Nonmonotonic Nominal Schemas Revisited

Matthias Knorr

NOVA LINCS, Departamento de Informática, Faculdade de Ciências e Tecnologia,
Universidade Nova de Lisboa

Abstract. Recently, a very general description logic (DL) that extends $\mathcal{SROIQ}$ (the DL underlying OWL 2 DL) at the same time with nominal schemas and epistemic modal operators has been proposed, which encompasses some of the most prominent monotonic and non-monotonic rule languages, including Datalog under the answer set semantics. A decidable fragment is also presented, but the restricted language does not fully cover all formalisms encompassed by the complete language. In this paper, we aim to remedy that by studying an alternative set of restrictions to achieve decidability, and we show that the existing embeddings of the formalisms covered by the full language can be adjusted accordingly.

1 Introduction

Extending Description Logics (DLs) with modeling features admitting non-monotonic reasoning has been frequently requested in many application domains (see, e.g., [14] for semantic matchmaking on annotations at electronic online marketplaces). In fact, the vast amount of work dedicated to the topic may serve as a witness in its own right. DLs have been extended, for example, with defaults [2], with notions of circumscription [4,33], and epistemic reasoning provided by the inclusion of modal[1] operators within the language [8] or only in queries [29]. In addition, a plethora of approaches combine DLs with (often non-monotonic) rules (see, e.g., [9,30,19] and references in their sections on related work). As these approaches are commonly of different expressivity and based on quite advanced different formal grounds, a uniform overarching formalism allowing the integration of possibly all the various modeling features is an extremely complicated problem.

In [20], a very general DL language is introduced that extends the expressive DL underlying OWL 2, $\mathcal{SROIQ}$, with nominal schemas [24] and epistemic operators as defined in [8] with the aim of integrating the W3C standards OWL [15] and (non-monotonic) RIF [18] and their underlying formalisms, DLs and rule languages respectively, thus contributing towards the goal of a unifying logic for the Semantic Web (as foreseen in the well-known Semantic Web stack). The full language is in fact very expressive, capturing a variety of different formalisms, among them two based on MKNF logics [27] that had been considered of different expressivity so far – MKNF DLs [8], i.e., the epistemic extension of DLs, and Hybrid MKNF, one very expressive combination of DLs and non-monotonic rules. Though not the full language of the latter is

[1] In the remainder of the paper, we use the terms modal and epistemic operator interchangeably to refer to the same notion.

considered, coverage of Answer Set Programming [11] is ensured, arguably the most widely used non-monotonic reasoning rule formalism.

A decidable fragment of the full language is also considered in [20], which is strongly related to the one presented in [8]. In fact, the restrictions are such that the tableau algorithm presented in [8] can in principle be re-used. As it turns out, however, the decidable language does not encompass all the formalisms for which coverage within the full language is shown. While this does not invalidate the approach as such, in particular, if one views such a unifying formalism mainly as a conceptual underpinning, it is certainly undesired if one rather wants to use it for modeling and reasoning.

In this paper, we aim to solve this problem, i.e., we consider a different set of restrictions, for which we show that reasoning is decidable and that, at the same time, encompasses all the formalisms discussed in [20] with only minor adjustments to the previously presented embeddings. The principal idea builds on the usage of nominals and nominal schemas, which are necessarily present in the language by design anyway, to limit the applicability of concept inclusions containing modal operators. As an additional result, we believe that the new restrictions are more succinct and that the resulting adaptation of the procedure for verifying the existence of models becomes less complicated. To further simplify notation, here we do not consider the full language presented in [20], which is based on $\mathcal{SROIQ}$, but rather a language based on $\mathcal{ALC}$ with only the minimally necessary extensions and we term this language $e\mathcal{ALCOV}$ (see Sect. 2 for a detailed explanation on the name). As we can show, such language is already expressive enough to cover the desired non-monotonic modeling features.

The remainder of the paper proceeds as follows. In Sect. 2, we recall the syntax and semantics of the DL $e\mathcal{ALCOV}$ we consider here. We then introduce the new alternative conditions of so-called safe $e\mathcal{ALCOV}$ KBs in Sect. 3 and we subsequently show that these do ensure decidability of reasoning, i.e., checking (MKNF-)satisfiability. In Sect. 4, we show that, with minor adaptations, the applied changes do now permit coverage of the discussed formalisms in [20] within the decidable fragment (of safe $e\mathcal{ALCOV}$ KBs), before we conclude and discuss future work in Sect. 5.

2 Epistemic DLs

In this section, we recall the syntax and semantics of epistemic description logics (DLs) from [20]. Here, we focus on a subset of the language considered in [20] to make the presentation more concise and to ease the reading. Namely, we consider the epistemic DL $\mathcal{ALCK}_{\mathcal{NF}}$ [8], which is $\mathcal{ALC}$ enhanced with epistemic operators, extended by nominals, nominal schemas [24], and the universal role. Nominal schemas represent variable nominals that can only be bound to known individuals, and the universal role can be represented using role hierarchies and negation on roles [23], but as we want to keep the presentation simple, we leave this implicit. Since the name of the resulting language $\mathcal{ALCOVK}_{\mathcal{NF}}$ (or even $\mathcal{ALCHOV}(\neg)\mathcal{K}_{\mathcal{NF}}$ for the implicit encoding of the universal role) following standard and historic patterns would be quite cumbersome, we propose using the name $e\mathcal{ALCOV}$ instead, which stands for epistemic $\mathcal{ALCOV}$ (including the universal role). The term epistemic originates from the two epistemic/modal operators $\mathbf{K}$ and $\mathbf{A}$, where $\mathbf{K}$ is interpreted in terms of minimal knowledge, while $\mathbf{A}$ is interpreted

Table 1. Syntax and semantics of $e\mathcal{ALCOV}$

Name	Syntax	Semantics
concept name	A	$A^{\mathcal{I}} \subseteq \Delta$
role name	V	$V^{\mathcal{I}} \subseteq \Delta \times \Delta$
individual name	a	$a^{\mathcal{I}} \in \Delta$
variable	x	$\mathcal{Z}(x) \in \Delta$
top	$\top$	Δ
bottom	$\bot$	$\emptyset$
nominal (schema)	$\{t\}$	$\{a \mid [a]_{\approx} \approx t^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
concept complement	$\neg C$	$\Delta \setminus C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
concept conjunction	$C \sqcap D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cap D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
concept disjunction	$C \sqcup D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \cup D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
existential restriction	$\exists R.C$	$\{\delta \in \Delta \mid \exists \epsilon \text{ with } (\delta,\epsilon) \in R^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \text{ and } \epsilon \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
universal restriction	$\forall R.C$	$\{\delta \in \Delta \mid (\delta,\epsilon) \in R^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \text{ implies } \epsilon \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}\}$
knowledge concept	$\mathbf{K}C$	$\bigcap_{\mathcal{J}\in\mathcal{M}} C^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
assumption concept	$\mathbf{A}C$	$\bigcap_{\mathcal{J}\in\mathcal{N}} C^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
universal role	U	$\Delta \times \Delta$
knowledge role	$\mathbf{K}V$	$\bigcap_{\mathcal{J}\in\mathcal{M}} V^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
assumption role	$\mathbf{A}V$	$\bigcap_{\mathcal{J}\in\mathcal{N}} V^{(\mathcal{J},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
role assertion	$V(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in V^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$
TBox axiom	$C \sqsubseteq D$	$C^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}} \subseteq D^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$

Interpretation $\mathcal{I}$; MKNF structure $(\mathcal{I},\mathcal{M},\mathcal{N})$; variable assignment $\mathcal{Z}$; $A \in N_C$; $C, D \in \mathsf{C}$; $V \in N_R$; $R \in \mathsf{R}$; $a, b \in N_I$; $x \in N_V$, and $t \in N_V \cup N_I$.

as autoepistemic assumption and corresponds to $\neg\mathbf{not}$, i.e., the classical negation of the negation as failure operator $\mathbf{not}$ used in [27] instead of $\mathbf{A}$.

We consider a signature $\Sigma = \langle N_I, N_C, N_R, N_V\rangle$ where N_I, N_C, N_R, and N_V are pairwise disjoint and finite sets of *individual names*, *concept names*, *role names*, and *variables*. In the following, we assume that Σ has been fixed. We define concepts and roles in $e\mathcal{ALCOV}$ by the following grammar.

$$\mathsf{R} ::= V \mid U \mid \mathbf{K}V \mid \mathbf{A}V$$

$$\mathsf{C} ::= \top \mid \bot \mid A \mid \{i\} \mid \{x\} \mid \neg\mathsf{C} \mid \mathsf{C} \sqcap \mathsf{C} \mid \mathsf{C} \sqcup C \mid \exists\mathsf{R}.\mathsf{C} \mid \forall\mathsf{R}.\mathsf{C} \mid \mathbf{K}\mathsf{C} \mid \mathbf{A}\mathsf{C}$$

where $V \in N_R$, $A \in N_C$, $i \in N_I$, and $x \in N_V$. The names of the individual concepts and roles can be found in Table 1. *Epistemic concepts* are knowledge and assumption concepts, while *epistemic roles* are knowledge and assumption roles.

As usual, a *TBox axiom* (or *general concept inclusion* (GCI)) is an expression $C \sqsubseteq D$ where $C, D \in \mathsf{C}$. An *ABox axiom* is of the form $C(a)$ or $V(a,b)$ where $C \in \mathsf{C}$,

$V \in N_R$, and $a, b \in N_I$. An $e\mathcal{ALCOV}$ *axiom* is any ABox or TBox axiom, and an $e\mathcal{ALCOV}$ *knowledge base* (KB) is a finite set of $e\mathcal{ALCOV}$ axioms.

The semantics of $e\mathcal{ALCOV}$ as recalled from [20] is an adaptation from [24] and [8]. As common, an *interpretation* $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ consists of a domain $\Delta^\mathcal{I} \neq \emptyset$ and a function $\cdot^\mathcal{I}$ that maps elements in N_I, N_C, and N_R to elements, sets, and relations of $\Delta^\mathcal{I}$ respectively. Additionally, nominal schemas require a *variable assignment* $\mathcal{Z}$ for an interpretation $\mathcal{I}$, which is a function $\mathcal{Z} : N_V \rightarrow \Delta^\mathcal{I}$ such that, for each $v \in N_V$, $\mathcal{Z}(v) = a^\mathcal{I}$ for some $a \in N_I$.

As common in MKNF-related semantics used to combine DLs with non-monotonic reasoning (see [8,17,19,30]), specific restrictions on interpretations are introduced to ensure that certain unintended logical consequences can be avoided (see, e.g., [30]). Here, we adopt the standard name assumption from [20]. An interpretation $\mathcal{I}$ (over Σ to which $\approx$ is added) employs the *standard name assumption* if

(1) N_I^* extends N_I with a countably infinite set of individuals that cannot be used in variable assignments, and $\Delta^\mathcal{I} = N_I^*$;
(2) for each i in N_I^*, $i^\mathcal{I} = i$; and
(3) equality $\approx$ is interpreted in $\mathcal{I}$ as a congruence relation – that is, $\approx$ is reflexive, symmetric, transitive, and allows for the replacement of equals by equals [10].

The first condition fixes the (infinite) universe, but limits the application of variable assignments to a finite subset, the second condition defines $\mathcal{I}$ as a bijective function, while the third ensures that we still can identify elements of the domain. As an immediate side-effect, the variable assignment is no longer tied to a specific interpretation and we can simplify notation by using Δ without reference to a concrete interpretation.

Now, the first-order semantics is lifted to satisfaction in MKNF structures that treat the modal operators w.r.t. sets of interpretations. An *MKNF structure* is a triple $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ where $\mathcal{I}$ is an interpretation, $\mathcal{M}$ and $\mathcal{N}$ are sets of interpretations, and $\mathcal{I}$ and all interpretations in $\mathcal{M}$ and $\mathcal{N}$ are defined over Δ. For any such $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ and assignment $\mathcal{Z}$, the function $\cdot^{(\mathcal{I},\mathcal{M},\mathcal{N}),\mathcal{Z}}$ is defined for arbitrary $e\mathcal{ALCOV}$ expressions as shown in Table 1. $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ and $\mathcal{Z}$ *satisfy* an $e\mathcal{ALCOV}$ axiom α, written $(\mathcal{I}, \mathcal{M}, \mathcal{N}), \mathcal{Z} \models \alpha$, if the corresponding condition in Table 1 holds. $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ *satisfies* α, written $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models \alpha$, if $(\mathcal{I}, \mathcal{M}, \mathcal{N}), \mathcal{Z} \models \alpha$ for all variable assignments $\mathcal{Z}$. A (non-empty) set of interpretations $\mathcal{M}$ *satisfies* α, written $\mathcal{M} \models \alpha$, if $(\mathcal{I}, \mathcal{M}, \mathcal{M}) \models \alpha$ holds for all $\mathcal{I} \in \mathcal{M}$, and $\mathcal{M}$ *satisfies* an $e\mathcal{ALCOV}$ knowledge base KB, written $\mathcal{M} \models KB$, if $\mathcal{M} \models \alpha$ for all axioms $\alpha \in KB$. Note the small deviation of the semantics of $\{t\}$ in Table 1 compared to that in [24], which is necessary to ensure that the semantics works as intended under standard name assumption.

It can be verified that the two sets of interpretations are each used to interpret one of the modal operators, but in the monotonic semantics above, they simply coincide. This changes with the non-monotonic MKNF model defined in the usual fashion [8,17,19,30]: $\mathcal{M}$ is fixed to interpret $\mathbf{A}$, and supersets $\mathcal{M}'$ of $\mathcal{M}$ are used to test whether the knowledge derived from $\mathcal{M}$ (via $\mathbf{K}$) is indeed minimal.

Definition 1. *Given an $e\mathcal{ALCOV}$ knowledge base KB, a (non-empty) set of interpretations $\mathcal{M}$ is an* MKNF model *of KB if (1) $\mathcal{M} \models KB$, and (2) for each $\mathcal{M}'$ with $\mathcal{M} \subset \mathcal{M}'$, $(\mathcal{I}', \mathcal{M}', \mathcal{M}) \not\models KB$ for some $\mathcal{I}' \in \mathcal{M}'$. KB is* MKNF-satisfiable *if an*

MKNF model of KB exists. An axiom α is MKNF-entailed by KB, *written $KB \models_{\mathbf{K}} \alpha$, if all MKNF models $\mathcal{M}$ of KB satisfy α.*

As noted in [8], since $\mathcal{M} \models KB$ is defined w.r.t. $(\mathcal{I}, \mathcal{M}, \mathcal{M})$, the operators $\mathbf{K}$ and $\mathbf{A}$ are interpreted in the same way, and so we can restrict instance checking $KB \models_{\mathbf{K}} C(a)$ and subsumption $KB \models_{\mathbf{K}} C \sqsubseteq D$ to C and D without occurrences of the operator $\mathbf{A}$. Also, in absence of modal operators in the $e\mathcal{ALCOV}$ KB, there is a unique MKNF model which simply contains all standard (first-order) models of KB as usual.

3 Reasoning in $e\mathcal{ALCOV}$

In [20], reasoning in $e\mathcal{SROIQ}$[2] is discussed following [8] for reasoning in $\mathcal{ALCK}_{\mathcal{NF}}$. The problem with this approach is that it is undecidable in general, so, as in [8], restrictions are applied in [20] to regain decidability, which in certain cases prevent coverage of the formalisms encompassed by the unrestricted language. To circumvent this, we still rely on the same idea in principle, but we revise the applied restrictions to achieve decidability making use of the gained expressiveness in $e\mathcal{ALCOV}$. In the following, we spell out the restrictions with some motivation right away, before we show that this indeed yields a decidable procedure for checking MKNF-satisfiability.

3.1 Safe $e\mathcal{ALCOV}$ KBs

Following [8], the overall idea is to reduce reasoning in $e\mathcal{ALCOV}$ to a number of reasoning tasks in non-modal $\mathcal{ALCOV}$ (again including U), for which each model of an $e\mathcal{ALCOV}$ KB is represented by means of an $\mathcal{ALCOV}$ KB. Formally, a set of interpretations $\mathcal{M}$ is $\mathcal{ALCOV}$ *representable* if there exists an $\mathcal{ALCOV}$ KB $KB_{\mathcal{M}}$ such that $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \text{ satisfies } KB_{\mathcal{M}}\}$. Then, undecidability can be caused by three sources. First, certain partially quantified expressions are not $\mathcal{ALCOV}$ representable (Theorem 4.1 in [8]), which is why we recall the notion of subjectively quantified KBs. For that purpose, we define that an $\mathcal{ALCOV}$ expression S is *subjective* if each $\mathcal{ALCOV}$ subexpression in S lies in the scope of at least one modal operator.

Definition 2. *An $e\mathcal{ALCOV}$ KB KB is* subjectively quantified *if each expression of the form $\exists R.C, \forall R.C$ occurring in KB satisfies one of the conditions: (1) R is an $\mathcal{ALCOV}$ role and C is an $\mathcal{ALCOV}$ concept, or (2) R and C are both subjective and C is of the form $\mathbf{K}D$, $\neg\mathbf{K}D$, $\mathbf{A}D$ or $\neg\mathbf{A}D$.*

There exists a slightly relaxed condition on subjectively quantified KBs [17], but for our purposes the original one suffices.

Second, even if subjectively quantified, certain nested expressions can be problematic, so we introduce (modally) flat concepts, that can be seen as a further restriction of simple concepts in [8], which prohibit such nesting altogether. Formally, an $e\mathcal{ALCOV}$ concept is *flat* if it does not contain any modal operator in scope of another, and an

[2] In [20], the term $\mathcal{SROIQV}(\mathcal{B}^s, \times)\mathcal{K}_{\mathcal{NF}}$ is used, but, for the sake of readability and with a slight abuse of notation, we follow our introduced naming scheme here.

$e\mathcal{ALCOV}$ KB KB is *flat* if each concept in it is flat. Thus, quantifier expressions of the form (2) in Def. 2 are flat as long as D does not contain further modal operators.

Third, intuitively, we have to make sure that GCIs involving modal operators cannot be used to derive an infinite number of true assertions (see also Theorem 4.10 in [8]). Rather than introducing simple KBs as in [8,20], we build on nominals and nominal schemas to introduce safe concepts.

Definition 3. *Given a subjectively quantified, flat $e\mathcal{ALCOV}$ KB KB, an $e\mathcal{ALCOV}$ concept C in KB is called* safe *if C is of the form $D \sqcap \{t\}$ for some* guard $t \in N_I \cup N_V$.

The idea is to use the nominal (schema) as a guard that restricts "applicability" of concepts involving modal operators to individuals occurring in KB. This all combines in the definition of safe $e\mathcal{ALCOV}$ KBs, for which we from now on consider two disjoint subsets of the $e\mathcal{ALCOV}$ TBox $\mathcal{T}$ of KB: $\mathcal{T}'$, the set of all axioms that contain no modal operators, and Γ, the set of all axioms that contain at least one modal operator.

Definition 4. *Let KB be an $e\mathcal{ALCOV}$ KB that is subjectively quantified and flat. Then, KB is* safe *if the following conditions are satisfied:*

1. *For each $C \sqsubseteq D \in \Gamma$, C is subjective and safe, D is safe for the same guard as C, and no operator $\mathbf{K}$ occurs in $\exists$ and $\forall$ restrictions in D;*
2. *There is no concept assertion in KB containing a subconcept of the form $\exists \mathbf{K}R.\mathbf{K}C$.*

Notably, due to KB also being subjectively quantified and flat, any $C \sqsubseteq D \in \Gamma$ in a safe KB can be rewritten into one such that all subjective subconcepts in it are safe. For example, the safe KB containing just the axiom

$$((\mathbf{K}(C \sqcup \exists R.D) \sqcap \exists \mathbf{K}R.\mathbf{K}G) \sqcup \neg \mathbf{A}E) \sqcap \{x\} \sqsubseteq \forall \mathbf{A}S.\mathbf{A}F \sqcap \{x\}$$

can be straightforwardly rewritten into

$$((\mathbf{K}(C \sqcup \exists R.D) \sqcap \{x\}) \sqcap (\exists \mathbf{K}R.\mathbf{K}G \sqcap \{x\})) \sqcup (\neg \mathbf{A}E \sqcap \{x\}) \sqsubseteq \forall \mathbf{A}S.\mathbf{A}F \sqcap \{x\}.$$

In the following, we assume that any $C \sqsubseteq D \in \Gamma$ in a safe $e\mathcal{ALCOV}$ KB is already rewritten this way, i.e., all subjective subconcepts in Γ are assumed safe.

Comparing to simple KBs (Def. 8 in [20]), intuitively, KB being flat covers condition 3. while 1. of Def. 4 covers to 1. and 2. there. Condition 2. in Def. 4 is not strictly required for decidability (as the case can be handled by finitely many models up to renaming of individuals [8]), but it will simplify the subsequent material without affecting coverage of related formalisms. These new conditions for safe KBs certainly have quite a different flavor compared to simple KBs in [8,20], but we believe that they are overall simpler and more easy to grasp, and at the same time not jeopardizing coverage of related formalisms. Of course, we still need to show that there is a decidable procedure for reasoning with safe $e\mathcal{ALCOV}$ KBs.

3.2 Determining MKNF Models

We start by grounding a given safe $e\mathcal{ALCOV}$ KB, i.e., we replace all occurring nominal schemas with nominals in all possible ways in the usual manner. This yields an $e\mathcal{ALCO}$ KB (again including U), which is trivially safe and obtained in finite time, though, in general, of exponential size in terms of the input KB.

From now on, we follow the principal argument from [8] as used in [20], but with some variations and simplifications due to our different restrictions and to some extent inspired by the reasoning algorithms for the related Hybrid MKNF [30].

First, we will collect a set of modal atoms based on the occurrence of epistemic concepts and roles in a given KB. In difference to [8], we only consider atoms over individuals occurring in the given KB. In the following, we use $\mathbf{M}$ to denote either $\mathbf{K}$ or $\mathbf{A}$, and $\mathbf{N}$ to denote either $\mathbf{M}$ or $\neg\mathbf{M}$, we assume $Q \in \{\exists, \forall\}$, and we remind that we consider that all subjective concepts in Γ are safe. Given a safe $e\mathcal{ALCO}$ KB KB, the set of *modal atoms* $MA(KB)$ is defined inductively as follows:

(1) if $\mathbf{M}D \sqcap \{a\}$ for some $a \in N_I$ occurs in KB, then $\mathbf{K}D(a) \in MA(KB)$;
(2) if $\mathbf{M}D$ occurs (non-safe) in concept assertion $C(a)$, then $\mathbf{K}D(a) \in MA(KB)$;
(3) if $Q\mathbf{M}R.\mathbf{N}D \sqcap \{a\}$ for some $a \in N_I$ occurs in KB, then $\mathbf{K}R(a,i), \mathbf{K}D(i) \in MA(KB)$ for all $i \in N_I$;
(4) if $Q\mathbf{M}R.\mathbf{N}D$ occurs (non-safe) in concept assertion $C(a)$, then $\mathbf{K}R(a,i), \mathbf{K}D(i) \in MA(KB)$ for all $i \in N_I$;
(5) nothing else belongs to $MA(KB)$.

A further difference to [8] is that we only collect modal atoms under $\mathbf{K}$. This is justified by the fact that for ensuring condition (1) of Def. 1, the same set of interpretations $\mathcal{M}$ is considered for evaluating formulas under $\mathbf{K}$ and $\mathbf{A}$. As an immediate benefit, when introducing partitions of these modal atoms next and guessing model candidates, we do not have to verify whether modal atoms under $\mathbf{K}$ and $\mathbf{A}$ are aligned.

We now introduce a *partition* of $MA(KB)$, which is a pair (P, N) of *positive modal atoms* P and *negative modal atoms* N such that $P \cap N = \emptyset$ and $P \cup N = MA(KB)$. As already mentioned, such partition can be understood as a guess about which modal atoms are supposed to be true (P) and false (N), and we can use it to simplify an $e\mathcal{ALCO}$ KB as follows. Given a safe $e\mathcal{ALCO}$ KB KB and a partition (P, N) of $MA(KB)$, $KB[P]$ denotes the $e\mathcal{ALCO}$ KB obtained from KB and (P, N) by:

1. replacing each occurrence of the form $\mathbf{M}D \sqcap \{a\}$ in KB and each (non-safe) occurrence of the form $\mathbf{M}D$ in a concept assertion $C(a) \in KB$ with $\top$ if $\mathbf{K}D(a) \in P$ and with $\bot$ otherwise;
2. replacing each occurrence of $\exists\mathbf{M}R.\mathbf{M}_1D \sqcap \{a\}$ ($\exists\mathbf{M}R.\neg\mathbf{M}_1D \sqcap \{a\}$) in KB and each (non-safe) occurrence of the form $\exists\mathbf{M}R.\mathbf{M}_1D$ ($\exists\mathbf{M}R.\neg\mathbf{M}_1D$) in a concept assertion $C(a) \in KB$ with $\top$ if there exists y such that $\mathbf{K}R(a,y) \in P$ and $\mathbf{K}D(y) \in P$ ($\mathbf{K}D(y) \notin P$) and with $\bot$ otherwise;
3. replacing each occurrence of $\forall\mathbf{M}R.\mathbf{M}_1D \sqcap \{a\}$ ($\forall\mathbf{M}R.\neg\mathbf{M}_1D \sqcap \{a\}$) in KB and each (non-safe) occurrence of the form $\forall\mathbf{M}R.\mathbf{M}_1D$ ($\forall\mathbf{M}R.\neg\mathbf{M}_1D$) in a concept assertion $C(a) \in KB$ with $\top$ if for each y such that $\mathbf{K}R(a,y) \in P$, $\mathbf{K}D(y) \in P$ ($\mathbf{K}D(y) \notin P$) and with $\bot$ otherwise.

Note that we leave N implicit here, as it is completely specified by P and the definition of a partition of $MA(KB)$. We generalize the notion of $KB[P]$, based on two partitions $(P, N), (P', N')$ of $MA(KB)$, to $KB[P'][P]$ which is obtained from KB in exactly the same way, only that if $\mathbf{M}$ or $\mathbf{M}_1$ is $\mathbf{K}$, then P' is used in the evaluation of the conditions, while for $\mathbf{M}$ or $\mathbf{M}_1$ being $\mathbf{A}$, P is used. In either case, it can readily be verified that the resulting KB does not contain any modal operators, hence is an $\mathcal{ALCO}$ KB (admitting U) for which satisfiability can be checked using standard DL reasoners.

With this in place, we can define an $\mathcal{ALCO}$ KB which takes the modal atoms guessed to be true into account, and use the resulting KB to check whether a guess is consistent with the original $e\mathcal{ALCO}$ KB. Let KB be a safe $e\mathcal{ALCO}$ KB and (P, N), (P', N') partitions of $MA(KB)$. Then, $Ob_{KB,P',P}$ denotes the following $\mathcal{ALCO}$ KB:

$$Ob_{KB,P',P} = KB[P'][P] \cup \{C(x) \mid \mathbf{K}C(x) \in P'\} \cup \{R(x,y) \mid \mathbf{K}R(x,y) \in P'\}$$

Then, partition (P, N) of $MA(KB)$ is *consistent with* the (safe) $e\mathcal{ALCO}$ KB if the following conditions hold:
(1) the $\mathcal{ALCO}$ KB $Ob_{KB,P,P}$ is satisfiable;
(2) $Ob_{KB,P,P} \not\models C(x)$ for each $\mathbf{K}C(x) \in N$;
(3) $Ob_{KB,P,P} \not\models R(x,y)$ for each $\mathbf{K}R(x,y) \in N$.
Basically, item (1) checks whether the guessed P does not yield contradictions w.r.t. KB, while (2) and (3) verify that no modal atom occurs wrongfully in N.

A link between a set of interpretations and partitions is established next. Let $\mathcal{M}$ be a set of interpretations over Δ. Then, $\mathcal{M}$ *induces* the partition (P, N) of $MA(KB)$:

$$\begin{aligned}
P = {} & \{\mathbf{K}C(x) \mid \mathbf{K}C(x) \in MA(KB) \text{ and } \mathcal{M} \models \mathbf{K}C(x)\} \\
& \cup \{\mathbf{K}R(x,y) \mid \mathbf{K}R(x,y) \in MA(KB) \text{ and } \mathcal{M} \models \mathbf{K}R(x,y)\} \\
N = {} & \{\mathbf{K}C(x) \mid \mathbf{K}C(x) \in MA(KB) \text{ and } \mathcal{M} \not\models \mathbf{K}C(x)\} \\
& \cup \{\mathbf{K}R(x,y) \mid \mathbf{K}R(x,y) \in MA(KB) \text{ and } \mathcal{M} \not\models \mathbf{K}R(x,y)\}
\end{aligned}$$

We can show that the intended correspondence indeed holds.

Lemma 1. *Let KB be a safe $e\mathcal{ALCO}$ KB, $\mathcal{M}$ a set of interpretations over Δ that satisfies KB such that $\mathcal{M} \models \mathbf{K}R(i_1, i_2)$ only if $i_1 \approx a \in N_I$ and $i_2 \approx b \in N_I$, and (P, N) the partition of $MA(KB)$ induced by $\mathcal{M}$. Then (P, N) is consistent with KB.*

Note that here, the particular restriction on $\mathcal{M}$ is necessary, otherwise the property would not hold. Take $(\exists \mathbf{A}R.\mathbf{A}C)(a)$. Then, $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models R(a,i) \wedge C(i)$ for some $i \in \Delta$ and $i \not\approx a\}$ clearly satisfies the assertion, yet the induced partition with $P = \emptyset$ is not consistent with KB (because of (1) and the fact that that $KB[P][P]$ only considers modal atoms in $MA(KB)$). The same restriction is no longer necessary for MKNF models for which the following one-to-one correspondence between every MKNF model $\mathcal{M}$ of KB and the partition induced by $\mathcal{M}$ can be shown.

Theorem 1. *A set $\mathcal{M}$ of interpretations over Δ is an MKNF model for a safe $e\mathcal{ALCO}$ KB KB iff the partition (P, N) of $MA(KB)$ induced by $\mathcal{M}$ satisfies the following:*
(1) (P, N) is consistent with KB;
(2) $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models Ob_{KB,P,P}\}$; and
(3) for each partition (P', N') of $MA(KB)$ such that $P' \subset P$, at least one of the following conditions does not hold:
(a) the $\mathcal{ALCO}$ KB $Ob_{KB,P',P}$ is satisfiable;
(b) $Ob_{KB,P',P} \not\models C(x)$ for each $\mathbf{K}C(x) \in N'$;
(c) $Ob_{KB,P',P} \not\models R(x,y)$ for each $\mathbf{K}R(x,y) \in N'$.

As an immediate consequence of this procedure, we can show that safe $e\mathcal{ALCOV}$ KBs are $\mathcal{ALCOV}$ representable.

Corollary 1. *Let KB be a safe $e\mathcal{ALCOV}$ KB, $\mathcal{M}$ an MKNF model of KB, and (P, N) be the partition of $MA_\Delta(KB)$ induced by $\mathcal{M}$. Then $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models Ob_{KB,P,P}\}$.*

4 Coverage within the Decidable Fragment

The established result in the previous section is certainly interesting in its own right, since, arguably, the imposed restrictions and the applied construction is considerably less complicated in terms of notation than the one applied in [8,20]. Anyway, the main outlined purpose of this revision is to ensure that the new decidable fragment encompasses all the formalisms for which coverage was shown in [20] only for the full language. In this section, we revisit this material and discuss relevant changes.

4.1 Monotonic Approaches

Naturally, our decidable language fragment of safe $e\mathcal{ALCOV}$ KBs covers $\mathcal{ALCOV}$ (with and without U) and all its sub-languages. This does not include $\mathcal{SROIQ}$, i.e., OWL 2 DL and its tractable profiles, but a trivial adjustment following the ideas in [20] where modal operators are limited to $\mathcal{ALCOV}$ concepts is easily conceivable.

Coverage of RIF-CORE [3], i.e., n-ary Datalog, interpreted as DL-safe Rules [31] carries over from [20] or alternatively from [25]. In fact, the latter does not even require the usage of the universal role U which is just fine if we only want to embed a Datalog program. However, if we want to cover an embedding of n-ary Datalog interacting with DLs, then the former is required: consider the Datalog rule $C(a) \rightarrow D(a)$ and a concept assertion $C(a)$. The rule can be translated to $\exists atom.(C \sqcap \{a\}) \sqsubseteq \exists atom.(D \sqcap \{a\})$ (slightly adjusted from [25]), but there is $\mathcal{I}$ such that $atom^{\mathcal{I}} = \emptyset$, i.e., $D(a)$ is no longer derivable. Thus, based on the former Datalog embedding, coverage of DL-safe SWRL [31], $\mathcal{AL}$-log [7], and CARIN [26] carries over, i.e., without much surprise all monotonic approaches as outlined in [20] are covered.

4.2 $\mathcal{ALCK}_{\mathcal{NF}}$

In [8], it is shown how several non-monotonic reasoning features (defaults, integrity constraints, and role and concept closure) can be modeled in the full language $\mathcal{ALCK}_{\mathcal{NF}}$ and it is argued that the restriction to simple KBs applied to achieve decidability does not impede coverage. The full $e\mathcal{ALCOV}$ language obviously includes $\mathcal{ALCK}_{\mathcal{NF}}$ by design, but since we have changed the restrictions to achieve decidability, these results do not carry over automatically, so we briefly discuss coverage of these features for safe $e\mathcal{ALCOV}$ KBs (and refer the reader for the detailed discussion to [8]).

First, closed $\mathcal{DL}$-defaults [2] of the form

$$d = \frac{\alpha : \beta_1, \ldots, \beta_n}{\gamma}$$

are covered in [8], where α, β_i, and γ are $\mathcal{DL}$ concepts and $n \geq 0$. Closed defaults are limited in their applicability to individuals explicitly mentioned in the knowledge base. This is achieved in [8] by using a new atomic concept I in each translation of a default and adding the assertions $I(a)$ for each a appearing in the knowledge base. Conceptually, this matches the idea of nominal schemas, so the translation of closed defaults can be presented as a safe $e\mathcal{ALCOV}$ axiom

$$\tau_{DK}(d) = \mathbf{K}\alpha \sqcap \neg\mathbf{A}\neg\beta_1 \sqcap \cdots \sqcap \neg\mathbf{A}\neg\beta_n \sqcap \{x\} \sqsubseteq \mathbf{K}\gamma \sqcap \{x\}$$

without the need to introduce new concepts or adding additional assertions, and it is easy to see that Theorem 3.1 from [8] can be adapted accordingly.

Theorem 2. *Let $\langle \Sigma, \mathcal{D} \rangle$ be an $\mathcal{ALC}$ KB with defaults, where Σ is an $\mathcal{ALC}$ KB and $\mathcal{D}$ is a set of $\mathcal{ALC}$-defaults. The $e\mathcal{ALCOV}$ KB $\tau(\Sigma, \mathcal{D})$ is such that, for every $\mathcal{ALC}$-concept C and every individual $a \in N_I$, it holds $\langle \Sigma, \mathcal{D} \rangle \models C(a)$ iff $\tau_{DK}(\Sigma, \mathcal{D}) \models C(a)$.*

Secondly, integrity constraints (ICs) are considered, and it is argued that ICs commonly apply to individuals explicitly mentioned in the considered KB and impose restrictions without changing the content of the KB. This is in line with our restrictions on safe $e\mathcal{ALCOV}$ KBs and it can be verified that all examples discussed in [8] can be made safe explicitly by introducing guards $\{x\}$ as for defaults. Finally, similar observations hold for the considerations on role and concept closure, i.e., all modeling features presented in [8] can indeed be adjusted to safe $e\mathcal{ALCOV}$ KBs without much effort.

4.3 Hybrid MKNF

Hybrid MKNF as a combination of DLs with non-monotonic rules is based on MKNF logics as well, but of different expressivity due to the different restrictions applied to the full MKNF language in each of the two approaches [30]. In [20], an embedding of hybrid MKNF into epistemic DLs is presented (we refer to that paper for the technical details). Though not the full language of hybrid MKNF is embedded, the presented fragment suffices to cover Answer Set Programming [11], i.e., disjunctive Datalog with classical negation and non-monotonic negation under the answer set semantics. Unfortunately, the presented embedding is in general not covered within the decidable fragment in [20] as shown with the simple example $\top \sqsubseteq \exists U.(\{a\} \sqcap C)$ and $\mathbf{K}D(a) \leftarrow \mathbf{K}C(a)$ as the latter would be embedded as $\mathbf{K}(\exists U.(\{a\} \sqcap C) \sqsubseteq \mathbf{K}(\exists U.(\{a\} \sqcap C)$ which is not simple [20]. This can be remedied with safe $e\mathcal{ALCOV}$ KB by changing the translation of MKNF rules $\mathsf{dl}(\mathbf{K}H_1 \vee \mathbf{K}H_l \leftarrow \mathbf{K}A_1, \ldots, \mathbf{K}A_n, \mathbf{not}B_1, \ldots, \mathbf{not}B_m)$ in [20] to

$$\mathbf{K}\mathsf{dl}(A_1) \sqcap \ldots \sqcap \mathbf{K}\mathsf{dl}(A_n) \sqcap \neg\mathbf{A}\mathsf{dl}(B_1) \sqcap \ldots \sqcap \neg\mathbf{A}\mathsf{dl}(B_m) \sqcap \{i\} \sqsubseteq \mathbf{K}\mathsf{dl}(H_1) \sqcup \ldots \sqcup \mathbf{K}\mathsf{dl}(H_l) \sqcap \{i\}$$

where i is a fresh individual and dl the translation function on (possibly classically negated) atoms defined in [20]. Essentially, in the original embedding, such translated concepts in GCIs would due to the universal role either be interpreted as Δ or $\emptyset$. Here, we introduce a nominal as guard that acts as a surrogate for all elements in Δ, thus reducing such interpretation to either i or $\emptyset$. An adaptation of the results in [20] (Lemma 3 and Theorem 4) are then straightforward.

Theorem 3. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF KB. $\mathcal{M}$ is an MKNF model of $\mathcal{K}$ iff $\mathcal{M}_1 = \{\mathcal{J} \mid \mathcal{J} \in fam(\mathcal{I})$ with $\mathcal{I} \in \mathcal{M}\}$ is a hybrid MKNF model of $\mathsf{dl}(\mathcal{K})$.*

This ensures that safe $e\mathcal{ALCOV}$ KBs in fact embed the restricted version hybrid MKNF and therefore also ASP.

5 Conclusions

We have studied epistemic extensions of DLs focusing here on $e\mathcal{ALCOV}$, i.e., $\mathcal{ALC}$ extended with nominals, nominals schemas, the universal role, and two epistemic operators for modeling non-monotonic reasoning. We have shown that this language encompasses all non-monotonic modeling features and approaches discussed in [20], and that an extension to a few missing monotonic languages (e.g., $\mathcal{SROIQ}$) is easily conceivable. We have introduced a set of restrictions on the general language which is different from that in [20], and we have shown that, under these restrictions, reasoning, i.e., checking MKNF-satisfiability becomes decidable, and, unlike in previous work, the restricted language still covers all the discussed modeling features.

An immediate matter for follow-up work is the computational complexity when reasoning with epistemic DLs, a question that has only received limited attention so far (in [8] a triple exponential space upper bound for reasoning with simple KBs has been pointed out, while no results are mentioned in [20]). It is clear that the complexity results established for reasoning with nominal schemas (without epistemic operators) [25] can serve as first necessary lower bounds, i.e., for $e\mathcal{ALCOV}$ in particular a minimal lower bound is established by the fact that reasoning in $\mathcal{ALCOV}$ is 2ExpTime-complete. This does neither account for the universal role nor the epistemic operators. Since $\mathcal{ALC}$ with arbitrary Boolean role constructors is NExpTime-complete [28,36] (as the restriction to safe Boolean role constructors in [36] does not suffice to cover U), and the decision procedure for MKNF-satisfiability requires nondeterministically guessing the right partition, by combining the different sources of complexity we conjecture a lower bound of at least N2ExpTime.

Another interesting topic for future work is to establish coverage for further related formalisms, for example by extending the expressiveness of rules permitted in the embedding of Hybrid MKNF, which then allows us to include already established embedding results for, e.g., [9,32] in [30] or by considering among others work on circumscription in DLs [4,33]. Building on the existing relation between epistemic extensions of DLs and Hybrid MKNF, we can also investigate the relation to parameterized logic programs [12,13], or multi-context systems [5] using the established connection between these and Hybrid MKNF [21]. Finally, an implementation may be considered given a) the more simple decision procedure proposed here and b) the recent work on implementing nominal schemas [22,37,6,34] and Hybrid MKNF [1,16]. In particular the encouraging results for Konclude [34,35] seem to indicate that this may in fact be achievable in a feasible manner.

Acknowledgments This work was partially supported by Fundação para a Ciência e a Tecnologia under project "ERRO – Efficient Reasoning with Rules and Ontologies" (PTDC/EIA-CCO/121823/2010) and strategic project PEst/UID/CEC/04516/2013, and by FCT grant SFRH/BPD/86970/2012.

References

1. Alferes, J.J., Knorr, M., Swift, T.: Query-driven procedures for hybrid MKNF knowledge bases. ACM Trans. Comput. Log. 14(2), 16 (2013)
2. Baader, F., Hollunder, B.: Embedding defaults into terminological representation systems. Journal of Automated Reasoning 14, 149–180 (1995)
3. Boley, H., Hallmark, G., Kifer, M., Paschke, A., Polleres, A., Reynolds, D. (eds.): RIF Core Dialect. W3C Recommendation 22 June 2010 (2010), available from http://www.w3.org/TR/rif-core/
4. Bonatti, P.A., Lutz, C., Wolter, F.: The complexity of circumscription in dls. J. Artif. Intell. Res. (JAIR) 35, 717–773 (2009)
5. Brewka, G., Eiter, T.: Equilibria in heterogeneous nonmonotonic multi-context systems. In: Proceedings of the Twenty-Second AAAI Conference on Artificial Intelligence. pp. 385–390. AAAI Press (2007)
6. Carral, D., Wang, C., Hitzler, P.: Towards an efficient algorithm to reason over description logics extended with nominal schemas. In: Faber, W., Lembo, D. (eds.) Web Reasoning and Rule Systems - 7th International Conference, RR 2013. Proceedings. LNCS, vol. 7994, pp. 65–79. Springer (2013)
7. Donini, F.M., Lenzerini, M., Nardi, D., Schaerf, A.: AL-log: Integrating datalog and description logics. Journal of Intelligent Information Systems 10(3), 227–252 (1998)
8. Donini, F.M., Nardi, D., Rosati, R.: Description logics of minimal knowledge and negation as failure. ACM Transactions on Computational Logic 3(2), 177–225 (2002)
9. Eiter, T., Ianni, G., Lukasiewicz, T., Schindlauer, R., Tompits, H.: Combining answer set programming with description logics for the Semantic Web. Artificial Intelligence 172(12–13), 1495–1539 (2008)
10. Fitting, M.: First-order logic and automated theorem proving. Springer, 2nd edn. (1996)
11. Gelfond, M., Lifschitz, V.: Classical negation in logic programs and disjunctive databases. New Generation Computing 9, 365–385 (1991)
12. Gonçalves, R., Alferes, J.J.: Parametrized logic programming. In: Janhunen, T., Niemelä, I. (eds.) Logics in Artificial Intelligence - 12th European Conference, JELIA 2010. Proceedings. LNCS, vol. 6341, pp. 182–194. Springer (2010)
13. Gonçalves, R., Alferes, J.J.: Decidability and implementation of parametrized logic programs. In: Cabalar, P., Son, T.C. (eds.) Logic Programming and Nonmonotonic Reasoning, 12th International Conference, LPNMR 2013. Proceedings. LNCS, vol. 8148, pp. 361–373. Springer (2013)
14. Grimm, S., Hitzler, P.: Semantic Matchmaking of Web Resources with Local Closed-World Reasoning. International Journal of Electronic Commerce 12(2), 89–126 (2008)
15. Hitzler, P., Krötzsch, M., Parsia, B., Patel-Schneider, P.F., Rudolph, S. (eds.): OWL 2 Web Ontology Language: Primer. W3C Recommendation 27 October 2009 (2009), available from http://www.w3.org/TR/owl2-primer/
16. Ivanov, V., Knorr, M., Leite, J.: A query tool for $\mathcal{EL}$ with non-monotonic rules. In: Alani, H., Kagal, L., Fokoue, A., Groth, P.T., Biemann, C., Parreira, J.X., Aroyo, L., Noy, N.F., Welty, C., Janowicz, K. (eds.) The Semantic Web - ISWC 2013 - 12th International Semantic Web Conference, Proceedings, Part I. LNCS, vol. 8218, pp. 216–231. Springer (2013)
17. Ke, P.: Nonmonotonic Reasoning with Description Logics. Ph.D. thesis, University of Manchester (2011)
18. Kifer, M., Boley, H. (eds.): RIF Overview. W3C Working Group Note 22 June 2010 (2010), available from http://www.w3.org/TR/rif-overview/
19. Knorr, M., Alferes, J.J., Hitzler, P.: Local closed world reasoning with description logics under the well-founded semantics. Artificial Intelligence 175(9–10), 1528–1554 (2011)

20. Knorr, M., Hitzler, P., Maier, F.: Reconciling OWL and non-monotonic rules for the semantic web. In: Raedt, L.D., Bessière, C., Dubois, D., Doherty, P., Frasconi, P., Heintz, F., Lucas, P.J.F. (eds.) ECAI 2012 - 20th European Conference on Artificial Intelligence. Proceedings. Frontiers in Artificial Intelligence and Applications, vol. 242, pp. 474–479. IOS Press (2012)
21. Knorr, M., Slota, M., Leite, J., Homola, M.: What if no hybrid reasoner is available? hybrid MKNF in multi-context systems. J. Log. Comput. 24(6), 1279–1311 (2014)
22. Krisnadhi, A., Hitzler, P.: A tableau algorithm for description logics with nominal schema. In: Web Reasoning and Rule Systems - 6th International Conference, RR 2012. Proceedings. LNCS, vol. 7497, pp. 234–237. Springer (2012)
23. Krötzsch, M.: Description Logic Rules, Studies on the Semantic Web, vol. 008. IOS Press/AKA (2010)
24. Krötzsch, M., Maier, F., Krisnadhi, A.A., Hitzler, P.: A better uncle for OWL: Nominal schemas for integrating rules and ontologies. In: Srinivasan, S., Ramamritham, K., Kumar, A., Ravindra, M.P., Bertino, E., Kumar, R. (eds.) Proceedings of the 20th International World Wide Web Conference, WWW2011. pp. 645–654. ACM (2011)
25. Krötzsch, M., Rudolph, S.: Nominal schemas in description logics: Complexities clarified. In: Baral, C., Giacomo, G.D., Eiter, T. (eds.) Principles of Knowledge Representation and Reasoning: Proceedings of the Fourteenth International Conference, KR 2014. AAAI Press (2014)
26. Levy, A.Y., Rousset, M.C.: Combining Horn rules and description logics in CARIN. Artificial Intelligence 104, 165–209 (1998)
27. Lifschitz, V.: Nonmonotonic databases and epistemic queries. In: Mylopoulos, J., Reiter, R. (eds.) Proceedings of the 12th International Joint Conferences on Artifical Intelligence, IJCAI'91. pp. 381–386. Morgan Kaufmann (1991)
28. Lutz, C., Sattler, U.: The complexity of reasoning with Boolean modal logics. In: Wolter, F., Wansing, H., de Rijke, M., Zakharyaschev, M. (eds.) Proc. of the 3rd Int. Conf. on Advances in Modal Logic (AiML 2000). pp. 329–348. World Scientific (2002)
29. Mehdi, A., Rudolph, S.: Revisiting semantics for epistemic extensions of description logics. In: Burgard, W., Roth, D. (eds.) Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2011. AAAI Press (2011)
30. Motik, B., Rosati, R.: Reconciling Description Logics and Rules. Journal of the ACM 57(5), 93–154 (2010)
31. Motik, B., Sattler, U., Studer, R.: Query answering for OWL-DL with rules. Journal of Web Semantics 3(1), 41–60 (2005)
32. Rosati, R.: DL+Log: A tight integration of description logics and disjunctive datalog. In: Doherty, P., Mylopoulos, J., Welty, C.A. (eds.) 10th International Conference on the Principles of Knowledge Representation and Reasoning, (KR'06), Proceedings. pp. 68–78. AAAI Press (2006)
33. Sengupta, K., Krisnadhi, A.A., Hitzler, P.: Local closed world semantics: Grounded circumscription for OWL. In: Aroyo, L., Welty, C., Alani, H., Taylor, J., Bernstein, A., Kagal, L., Noy, N.F., Blomqvist, E. (eds.) The Semantic Web - ISWC 2011 - 10th International Semantic Web Conference, Proceedings. LNCS, vol. 7031, pp. 617–632. Springer (2011)
34. Steigmiller, A., Glimm, B., Liebig, T.: Reasoning with nominal schemas through absorption. J. Autom. Reasoning 53(4), 351–405 (2014)
35. Steigmiller, A., Liebig, T., Glimm, B.: Konclude: System description. J. Web Sem. 27, 78–85 (2014)
36. Tobies, S.: Complexity results and practical algorithms for logics in knowledge representation. Ph.D. thesis, LuFG Theoretical Computer Science, RWTH-Aachen, Germany (2001)
37. Wang, C., Hitzler, P.: A resolution procedure for description logics with nominal schemas. In: Takeda, H., Qu, Y., Mizoguchi, R., Kitamura, Y. (eds.) Second Joint International Conference, JIST 2012. Proceedings. LNCS, vol. 7774, pp. 1–16. Springer (2012)