 0). Let's define the fuzzy residual capacities of the remaining arcs of augmenting chain. The arc (x_1, x_3) has fuzzy residual capacity equals to (45, 8, 8) – (28, 5, 5). Thus, the central value of the resulting number is 17. It is located between adjacent arc capacities: (16, 2, 2) and (20, 2, 3) of the original graph as shown in Fig. 4.

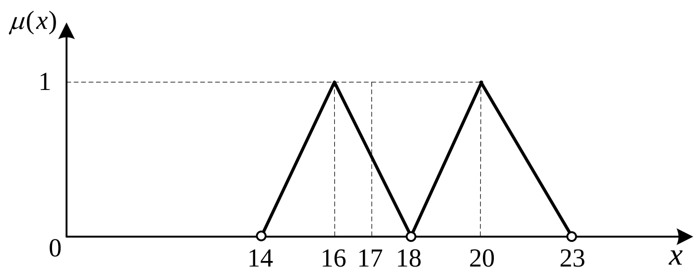

Fig. 4. Fuzzy triangular number with a center equals to 17 and its adjacent numbers.

Compute the left and the right deviation borders of the fuzzy triangular number with a center of 17 according to (3). Thus, we obtain a fuzzy triangular number of the form (17, 2, 2.25) units.

Fuzzy residual capacity of the arc (x_3, x_8) is (30, 5, 6) – (28, 5, 5). Consequently, we obtain a fuzzy triangular number with a center of 2, located to the left of fuzzy triangular number of the form (16, 2, 2). Deviation borders of the required number coincide with deviation borders of the number (16, 2, 2). Thus, we obtain a fuzzy triangular number of a type (2, 2, 2) units.

Fuzzy residual capacity of the arc (x_8, x_9) equals to (17, 2, 2.25) units, similarly with the arc (x_1, x_3).

Finally, fuzzy residual capacity of the arc (x_{11}, x_{12}) is equal to (32, 6, 7) – (28, 5, 5), i.e. we obtain fuzzy number (4, 2, 2) units and fuzzy residual capacity of the arc (x_{12}, x_{11}) equals to (28, 5, 5) units. Fuzzy residual capacities of the arcs $(x_3, x_1), (x_8, x_3), (x_9, x_8), (x_{11}, x_9), (x_{12}, x_{11})$ are (28, 5, 5) units.

The second iteration of the algorithm gives the augmenting chain $x_1x_2x_4x_5x_6x_{10}x_{12}$. Push the flow equals to (20, 2, 3) units along it. The arc (x_1, x_2) becomes saturated, consequently, fuzzy residual capacity of the arc (x_1, x_2) equals to (0, 0, 0). Fuzzy residual capacity of the arc (x_2, x_4) is (40, 7, 7) – (20, 2, 3), i.e. we obtain a fuzzy triangular number (20, 2, 3) units. Fuzzy residual capacity of the arc (x_4, x_5) is the difference between the numbers (25, 5, 4) and (20, 2, 3). Thus, we get a number with a center of 5, located to the left of the number (16, 2, 2), i.e. (5, 2, 2) units. Fuzzy residual capacity of the arc (x_5, x_6) is equal to (22, 4, 4) – (20, 2, 3), i.e., (2, 2, 2) units. Fuzzy residual capacity of the arc (x_6, x_{10}) is equal to (32, 6, 7) – (20, 2, 3), i.e. (12, 2, 2) units. Fuzzy residual capacity of the arc (x_{10}, x_{12}) is (45, 8, 8) – (20, 2, 3), i.e., (25, 5, 4) units. Fuzzy residual capacities of the arcs $(x_2, x_1), (x_4, x_2), (x_5, x_4), (x_6, x_5), (x_{10}, x_6), (x_{10}, x_{12})$ are (20, 2, 3) units.

The third iteration of the algorithm performs the augmenting chain $x_1x_3x_4x_5x_6x_{10}x_{12}$. Push the flow equals to (2, 2, 2) units along. The arc (x_5, x_6) becomes saturated. Let's define fuzzy residual capacities of the remaining arcs of the augmenting chain. Fuzzy residual capacity of the arc (x_1, x_3) is (17, 2, 2.25) – (2, 2, 2), i.e. (15, 2, 2) units. Fuzzy residual capacity of the arc (x_3, x_1) is (30, 5, 6) units. Fuzzy residual capacity of the arc (x_3, x_4) is equal to (28, 5, 5) – (2, 2, 2). We get the number with a center of 26, located between adjacent values (25, 5, 4) and (28, 5, 5).

Compute the left and the right deviation borders of the fuzzy triangular number with a center of 26 according to (3). Thus, we obtain a fuzzy triangular number of the form (26, 5, 4.33) units.

Fuzzy residual capacity of the arc (x_4, x_5) is equal to (5, 2, 2) – (2, 2, 2), i.e. (3, 2, 2) units. Fuzzy residual capacity of the arc (x_6, x_{10}) is equal to (12, 2, 2) – (2, 2, 2), i.e. (10, 2, 2) units. Fuzzy residual capacity of the arc (x_{10}, x_{12}) is equal to (25, 5, 4) – (2, 2, 2), i.e. we obtain a fuzzy number with a center of 23, located between adjacent values (22, 4, 4) and (25, 5, 4), therefore, the left deviation border of the number with a center of 23 equals to 4.33, the right deviation border is 4. We obtain fizzy triangular number (23, 4.33, 4) units. Fuzzy residual capacities of the arcs $(x_5, x_4), (x_6, x_5), (x_{10}, x_6), (x_{10}, x_{12})$ are (22, 4, 4) units.

After execution of three iterations of the algorithm it is impossible to pass any single additional flow unit. The total flow is (28, 5, 5) + (20, 2, 3) + (2, 2, 2) units. Therefore, we obtain a fuzzy triangular number with a center of 50, located to the right of the number (45, 8, 8) with the borders, repeated deviations of the number 45: (50, 8, 8) units.

Thus, the maximum flow value between the stations "Kemerovo" and "Novosibirsk-Gl." is (50, 8, 8) units. Let us carry out an interpretation of the results: the maximum flow between the given stations can not be less than 42 and more than 58 units, with the highest degree of confidence it will be equal to 50 units. But with changes in the environment, repairs on the roads, traffic congestions the flow is guaranteed to lie in the range from 42 to 58 units. Fuzzy optimal flow distribution along the arcs and labels of the nodes is shown in Fig. 5. Saturated arcs are bold.

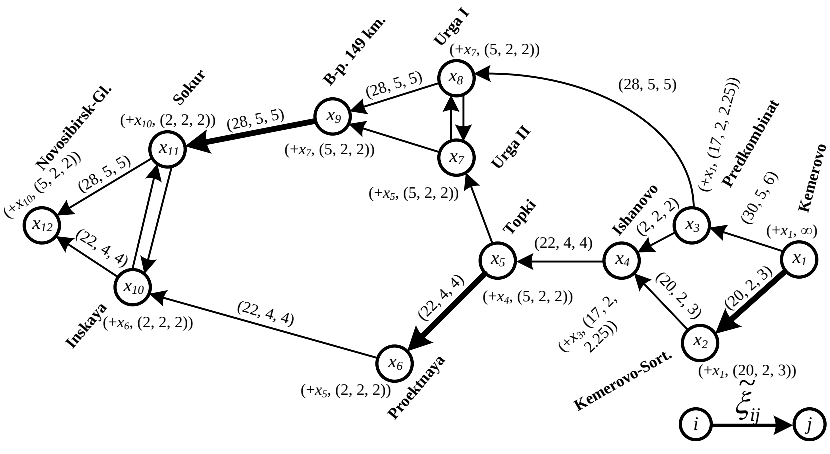

Fig. 5. Network with maximum flow of (50, 8, 8) units.

5 Solving the Task of Minimum Cost Flow Determining in Fuzzy Network

Consider the problem of minimum cost flow finding in a network according to fuzzy values of arc capacities, flows and transmission costs of one flow unit.

$$\sum_{(x_i,x_j)\in A} \tilde{c}_{ij} \cdot \tilde{\xi}_{ij} \to \min,$$

$$\sum_{x_j\in\Gamma(x_i)} \tilde{\xi}_{ij} - \sum_{x_k\in\Gamma^{-1}(x_i)} \tilde{\xi}_{ki} = \begin{cases} \tilde{\omega},\ x_i = s, \\ -\tilde{\omega},\ x_i = t, \\ \tilde{0},\ x_i \neq s,t, \end{cases} \tag{5}$$

$$\tilde{0} \le \tilde{\xi}_{ij} \le \tilde{q}_{ij},\ \forall (x_i, x_j) \in A.$$

In (5) $\tilde{c}_{ij}$ – fuzzy transmission cost of one flow unit along the arc (x_i, x_j), $\tilde{\omega}$ – given fuzzy flow value, that doesn't exceed the maximum flow $\tilde{v}$ in the network.

Let us turn to the graph, shown in Fig. 3. Fuzzy values of transmission costs in addition to fuzzy arc capacities are given in this task:

$\tilde{c}_{x_1x_2} = (12,3,3)$; $\tilde{c}_{x_1x_3} = (6,1,2)$; $\tilde{c}_{x_3x_4} = (10,2,3)$; $\tilde{c}_{x_2x_4} = (18,4,5)$; $\tilde{c}_{x_3x_8} = (4,1,1)$;
$\tilde{c}_{x_4x_5} = (12,3,3)$; $\tilde{c}_{x_5x_7} = (20,5,6)$; $\tilde{c}_{x_8x_7} = (15,4,4)$; $\tilde{c}_{x_7x_8} = (15,4,4)$; $\tilde{c}_{x_8x_9} = (21,6,7)$;
$\tilde{c}_{x_7x_9} = (10,2,3)$; $\tilde{c}_{x_5x_6} = (30,8,9)$; $\tilde{c}_{x_6x_{10}} = (8,2,2)$; $\tilde{c}_{x_9x_{11}} = (19,5,5)$;
$\tilde{c}_{x_{11}x_{10}} = (32,7,12)$; $\tilde{c}_{x_{10}x_{11}} = (32,7,12)$; $\tilde{c}_{x_{11}x_{12}} = (25,7,8)$; $\tilde{c}_{x_{10}x_{12}} = (20,5,6)$.

It is necessary to find a flow value $\tilde{\omega}$ of (45, 8, 8) units from the source to the sink, which has a minimal cost. Consider the Busacker-Gowen's [2] algorithm, taking into account the fuzzy capacities and costs to solve this problem:

Step 1. Assign all arc flows and the flow rate equal to zero.

Step 2. Determine the modified arc costs $\tilde{c}^*_{ij}$ that depend on the value of the already found flow as follows:

$$\tilde{c}^*_{ij} = \begin{cases} \tilde{c}_{ij},\ if\ \tilde{0} \le \tilde{\xi}_{ij} \le \tilde{q}_{ij}, \\ \infty,\ if\ \tilde{\xi}_{ij} = \tilde{q}_{ij}, \\ -\tilde{c}_{ji},\ if\ \tilde{\xi}_{ij} > 0. \end{cases}$$

Step 3. Find the shortest chain (in our case – the chain of minimal cost) [2] from the source to the sink taking into account arc costs $\tilde{c}^*_{ij}$, found in the step 1. Push the flow along this chain until it ceases to be the shortest. Receive the new flow value by adding the new flow value, passing along the considered chain, to the previous one. If the new flow value equals to $\tilde{\omega}$, then the end. Otherwise, go to the step 2.

Solve this problem, taking into account fuzzy arc capacities costs.

Step 1. Assign all $\tilde{\xi}_{ij} = 0$.

Step 2. Determine $\tilde{c}^*_{ij} = \tilde{c}_{ij}$.

Step 3. Find the shortest path by the Ford's algorithm [1]: $x_1x_3x_8x_9x_{11}x_{12}$ of the total cost of (75, 8, 8) standard units. Push the flow, equals to (28, 5, 5) units along this chain.

Step 2. Define the new modified fuzzy arc costs: $\tilde{c}^*_{x_1x_3} = (6,1,2)$; $\tilde{c}^*_{x_3x_1} = -(6,1,2)$; $\tilde{c}^*_{x_3x_8} = (4,1,1)$; $\tilde{c}^*_{x_8x_3} = -(4,1,1)$; $\tilde{c}^*_{x_8x_9} = (21,6,7)$; $\tilde{c}^*_{x_9x_8} = -(21,6,7)$; $\tilde{c}^*_{x_9x_{11}} = \infty$; $\tilde{c}^*_{x_{11}x_9} = -(19,5,5)$; $\tilde{c}^*_{x_{11}x_{12}} = (25,7,8)$; $\tilde{c}^*_{x_{12}x_{11}} = -(25,7,8)$.

Step 3. Find the shortest path using the obtained modified costs: $x_1x_3x_4x_5x_6x_{10}x_{12}$ of the total cost of (86, 8, 8) standard units. Push the flow, equals to (17, 2, 2.25) units along this chain. As a result, we obtain the total flow equals to (45, 8, 8) units, having a total transmission cost along the network, equals to (28, 5, 5) × ((75, 8, 8) + (86, 8, 8)) = (3562, 8, 8) standard units. There are fuzzy flow values $\tilde{\xi}_{ij}$ under the arcs and fuzzy transmission costs $\tilde{c}_{ij}$ of optimal fuzzy flow values $\tilde{\xi}_{ij}$ above the arcs of the graph, saturated arcs are bold as shown in Fig. 6.

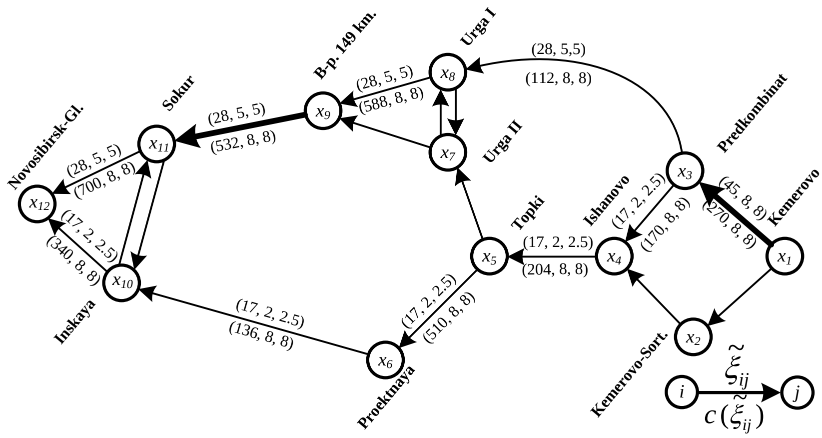

Fig. 6. Network with the flow of (45, 8, 8) units and transmission costs of each arc of the total cost (3562, 8, 8) standard units.

6 Conclusion

This paper examines the problems of maximum and minimum cost flow determining in networks in terms of uncertainty, in particular, the arc capacities, as well as the transmission costs of one flow unit are represented as fuzzy triangular numbers. The technique of addition and subtraction of triangular numbers is considered. Presented technique suggests calculating the deviation borders of fuzzy triangular numbers based on the linear combinations of the deviation borders of the adjacent values. The fact that the limits of uncertainty of fuzzy triangular numbers should increase with the increasing of central values is taken into account. Advantage of the proposed method lies in the fact that operations with fuzzy triangular numbers don’t lead to a strong "blurring" of their deviation borders, it makes calculations with such numbers more effective.

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