A Collaborative Model for Participatory Load Management in the Smart Grid*

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Abstract. The major task of a grid operator is perfectly balancing the demand of all customers at any instant with supply. One of the facets of the Smart Grid vision is tackling the problem of balancing demand with supply using strategies that act on the demand side. With the deployment of intelligent ICT devices in domestic environments, homes are becoming smarter and able to optimise the electricity consumption to minimise costs and/or meet supply constraints. In this work, we envision a scenario where smart homes actively participate in the balancing of demand with supply by forming groups of electricity consumers that agree on a joint demand profile to be contracted with a retail electricity provider. We put forward a novel business model as well as an optimisation model for collaborative load management, showing the economic benefits for the participants.

1 INTRODUCTION

A power system needs to perfectly balance at any instant the demand of all customers with supply in order to keep voltage and frequency stable and guarantee a safe functioning of the system. This task is carried out by the grid operator. The traditional approach is intervening from the supply side, by increasing or decreasing the supply to continuously match demand. Base load demand (i.e., the amount of electricity required on a continuous basis) is usually covered by power stations with low generation costs, but long start-up times. These power stations are therefore not able to quickly adjust their generation capacity to match unexpected peak load demand. Balancing power is therefore provided by expensive and carbon-intensive power plants, which are responsible for most part of consumer electricity bill.

One of the facets of the Smart Grid vision is tackling the problem of balancing demand with supply using strategies that act on the demand side [6][7]. For instance, the grid operator may use demand dispatch schemes that remotely turn (industrial) intensive loads off for a limited period of time in order to reduce demand. Also peak-shaving strategies, such as real-time pricing, may be used to encourage off-peak consumption, thus flattening demand [1].

All these strategies do not conceive an active and participatory role for the consumers. With the deployment of intelligent ICT devices in domestic environments, homes are becoming smarter and able to optimise the electricity consumption to minimise costs and/or meet supply constraints [8]. The participation of consumers into the management of demand is quite a recent line of research. For instance, Vinyals *et al.*

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proposed the formation of coalitions among energy consumers with near-complementary consumption restrictions [9]. In this work, a coalition of consumers can act in the market as a single virtual energy consumer (VEC), buying electricity directly from the day-ahead market and the future market. The experimental results show that the coalition of consumers obtains noticeable gains if the (average) price of electricity in the future market is half the (average) price of electricity in the day-ahead market, while using realistic prices the gains account only for less than 2%. Nevertheless, there is a growing consensus towards a more active role for the consumers.

In this work, we envision a business model where smart homes actively participate in the balancing of demand with supply by forming groups of electricity consumers that agree on a joint demand profile to be contracted with a retail electricity provider. By doing so, the consumers are able to get better prices from the retail electricity provider, since the management of balancing the demand with the contracted supply (and the eventual penalties) is responsibility of the consumers.

This paper is structured as follows: Section 2 presents our optimisation model and the computation of payments and penalties; Section 3 defines the scenario used for the evaluation of the model; in Section 4 the experimental results are reported; finally we conclude in Section 5.

2 COLLABORATIVE LOAD MANAGEMENT MODEL

This work envisions a scenario such as that depicted in Figure 1. Let $\mathcal{H}$ be a set of smart homes, represented by an aggregator, which interacts with a retail electricity provider (REP) to contract power on a daily basis. On a given day, each smart home is assumed to estimate its power consumption for the next day. Provided with the data of each home, the aggregator optimises the energy consumption of the whole group of smart homes and purchases the power to be delivered the next day from the REP.

2.1 Consumer load

We classify loads into two categories: those that can be shifted in time and those that cannot. The sum of the power consumption of the latter type of loads forms the *base load*, while all the others loads are individually modelled as *shiftable loads*. Each consumer has exactly one base load and several shiftable loads. Shiftable loads are further classified in loads that can be interrupted and resumed (*shiftable interruptible loads*) and loads that can be shifted but once they start they cannot be interrupted (*shiftable atomic loads*)[1]. Let $\mathcal{S} = \mathcal{I} \cup \mathcal{A}$ be the set of shiftable loads of a consumer, where $\mathcal{I}$ is the set of shiftable loads that can be interrupted and resumed, while $\mathcal{A}$ is the set of shiftable atomic loads.

[1] Examples of shiftable interruptible loads are plug-in (hybrid) electric vehicles, or heating/air conditioning (AC) devices, which can be switched on and off while maintaining the temperature between the desired limits. Examples of shiftable atomic loads are washing machines or dryers.

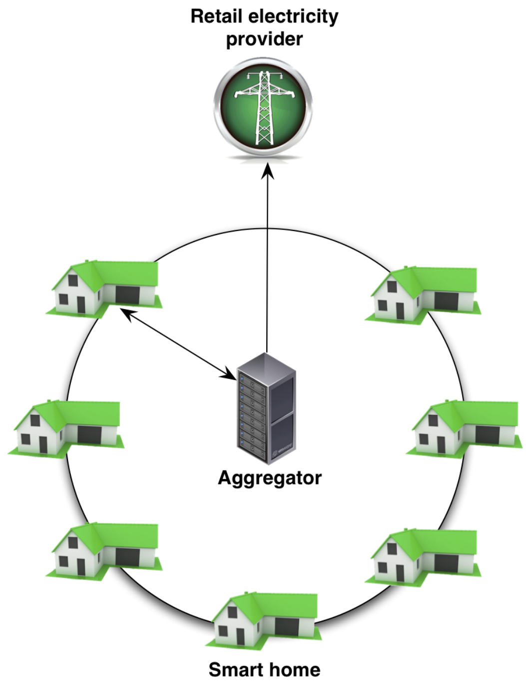

Fig. 1. Collaborative load management scenario

Definition 1: base load
The base load is defined as:

$$\mathbf{w}^B = [w_1^B \; w_2^B \; \dots \; w_N^B]^T$$

where $\mathcal{T} = \{1, \dots, N\}$ *is the number of time slots in a day, and* $w_t^B \in \mathbb{R}^+$ *is the base load power (expressed in kW) for time slot* t.

Definition 2: shiftable interruptible load
A shiftable interruptible load is defined as:

$$\mathbf{x}^{S_I} = [x_1^{S_I} \; x_2^{S_I} \; \dots \; x_N^{S_I}]^T, W^{S_I}, d^{S_I}, t_s^{S_I}, t_f^{S_I}$$

where $x_t^{S_I} \in \{0, W^{S_I}\}$ *is the load power for time slot* t, W^{S_I} *is the power rate (expressed in kW) of the load,* $d^{S_I} \in \{1, \dots, |\mathcal{T}|\}$ *is the duration of the load,* $t_s^{S_I} \in \mathcal{T}$ *is the earliest time slot for the load to start, and* $t_f^{S_I} \in \mathcal{T}$ *is the latest time slot for the load to finish,*

subject to

$$t_f^{S_I} - t_s^{S_I} + 1 \geq d^{S_I} \quad (1)$$

$$\sum_{t \in \mathcal{T}} x_t^{S_I} = d^{S_I} W^{S_I} \tag{2}$$

$$\sum_{\substack{t < t_s^{S_I} \\ t > t_f^{S_I}}} x_t^{S_I} = 0 \tag{3}$$

Constraint (1) ensures that the number of available time slots between the earliest time slot and the latest time slot is enough for the shiftable load to run for its entire duration d^{S_I}. Constraint (2) ensures that the shiftable load runs for d^{S_I} time slots. Constraint (3) prevents the shiftable load from running before the earliest time slot $t_s^{S_I}$ or after the latest time slot $t_f^{S_I}$.

Definition 3: shiftable atomic load
A shiftable atomic load is defined as:

$$\mathbf{x}^{S_A} = [x_1^{S_A} \ x_2^{S_A} \ \dots \ x_N^{S_A}]^T, W^{S_A}, d^{S_A}, t_s^{S_A}, t_f^{S_A}$$

where $x_t^{S_A} \in \{0, W^{S_A}\}$ is the load power for time slot t, W^{S_A} is the power rate (expressed in kW) of the load, $d^{S_A} \in \{1, \dots, |\mathcal{T}|\}$ is the duration of the load, $t_s^{S_A} \in \mathcal{T}$ is the earliest time slot for the load to start, and $t_f^{S_A} \in \mathcal{T}$ is the latest time slot for the load to finish,

subject to Constraints (1), (2), (3) and

$$x_t^{S_A} + x_{t+n}^{S_A} \leq W^{S_A} + x_{t+1}^{S_A} \tag{4}$$

$$\forall n \in \{2, \dots, N-1\}, \ \forall t \in \{t_s^{S_A}, \dots, t_f^{S_A} - n\}$$

Constraint 4 ensures that there exists a set of d^{S_A} consecutive slots when the load is running[2].

Definition 4: overall shiftable loads
We define the overall shiftable loads vector $\mathbf{x}$ *as:*

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_N]^T = \tag{5}$$

$$= \left[\left(\sum_{S_k \in \mathcal{S}} x_1^{S_k} \right) \left(\sum_{S_k \in \mathcal{S}} x_2^{S_k} \right) \ \dots \ \left(\sum_{S_k \in \mathcal{S}} x_N^{S_k} \right) \right]^T$$

[2] Less formally, Constraint 4 ensures that if two slots are equal to W^{S_A}, then there is no slot in between that is equal to 0.

2.2 Joint load optimisation

The economic model for participatory load management that we proposed is based on two components: energy and power. The group of smart homes, represented by the aggregator, must pay the REP for the purchased energy as well as for the power capacity that is needed by the smart homes.

Let $\mathbf{p}^e = [p^e_1\ p^e_2\ \ldots\ p^e_N]^T$ be the price of electricity (expressed in €/kWh) defined by the REP to supply energy over the N time slots. Let p^c be the price that is charged for the required capacity (expressed in €/kW). The goal of the aggregator is defining the consumer load of each smart home so as to minimise the total cost. This goal is defined by the following optimisation problem[3]:

$$\underset{\mathbf{x}^1,\mathbf{x}^2,\ldots,\mathbf{x}^m,x^c}{\text{minimise}} : \sum_{t\in\mathcal{T}} p^e_t \Delta t \sum_{i\in\mathcal{H}} \left(w^{B^i}_t + \sum_{S^i_k\in\mathcal{S}^i} x^{S^i_k}_t \right) + p^c x^c \tag{6}$$

subject to

$$x^c \geq \sum_{i\in\mathcal{H}} \left(w^{B^i}_t + \sum_{S^i_k\in\mathcal{S}^i} x^{S^i_k}_t \right),\ \forall t\in\mathcal{T} \tag{7}$$

where Δt is the duration of a time slot. Constraint (7) sets the variable x^c to the peak power consumption of the group of smart home throughout the set of time steps, which represents the required power capacity.

2.3 Computing day-ahead payments

The value of the objective function described in Eq. 6 is the total cost $c(\mathcal{H})$ incurred by the group of smart homes $\mathcal{H}$. This cost must be shared among the participants. To do that, we have to model what would be the cost that an individual home with the same demand would pay if it did not participate in the group. In this case, we assume a situation where electricity is paid for at a fixed per unit price p^{fix} (expressed in €/kWh), as it happens with the regulated tariffs currently used in most countries. In this case, there is no need to defer loads, since the price of electricity is fixed.

Let $\widehat{\mathbf{w}}^i = [\widehat{w}^i_1\ \widehat{w}^i_2\ \ldots\ \widehat{w}^i_N]^T$ be the consumer load vector such that:

$$\widehat{w}^i_t = \begin{cases} w^{B^i}_t + \sum_{S^i_k\in\mathcal{S}^i} W^{S^i_k} & \text{if } t\in\{t^{S^i_k}_s,\ldots,t^{S^i_k}_s + d^{S^i_k} - 1\} \\ w^{B^i}_t & \text{otherwise} \end{cases} \tag{8}$$

Constraint (8) ensures that each shiftable load of i is executed at the earliest time slot without interruption. The load vector $\widehat{\mathbf{w}}^i$ could then be considered as the preferred load of a home that does not join the collaborative group. Let $c(\{i\})$ be the cost incurred by smart home i for demanding the load vector $\widehat{\mathbf{w}}^i$:

[3] The problem described in Eq. 6 can be modelled as a standard mixed integer linear programming problem, which has been solved with IBM ILOG CPLEX 11.0

$$c(\{i\}) = p^{\text{fix}} \Delta t \sum_{t \in \mathcal{T}} \widehat{w}_t^i \tag{9}$$

The task of the aggregator is therefore defining a vector of payments $\mathbf{z} = [z^1 \; z^2 \; \ldots \; z^m]^T$, such that:

$$\sum_{i \in \mathcal{H}} z^i = c(\mathcal{H}) \tag{10}$$

$$z^i \leq c(\{i\}), \; \forall i \in \mathcal{H} \tag{11}$$

Constraint (10) ensures that the sum of the payments equal the total cost. Constraint (11) is needed to satisfy the *individual rationality* condition (i.e., a smart home will not join the group if the cost of doing that is greater than the cost of acting on its own).

In cooperative game theory, the set of all the vectors $\mathbf{z}$ that satisfy constraints (10) and (11) is a solution concept called Core. We remark that in this model we assume that there is no discomfort cost derived from running a shiftable load over any set of d^S time slots between t_s^S and t_f^S. In fact, if a consumer is sensitive to discomfort, they may impose $t_f^S = t_s^S + d^S - 1$, so that the load cannot be shifted at all.

2.4 Computing imbalance penalties

Once the aggregator has solved the optimisation problem described in Section 2.2, it contracts with the retail electricity provider a certain power profile $\mathbf{w}^{\text{buy}} = [w_1^{\text{buy}} \; w_2^{\text{buy}} \; \ldots \; w_N^{\text{buy}}]^T$, where:

$$w_t^{\text{buy}} = \sum_{i \in \mathcal{H}} \left(w_t^{B^i} + \sum_{S_k^i \in \mathcal{S}^i} x_t^{S_k^i} \right) \tag{12}$$

The group of smart homes is therefore committed to consume exactly the contracted amount of power. However, on the day of the delivery of the contracted power, it is possible that the real consumption differs from the contracted one. Let $\mathbf{w}^{\text{real}} = [w_1^{B^i_{\text{real}}} \; w_2^{B^i_{\text{real}}} \; \ldots \; w_N^{B^i_{\text{real}}}]^T$ be the real base load of a smart home during the day of the delivery. In this work we assume that only the base load may differ from the predicted one, since all the shiftable loads are scheduled automatically according to the optimal plan. The power consumption mismatch is therefore defined as:

$$\boldsymbol{\epsilon} = [\epsilon_1 \; \epsilon_2 \; \ldots \; \epsilon_N]^T = \tag{13}$$
$$= \left[\left(w_1^{B^i_{\text{real}}} - w_1^{B^i} \right) \; \left(w_2^{B^i_{\text{real}}} - w_2^{B^i} \right) \; \ldots \; \left(w_N^{B^i_{\text{real}}} - w_N^{B^i} \right) \right]^T$$

If $\epsilon_t > 0$, the smart home is in a short position (i.e., it has been contracted less power than what is needed), while if $\epsilon_t < 0$, the smart home is in a long position (i.e., it has been contracted more power than what is needed). In the first case, the aggregator may be required to buy the missing power in the balancing market, while in the second case the aggregator may be required to sell the excess power in the balancing market.

Let $\mathbf{p}^{\text{bal-up}} = [p_1^{\text{bal-up}} \; p_2^{\text{bal-up}} \; \dots \; p_N^{\text{bal-up}}]^T$ be the price of electricity for "balancing-up" adjustments (i.e., when more power must be purchased in the balancing market), and let $\mathbf{p}^{\text{bal-down}} = [p_1^{\text{bal-down}} \; p_2^{\text{bal-down}} \; \dots \; p_N^{\text{bal-down}}]^T$ be the price of electricity for "balancing-down" adjustments (i.e., when excess power must be sold in the balancing market). During the day of the delivery of electricity, each smart home must pay the aggregator, as imbalance penalty, the following amount:

$$p_t^{\text{bal}} \Delta t \sum_{t \in \mathcal{T}} |\epsilon_t| \tag{14}$$

$$p_t^{\text{bal}} = \begin{cases} p_t^{\text{bal-up}} & \text{if } \epsilon_t \geq 0 \\ p_t^{\text{bal-down}} & \text{if } \epsilon_t < 0 \end{cases}$$

The imbalance penalty is intended to incentivise smart homes to adhere to their contracted power, by better predicting the day-ahead consumption. We remark that the fact that a smart home pays the aggregator for its short (or long) position does not automatically imply that the aggregator on its turn will cover the position in the balancing market. For example, it is possible that a short position of a smart homes is cancelled out by a long position of another smart home, for the same amount of kW. Therefore, although both smart homes pay the aggregator for their mismatches, the aggregator is not required to buy or sell power in the balancing market. In this case, we assume that the aggregator keeps the money that has been paid as imbalance penalty by the two smart homes.

3 EVALUATION SCENARIO

We define the evaluation scenario as follows. The duration Δt of a time slot is 10 minutes, and the number of smart homes in $\mathcal{H}$ is 10. For reasons of computational complexity, we kept the number of homes relatively small in order to solve optimally the problem described in Eq. 6. In this work, we assume that $\mathbf{p}^e = (1 + \alpha)\mathbf{p}^{\text{mkt}}$, where $\mathbf{p}^{\text{mkt}}$ is the price of electricity in the day-ahead electricity market and $\alpha > 0$ is a parameter that ensures a profit margin for the retail electricity provider. The price of electricity of the day-ahead ($\mathbf{p}^{\text{mkt}}$) and balancing ($\mathbf{p}^{\text{bal-up}}$ and $\mathbf{p}^{\text{bal-down}}$) markets are taken from the July 2012 and January 2012 bulletin of the Spanish market operator[4]. The capacity price p^c is set to 0.07 €/kW, which is the capacity price in Spain for power delivery greater than 15 kW.

Each smart home is equipped with a certain number of electric equipments, such as heaters, washing machines, plug-in electric vehicles, etc. The probability that a smart home has a particular electric equipment has been obtained from a study of the Institute for Diversification and Energy Saving, in collaboration with Eurostat [5]. This study analysed the electricity consumption of the residential sector in Spain. Table 1 resumes, for each electric equipment, the associated probability of being present in a smart home. The type of load can be base (B), shiftable interruptable (I) or shiftable atomic (A). Although nowadays the penetration of the plug-in electric vehicle (EV) is negligible,

[4] http://www.omie.es

we assume a scenario where 10% of households has a plug-in (hybrid) electric vehicle, which is the projected penetration by 2020 that is reported by many studies [3, 4]. For simplicity we also assume that the probability of an equipment of being present in a smart home is statistically independent of the presence of other equipments[5].

Once the electric equipments that are present in a smart home has been defined, it is necessary to instantiate the *predicted* base load and shiftable loads for the next day, used by the aggregator to define the optimal consumer load $\mathbf{w}$, and the *real* base load $\mathbf{w}^{\mathrm{real}}$ during the day of the delivery. These instantiations are based on an elaboration of the results of the INDEL project, carried out by the Spanish grid operator, which assessed the electric demand of the residential sector in Spain [2].

Table 1. Loads and probability ($p(load)$) of being present in a smart home.

Type of load	**Load**	$p(load)$
B	Water	0.2
B	Lighting	1
B	Kitchen	0.53
B	Fridge	1
B	Freezer	0.23
B	Oven	0.77
B	Microwave	0.9
B	TV	1
B	Desktop computer	0.52
B	Laptop computer	0.41
I	Heating	0.41
I	AC	0.49
I	EV	0.1
A	Washing machine	0.93
A	Dryer	0.28
A	Dishwasher	0.53

3.1 Base load

Water The power demand of an electric water heater is characterised by high peaks of power at regular intervals. The typical consumption cycle is turning the water heater on for half an hour (or 3 time slots) every two hours between 0:00 and 18:00. Between 18:00 and 24:00 the interval between two consecutive half-hour of usage decreases to one hour. The reason of this functioning is that the heat loss are negligible when the consumer does not use hot water, while during intense usage (between 18:00 and 24:00) the equipment needs to heat water more frequently. The water heater contribution to the base load is modelled as follows. An initial time slot t is randomly selected from the set $\{1,\ldots,6\}$ (i.e., the first hour of the day). Starting from time slot t, the water heater is

[5] In reality, this may not be the case. For example, usually the presence of a dryer is conditioned to the presence of the washing machine.

turned on at regular intervals for 3 consecutive time slots, consuming 1.2 kW for every time slot it is turned on. The regular interval is set to 12 time slots (i.e., 2 hours) between 0:00 and 18:00, and 6 time slots (i.e., 1 hour) between 18:00 and 24:00.

Lighting The average power consumption of lighting in a working day is taken from [2], using for every time slot a normal distribution with mean equal to the average consumption and variance equal to 5% of the average.

Kitchen The average power consumption of an electric kitchen in a working day is taken from [2]. The electric kitchen is not used at all until 6 in the morning. A normal distribution with mean equal to the average consumption and variance equal to 5% of the average is used to stochastically generate different power consumptions.

Fridge and freezer The fridge and the freezer are two appliances that are always running at a constant power rate. For these two loads, we use a fixed power rate of 0.08 kW and 0.07 kW respectively.

Oven According to the surveys collected in [2], when the electric oven is used it runs between 20 minutes and 1 hour, around 14:00 $\pm$ 1h (lunch time) and/or around 21:00 $\pm$ 1h (dinner time). The probability of using the oven at lunch time is 0.8, while the probability of using it at dinner time is 0.2. The oven is used on average 2 times a week, and its power rate is 1.2 kW.

Microwave The microwave is used repeatedly throughout a day for short periods of time (10 minutes). Analysing the data of [2], the microwave is mainly used around 9:00 $\pm$ 1h, 11:00 $\pm$ 1h, 15:00 $\pm$ 1h and 22:00 $\pm$ 1h, with probability 0.12, 0.2, 0.25 and 0.43 respectively. The microwave is used every day, and its power rate is 1.3 kW.

TV, desktop computer and laptop computer The study carried out in [2] did not analyse the usage of TVs, desktop computers or laptop computers. In this works we assume that each device is used twice a day, at 14:00 $\pm$ 1h and at 20:00 $\pm$ 1h, and each usage takes between 1 and 3 hours. The power rate of the TV, the desktop computer and the laptop computer is set to 0.01, 0.1 and 0.02 kW respectively.

3.2 Shiftable interruptible loads

Heating The power demand of an electric heating system with a thermostat is characterised by high peaks of active power. A typical heater is usually off before 8:00 in the morning and after 23:00 in the night. Between 8:00 and 23:00 the heater is turned on for a total of 3.5 hours. Although the functioning of the heater depends on the number of people inside the home, the external weather conditions and the thermal leakage of the home, in this work we rely on the typical power consumption reported in [2]. A heater must be on for 10 minutes in every hour (1 time slot out of 6) between 8:00 and 20:00, and for 30 minutes in every hour (3 time slot out of 6) between 20:00 and 23:00. Thus between 8:00 and 20:00 a heater is on for 2 hours of the total usage of 3.5 hours, and the remaining 1.5 hours is placed between 20:00 and 23:00. Each smart home is equipped with 1 to 3 heaters, each of them with a power consumption of 1 kW.

On the basis of the aforementioned assumption, we instantiated in our model the load of each heater as follows. For each heater we define 15 shiftable loads, representing each hour τ between 8:00 and 22:00 inclusive. Each load $\mathbf{w}^{S_{H_\tau}}$ is characterised by a power rate of $W^{S_{H_\tau}} = 1$kW, a earliest time slot $t_s^{S_{H_\tau}} = 6\tau + 1$, a latest time slot $t_f^{S_{H_\tau}} = 6\tau + 6$, and a duration equal to:

$$d^{S_{H_\tau}} = \begin{cases} 1 & \text{if 8:00} \leq \tau \leq \text{20:00} \\ 3 & \text{if 20:00} < \tau \leq \text{22:00} \end{cases} \tag{15}$$

AC A typical AC system is turned on for a certain amount of time between 13:00 and 18:00, consuming an amount of energy that varies between 1.6 and 5.6 kWh per day. Similarly to a heating system, the power consumption of an AC system depends on environmental conditions. However, for simplicity we define the load $\mathbf{w}^{S_{AC}}$ of an AC system as follows. The earliest time slot is set to $t_s^{S_{AC}} = 6 \cdot 13 + 1$, while the latest time slot is set to $t_f^{S_{AC}} = 6 \cdot 18 + 6$. We then draw the amount of energy e that is consumed from a uniform distribution over the interval $[1.6, 5.6]$ kWh. Given the power rate $W^{S_{AC}} = 1.5$ kW, the number of time slots when the AC is running is therefore defined as:

$$d^{S_{AC}} = \frac{e}{W^{S_{AC}} \Delta t} \tag{16}$$

EV In this work we assume that a plug-in electric vehicle uses Level 1 charging, with a power rate of $W^{S_{EV}} = 1.92$ kW. We assume that the EV owner arrives at home at 19:00 $\pm$ 1h, and needs the EV charged at 8:00 $\pm$ 1h of the next day. We assume a battery size of 24kWh (such as that of the Nissan Leaf), and state of charge SOC at the time the EV is plugged-in uniformly distributed between $[0.3, 0.8]$. Since the charging is spread over two consecutive days, for every EV we instantiate two shiftable interruptible loads: one for the charging between the arrival time and 24:00 ($\mathbf{w}^{S_{EV_1}}$) and one for the charging between 24:00 and the departure time ($\mathbf{w}^{S_{EV_2}}$). For $\mathbf{w}^{S_{EV_1}}$, the earliest time slot $t_s^{S_{EV_1}}$ is equal to the time slot corresponding to the arrival time, while the latest time slot $t_f^{S_{EV_1}}$ is equal to N (i.e., the last time slot in $\mathcal{T}$). For $\mathbf{w}^{S_{EV_2}}$, the earliest time slot $t_s^{S_{EV_2}}$ is equal to 1, while $t_f^{S_{EV_2}}$ is equal to the time slot corresponding to the departure time. For the definition of the two durations, $d^{S_{EV_1}}$ and $d^{S_{EV_2}}$, we use the following heuristic. Let $k_1 = 144 - t_s^{S_{EV_1}} + 1$ be the number of time slots between the arrival time and 24:00, and let $k_2 = t_f^{S_{EV_2}}$ be the number of time slots between the 0:00 of the next day and the departure time. Given that the amount of energy needed by the EV is $e = 24(1 - SOC)$ kWh, the EV tries to charge $e_1 = ek_1/(k_1 + k_2)$ kWh between the arrival time and 24:00, and the remaining $e_2 = ek_2/(k_1 + k_2)$ between 0:00 and the departure time. The durations of the two loads are therefore:

$$d^{S_{EV_1}} = \frac{e_1}{W^{S_{EV}} \Delta t} \quad d^{S_{EV_2}} = \frac{e_2}{W^{S_{EV}} \Delta t} \tag{17}$$

3.3 Shiftable atomic loads

Washing machine According to the study we use as a reference [2], the washing machine is run on average 3 times a week. The earliest time slot $t_s^{S_{WM}}$ is set at 11:00 ± 1h (with probability 0.78) or at 19:00 ± 1h (with probability 0.22). In this work we assume that the latest time slot $t_f^{S_{WM}}$ is set at 15:00 ± 1h (if the washing machine is run in the morning), or at 23:00 ± 1h (if the washing machine is run in the evening). A typical washing machine operates for one to two hours at a power rate $W^{S_{WM}}$ of 0.19 kW. The duration $d^{S_{WM}}$ (in time slots) is therefore drawn uniformly from the set $\{6...,12\}$ (from 1 to 2 hours).

Dryer The smart homes that have a dryer installed are assumed to run this device 3 times a week on average. The earliest time slot $t_s^{S_D}$ is set at 17:00 ± 1h, while the latest time slot $t_f^{S_D}$ is set at 21:00 ± 1h. A typical dryer operates at a power rate W^{S_D} of 1.24 kW, while the duration d^{S_D} (in time slots) is drawn uniformly from the set $\{6...,9\}$ (1 to 1.5 hours).

Dishwasher A dishwasher is run on average 4 times per week, either at 15:00 ± 1h (with probability 0.5) or at 19:00 ± 1h (with probability 0.5). Here we assume that the latest time slot $t_f^{S_{DW}}$ is set at 19:00 ± 1h (if the dishwasher is run in the afternoon), or at 23:00 ± 1h (if the dishwasher is run in the night). A typical dishwasher operates at a power rate $W^{S_{DW}}$ of 0.66 kW. The duration $d^{S_{DW}}$ (in time slots) is drawn uniformly from the set $\{6...,12\}$ (from 1 to 2 hours).

Table 2. Experimental results

	w/o load mgmt. (€)	**REP margin**	**with load mgmt. (€)**	**gain (€)**
Winter	50.65 ± 1.39	$\alpha = 0.1$	32.04 ± 1.79	18.60
		$\alpha = 0.3$	35.97 ± 2.51	14.68
		$\alpha = 0.5$	43.91 ± 2.28	6.74
		$\alpha = 0.7$	45.18 ± 2.76	5.46
Summer	43.02 ± 3.46	$\alpha = 0.1$	16.37 ± 1.16	26.63
		$\alpha = 0.3$	32.31 ± 2.55	10.69
		$\alpha = 0.5$	40.88 ± 2.86	2.11
		$\alpha = 0.7$	42.56 ± 2.68	0.44

4 EXPERIMENTAL RESULTS

We compute the average monthly payment that an individual home will pay if it does not participate in a load management collaborative group. Then, depending on the REP's

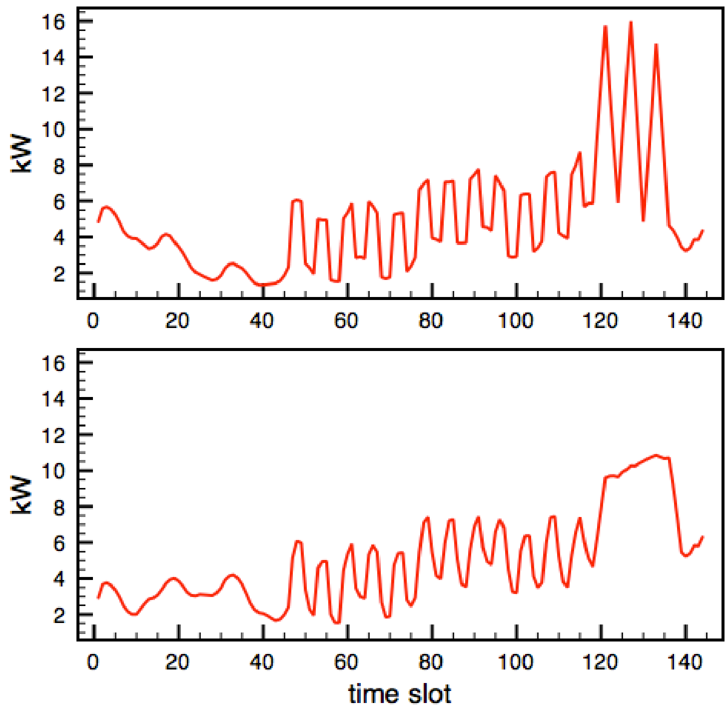

Fig. 2. Load profile with (bottom) and without (top) load management

profit margin (α), we compute the average monthly payment of an individual home that participates in a load management collaborative group. The difference between the payments without and with load management gives us the monthly monetary gain of a individual home. Table 2 shows the results of the experimental simulations. In winter, the average monthly payment without load management is about 50€. With participatory load management, an individual home is able to save 18€ per month, when REP's margin over the spot price of electricity is 10% ($\alpha = 0.1$). For bigger profit margins, of course the advantages of a load management scheme decrease, and the gain falls to 5€ per month, when REP's profit margin is 70% ($\alpha = 0.7$). In summer, the average monthly payment without load management is 43€, slightly lower than winter's payment. The gain obtained from participating in a load management group spans from about 26€ per month ($\alpha = 0.1$) to about 0€ when REP's profit margin very high ($\alpha = 0.7$).

We are also interested in computing the gains that the aggregator obtains. We define the daily gain of the aggregator as the difference between the imbalance penalties paid by the smart homes to the aggregator during the day of delivery and the net financial position of the aggregator after covering short and long positions of the group of smart homes in the balancing markets (see Eq. 18).

$$G = \overbrace{\sum_{t \in \mathcal{T}} p_t^{\text{bal}} \Delta t \sum_{i \in \mathcal{H}} |\epsilon_t^i|}^{\text{home}\rightarrow\text{aggregator}} - \overbrace{\sum_{t \in \mathcal{T}} p_t^{\text{bal}} \Delta t \sum_{i \in \mathcal{H}} \epsilon_t^i}^{\text{aggregator}\rightarrow\text{balancing market}} \tag{18}$$

The monthly net financial position of the aggregator after after covering short and long positions in the balancing markets is on average negative, accounting for a loss of -5.79€ ± 0.52. Nevertheless, since smart homes pay the aggregator for their imbalances, even if they may cancel out and therefore do not require any buying or selling the balancing market, the average monthly gain of the aggregator is 38.12€ ± 6.4. Since the aggregator could be a mere coordinating entity with no profit maximising interests, this gain could be shared among the smart homes and therefore increase the benefit of each participant.

Figure 2 plots the winter load profile when the smart homes are organised in a collaborative group (bottom), compared to the same set of homes that do not participate in the load management (top). It is possible to appreciate how load management smooths the evening power peak, since power capacity is part of the cost function to be minimised (see Eq. 6). This fact does not only translate into lower costs for the consumers, but also lower installation costs for the grid operator, since less capacity is needed to serve the set of homes involved in the participatory load management scheme.

5 CONCLUSIONS

In this work, we put forward a model for participatory load management, where smart homes actively participate in the balancing of demand with supply by forming groups of electricity consumers that agree on a joint demand profile to be contracted with a REP. We defined an economic model where electricity is priced by the REP above the spot market price but below the fixed per unit price paid by conventional consumers. In this way the REP obtain a profit margin and it does not have to take care of balancing the demand of its consumers with supply, since it is direct responsibility of the collaborative group of smart homes. These homes, represented by an aggregator, optimise electricity consumption and power capacity, while trying to sticking to the contracted supply on the day of the delivery. The experimental evaluation shows that an individual smart home may gain up to 18€ per month (in winter) and up to 26€ per month (in summer). At the same time, by putting a price on the needed power capacity, the group of smart homes is able to shave the peak power consumption, thus reducing installation costs.

As future work, the complexity of the optimisation model must be tackled in order to increase scalability, either by distributing the optimisation or by means of metaheuristics methods. Furthermore, more sophisticate techniques to model the stochasticity of the problem can be employed, such as agent-based simulations.

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