Interval Temporal Description Logics*

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1 Introduction

In this paper, we construct a combination $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ of the Halpern-Shoham interval temporal logic $\mathcal{HS}$ [15] with the description logic $\mathit{DL}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ [12, 1], which is a Horn extension of the standard language OWL 2 QL. The temporal operators of $\mathcal{HS}$ are of the form $\langle \mathsf{R} \rangle$ ('diamond') and $[\mathsf{R}]$ ('box'), where R is one of Allen's interval relations *After*, *Begins*, *Ends*, *During*, *Later*, *Overlaps* and their inverses ($\bar{A}$, $\bar{B}$, $\bar{E}$, $\bar{D}$, $\bar{L}$, $\bar{O}$). The propositional variables of $\mathcal{HS}$ are interpreted by sets of closed intervals $[i, j]$ of some flow of time (e.g., $\mathbb{Z}$, $\mathbb{R}$), and a formula $\langle \mathsf{R} \rangle \varphi$ ($[\mathsf{R}]\varphi$) is regarded to be true in $[i, j]$ iff φ is true in some (respectively, all) interval(s) $[i', j']$ such that $[i, j]\mathsf{R}[i', j']$ in Allen's interval algebra.

In $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$, we represent temporal data by means of assertions such as $\mathit{SummerSchool}(\mathit{RW}, t_1, t_2)$ and $\mathit{teaches}(\mathit{US}, \mathit{DL}, s_1, s_2)$, which say that RW is a summer school that takes place in the time interval $[t_1, t_2]$ and US teaches DL in the time interval $[s_1, s_2]$. Note that temporal databases store data in a similar format [17]. Temporal concept and role inclusions are used to impose constraints on the data and introduce new concepts and roles. For example, $\mathit{AdvCourse} \sqcap \langle \bar{D} \rangle \mathit{MorningSession} \sqsubseteq \bot$ says that advanced courses are not given in the morning sessions described by $\langle \bar{B} \rangle \mathit{LectureDay} \sqcap \langle A \rangle \mathit{Lunch} \sqsubseteq \mathit{MorningSession}$; $\mathit{teaches} \sqsubseteq [D]\mathit{teaches}$ claims that the role *teaches* is downward hereditary (or stative) in the sense that if it holds in some interval then it also holds in all of its sub-intervals; $[D](\langle O \rangle \mathit{teaches} \sqcup \langle \bar{D} \rangle \mathit{teaches}) \sqcap \langle B \rangle \mathit{teaches} \sqcap \langle E \rangle \mathit{teaches} \sqsubseteq \mathit{teaches}$, on the contrary, states that *teaches* is coalesced (or upward hereditary). The inclusions $\mathit{teaches} \sqsubseteq [D]\mathit{teaches}$ and $[D](\langle O \rangle \mathit{teaches} \sqcup \langle \bar{D} \rangle \mathit{teaches}) \sqsubseteq \mathit{teaches}$ ensure that *teaches* is both upward and downward hereditary. On the other hand, 'rising stock market' and 'high average speed' are typical examples of concepts that are not downward hereditary; for a discussion of these notions see [6, 21, 18].

Although the complexity of full $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ remains unknown, in this paper we define two fragments, $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}/\mathit{flat}}_{horn}$ and $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}[G]}_{horn}$, where satisfiability and instance checking are P-complete for both combined and data complexity.

Our interest in tractable description logics with interval temporal operators is motivated by possible applications in ontology-based data access (OBDA) [12] to *temporal databases*. In this context, we naturally require reasonably expressive yet tractable ontology and query languages with temporal constructs (although

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some authors advocate the use of standard atemporal OWL 2 QL with temporal queries [16, 7]). Our choice of $\mathcal{HS}$ as the temporal component of $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ is explained by the fact that modern temporal databases adopt the (downward hereditary) interval-based model of time [17, 13] and use coalescing to group time points into intervals [6]. We show that, unfortunately, the logics $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}/flat}_{horn}$ and $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}[G]}_{horn}$ cannot guarantee first-order rewritability of even atomic queries, though we conjecture that datalog rewritings are possible.

2 Description Logic $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$

The language of $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ contains *individual names* $a_0, a_1, \dots$, *concept names* $A_0, A_1, \dots$, and *role names* $P_0, P_1, \dots$. *Basic roles* R, *basic concepts* B, *temporal roles* S and *temporal concepts* C are given by the grammar

$$R \;::=\; P_k \;\mid\; P_k^-, \qquad\qquad B \;::=\; A_k \;\mid\; \exists R,$$
$$S \;::=\; R \;\mid\; [\mathsf{R}]S \;\mid\; \langle\mathsf{R}\rangle S, \qquad\qquad C \;::=\; B \;\mid\; [\mathsf{R}]C \;\mid\; \langle\mathsf{R}\rangle C,$$

where R is one of Allen's interval relations or the universal relation G. Over the closed intervals $[i, j] = \{n \in \mathbb{Z} \mid i \le n \le j\}$, for $i \le j$, we set:

- $[i, j]A[i', j']$ iff $j = i'$, (After)
- $[i, j]B[i', j']$ iff $i = i'$ and $j \ge j'$, (Begins)
- $[i, j]E[i', j']$ iff $i \le i'$ and $j = j'$, (Ends)
- $[i, j]D[i', j']$ iff $i \le i'$ and $j' \le j$, (During)
- $[i, j]L[i', j']$ iff $j \le i'$, (Later)
- $[i, j]O[i', j']$ iff $i \le i' \le j \le j'$ (Overlaps)

and define their inverses in the standard way. Note that we allow single-point intervals $[i, i]$ and use non-strict $\le$ instead of the more common $<$ (in fact, one can show that the use of $<$ would make reasoning non-tractable). An $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ TBox is a finite set of *concept* and *role inclusions* and *disjointness constraints* of the form

$$C_1 \sqcap \dots \sqcap C_k \sqsubseteq C^+, \qquad\qquad S_1 \sqcap \dots \sqcap S_k \sqsubseteq S^+,$$
$$C_1 \sqcap \dots \sqcap C_k \sqsubseteq \bot, \qquad\qquad S_1 \sqcap \dots \sqcap S_k \sqsubseteq \bot,$$

where C^+, R^+ denote temporal concepts and roles without diamond operators $\langle\mathsf{R}\rangle$. An $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ ABox is a finite set of atoms of the form $A_k(a, i, j)$ and $P_k(a, b, i, j)$ in which *temporal constants* $i \le j$ are given in binary. An $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ *knowledge base* (KB) is a pair $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ an ABox.

An $\mathcal{HS}\text{-}\mathit{Lite}^{\mathcal{H}}_{horn}$ *interpretation*, $\mathcal{I}$, consists of a family of standard (atemporal) DL interpretations $\mathcal{I}[i, j] = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}[i,j]})$, for all $i, j \in \mathbb{Z}$ with $i \le j$, in which $\Delta^{\mathcal{I}} \neq \emptyset$, $a_k^{\mathcal{I}[i,j]} = a_k^{\mathcal{I}}$ for some (fixed) $a_k^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $A_k^{\mathcal{I}[i,j]} \subseteq \Delta^{\mathcal{I}}$ and $P_k^{\mathcal{I}[i,j]} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The role and concept constructs are interpreted in $\mathcal{I}$ as follows:

$$(P_k^-)^{\mathcal{I}[i,j]} = \big\{(x, y) \mid (y, x) \in P_k^{\mathcal{I}[i,j]}\big\}, \qquad (\exists R)^{\mathcal{I}[i,j]} = \big\{x \mid (x, y) \in R^{\mathcal{I}[i,j]}\big\},$$
$$([\mathsf{R}]S)^{\mathcal{I}[i,j]} = \bigcap_{[i,j]\mathsf{R}[i',j']} S^{\mathcal{I}[i',j']}, \qquad ([\mathsf{R}]C)^{\mathcal{I}[i,j]} = \bigcap_{[i,j]\mathsf{R}[i',j']} C^{\mathcal{I}[i',j']}$$

and dually for the 'diamond' operators $\langle\mathsf{R}\rangle$.

The *satisfaction relation* $\models$ is defined by taking:

$$\begin{array}{rcl}
\mathcal{I} \models A(a,i,j) & \text{iff} & a^{\mathcal{I}} \in A^{\mathcal{I}[i,j]}, \\
\mathcal{I} \models P(a,b,i,j) & \text{iff} & (a^{\mathcal{I}}, b^{\mathcal{I}}) \in P^{\mathcal{I}[i,j]}, \\
\mathcal{I} \models \bigsqcap_k C_k \sqsubseteq C & \text{iff} & \bigcap_k C_k^{\mathcal{I}[i,j]} \subseteq C^{\mathcal{I}[i,j]}, \quad \text{for } \textit{all} \text{ intervals } [i,j], \\
\mathcal{I} \models \bigsqcap_k S_k \sqsubseteq S & \text{iff} & \bigcap_k S_k^{\mathcal{I}[i,j]} \subseteq S^{\mathcal{I}[i,j]}, \quad \text{for } \textit{all} \text{ intervals } [i,j],
\end{array}$$

and similarly for disjointness constraints. Note that concept and role inclusions as well as disjointness constraints are interpreted *globally*. For a TBox inclusion or an ABox assertion α, we write $\mathcal{K} \models \alpha$ if $\mathcal{I} \models \alpha$, for all models $\mathcal{I}$ of $\mathcal{K}$.

3 Propositional $\mathcal{HS}_{horn}$ is Tractable

Denote by $\mathcal{HS}_{horn}$ the fragment of $\mathcal{HS}$-$Lite^{\mathcal{H}}_{horn}$ *without* role names and with ABoxes that contain a *single* individual name. TBoxes in this restricted language can be regarded as Horn formulas of the propositional interval temporal logic $\mathcal{HS}$, which is notorious for its nasty computational behaviour; for results on the (un)decidability of various fragments of $\mathcal{HS}$, see, e.g., [14, 10, 9, 8, 19, 11, 20]. The designed logic $\mathcal{HS}_{horn}$ appears to be the first *tractable* fragment of $\mathcal{HS}$:

Theorem 1. *$\mathcal{HS}_{horn}$ is* P*-complete for both combined and data complexity.*

Membership in P follows from the polynomial canonical model and P-hardness for (data) complexity is by reduction of the monotone circuit value problem.

So far, we have managed to lift this result to two proper interval temporal description logics, both of which are fragments of $\mathcal{HS}$-$Lite^{\mathcal{H}}_{horn}$.

4 Tractability of $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$ and $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$

The first fragment, denoted $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$, only allows those $\mathcal{HS}$-$Lite^{\mathcal{H}}_{horn}$ TBoxes that are *flat* in the sense that their concept inclusions do not contain $\exists R$ on the right-hand side. Our second fragment, denoted $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$, allows only the operator $[G]$ in the definition of temporal roles S (with no restrictions imposed on temporal concepts). Thus, unlike $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$, the fragment $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$ contains full DL-$Lite^{\mathcal{H}}_{horn}$.

Theorem 2. (i) *The satisfiability problem for $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$ and $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$ KBs is* P*-complete for combined complexity.*

(ii) *Instance checking for $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$ and $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$ is* P*-complete for data complexity.*

This result contrasts with the lower data complexity (AC^0 and NC^1) of instance checking with point-based temporal *DL-Lite* [5, 3, 2].

In view of Theorem 2 (ii), the temporal ontology languages $\mathcal{HS}$-$Lite^{\mathcal{H}/flat}_{horn}$ and $\mathcal{HS}$-$Lite^{\mathcal{H}[G]}_{horn}$ cannot guarantee first-order rewritability of even atomic queries, though we believe that datalog rewritings are possible. We leave the query rewritability issues, in particular, the design of DL-$Lite^{\mathcal{H}}_{core}$-based fragments supporting first-order rewritability as well as temporal extensions of the OWL 2 EL and OWL 2 RL profiles of OWL 2 for future research.

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