Uncertain Conditionals and Counterfactuals in (Non-)Causal Settings

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Abstract

Conditionals are basic for human reasoning. In our paper, we present two experiments, which for the first time systematically compare how people reason about indicative conditionals (Experiment 1) and counterfactual conditionals (Experiment 2) in causal and non-causal task settings ($N = 80$). The main result of both experiments is that conditional probability is the dominant response pattern and thus a key ingredient for modeling causal, indicative, and counterfactual conditionals. In the paper, we will give an overview of the main experimental results and discuss their relevance for understanding how people reason about conditionals.

Keywords: Causality; Conditionals; Conditional Probability; Counterfactuals; Reasoning; Uncertainty

Introduction

Classical logic used to be the dominating rationality framework for psychological reasoning research in the 20th century. To deal with the defeasibility and uncertainty of everyday life inference, probabilistic rationality norms have gained popularity (e.g., Baratgin, Over, & Politzer, 2014; Elqayam & Over, 2012; Evans & Over, 2004; Oaksford & Chater, 2009; Pfeifer, 2013; Pfeifer & Douven, 2014). This development has influenced how the quality of human inference has been evaluated. Correspondence between human inference about indicative conditionals and the semantics of the conditional event,[1] for example, is nowadays regarded by most psychologists of reasoning as *rational*, whereas the semantics of the material conditional[2] was regarded as the normative gold standard in the last century. For this reason, the majority of human responses in truth table tasks was labeled *defective truth table*, whereas it is broadly regarded as rational today, since this response is not defective. Rather, it corresponds to the *de Finetti table* (i.e., the truth table of the conditional event).

Conditionals and reasoning about conditionals are basic for human reasoning. Among other things, conditionals can not only express abstract relationships but allow also for representing causal information: If some cause (e.g., *taking aspirin*) is present, then an effect occurs (*alleviates headache*). Such causal conditionals are closely related with counterfactuals. When people think about whether *taking aspirin* and *headache* are causally related, they ask whether the corresponding counterfactual "If aspirin were taken, headache would be alleviated" holds. Thus, understanding how people reason about causal and counterfactual conditionals is crucial for understanding causal cognition. Compared to the vast psychological literature on indicative conditionals (for an overview see, e.g., Evans & Over, 2004), studies on adult reasoning about counterfactuals are surprisingly rare. Within the probabilistic truth table task paradigm, counterfactuals were investigated only by Over et al. (2007). In our paper, we present two experiments, which for the first time systematically compare how adults reason about indicative conditionals (Experiment 1) and counterfactual conditionals (Experiment 2) in causal and non-causal probabilistic truth table task settings.

Both experiments are designed to investigate the following key questions: Are there any differences in the probabilistic interpretations of conditionals, comparing indicative and counterfactual conditionals in causal and non-causal settings? How do people draw inferences from argument forms involving counterfactuals?

Experiment 1: Indicative Conditionals

Participants Forty students of Protestant Theology at Augustana-Hochschule Neuendettelsau (Germany) were assigned equally to a non-causal and a causal conditional task set. Participants were payed 10€.

Task Materials The materials were adapted from the probabilistic truth table tasks used in Fugard, Pfeifer, Mayerhofer, and Kleiter (2011). Materials were presented in two pen and paper task sets. In each task, a short cover story introduced the domain of the task. For the non-causal conditions, we used pictures of six-sided dice with black or white geometric figures. The target sentences had the form "If the side shows *white*, then the side shows a *triangle*." For the causal

[1] The conditional event $C|A$ ("C given A") is *true* if $A \wedge C$ ("A and C") is true, *false* if $A \wedge \neg C$ ("A and not-C") is true, and *void* if $\neg A$ is true.

[2] The material conditional $A \supset C$ ("A implies C", i.e., "not A or C") is *false* if $A \wedge \neg C$ is true, but *true* otherwise.

condition, we used stylized pictures of six medical data sheets detailing the (purely fictional) name of a drug and the medication's effect on a patient's symptoms. Target sentences had the form "If a patient takes *Ambutal*, then the symptoms *diminish*." Participants were asked how sure they could be that the target sentence holds. They responded by ticking boxes in a "x out of y" format. Also, participants gave a rating for their confidence in the correctness of their response for each task, which we gathered to check for possible changes in confidence accompanying shifts of interpretation of the conditional. The target sentence was formulated in the indicative "If A, then C"-form for the first 19 tasks. Task 20 and 21 formulated a disjunction of the negated antecedent ($\neg A$) and consequent (C) of a corresponding (and logically equivalent) material conditional ($A \supset C$).

Procedure Each participant was tested individually. After the pen and paper tasks, we collected qualitative data on how they interpreted the conditionals and the respective role of cause and effect by a structured interview.

Results After performing Holm-Bonferroni corrections for multiple significance tests, the probability response patterns of the first 19 tasks did not differ significantly between both groups. Participants in both groups predominantly chose the Conditional Event interpretation (see Figure 1 for details). The probability responses according to the three main interpretations of the conditional for tasks 1-19 (the tasks with "If A, then C" target sentences) were distributed as follows: In the non-causal group ($n_1 = 20$), out of 380 responses, 81% were Conditional Event responses, 15% were Conjunction responses, 1% were Material Conditional responses, and 4% were "other" responses. In the causal group ($n_2 = 20$), 95% were Conditional Event responses, 1% were Conjunction responses, 1% were Material Conditional responses, and 3% were other responses. Across both groups, 35 participants responded by the Conditional Event in at least 78% of the tasks.

We observed statistically significant differences between the non-causal and the causal group with regard to the probability responses for the pooled data from tasks 20 and 21 (the tasks with disjunctions as target sentences), as determined by Fisher's Exact test ($p = .04$). In the non-causal group, 15% of responses were consistent with the Conditional Event response, 48% of responses were consistent with the Material Conditional responses, and 38% were other responses. In the causal group, 38% of responses were consistent with the Conditional Event response, 25% of responses were consistent with the Material Conditional responses, and 38% were other responses.

In total, eight participants shifted their interpretation

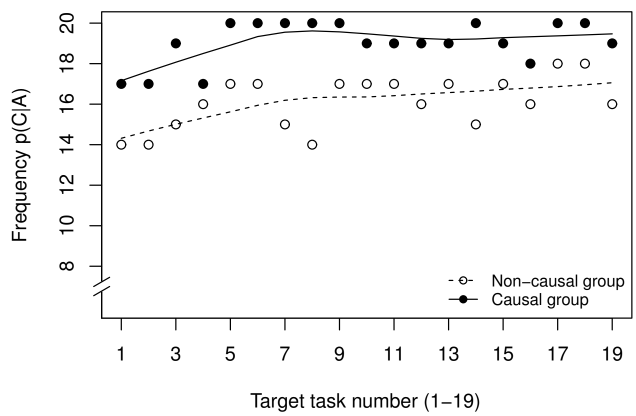

Figure 1: Frequency of participants per task giving the Conditional Event response in the non-causal ($n_1 = 20$) and the causal group ($n_2 = 20$), Experiment 1. The dashed/solid lines were generated using the locally weighted scatter plot smoother method (`lowess`, implemented in `R`).

to the Conditional Event within the first 19 tasks and 38% of participants reported higher confidence values within the three tasks after the shift.

In the structured interview at the end of the experiment, participants' responses confirmed the results reported above. Thirty-six participants explained their solution by appeal to features of reasoning with the Conditional Event interpretation, such as only counting the objects mentioned in the antecedent of the target sentence and then using this as the relevant set from which to count the objects that fit the consequent. When participants were asked to construct a consistent premise set based on a given degree of belief in a conclusion, 27 participants gave a set that corresponds unequivocally to the Conditional Event interpretation. Eleven participants produced sets that could fit either the Conjunction or the Conditional Event interpretation. Only one participant gave a set that corresponds unequivocally to the Conjunction interpretation.

75% of participants in the causal group judged the "symptoms diminish" target sentence to be an example of a relation of cause and effect, compared to 50% of participants for the "no influence" target sentence. By comparison, only 30% of participants in the non-causal group judged the "dice" target sentences to be examples of a relation of cause and effect. This validates the assumption that the medical task material triggered primarily causal reasoning whereas the dice task material did not do so.

Discussion The findings clearly show that the Conditional Event interpretation was the dominant response across both groups. Furthermore, participants in the causal group more frequently mentioned "cause" and "effect" in the interview, while the non-causal group did not do so: This can be interpreted as an indicator for causal reasoning in the causal group.

Experiment 2: Counterfactual Conditionals

Participants Forty students of Protestant Theology at Augustana-Hochschule Neuendettelsau (Germany) were assigned equally to a non-causal and a causal conditional task set. Participants were paid 15€ for their time. We ensured that no participant of Experiment 1 took part in Experiment 2.

Task Materials We used the same materials as in Experiment 1, with the difference that the target conditionals were replaced by corresponding counterfactual conditionals, such as "If the patient were to take *Raverat*, then it would have *no influence* on the symptoms" ("*Wenn der Patient Raverat nehmen würde, dann hätte es keinen Einfluss auf die Symptome*"). To clearly mark the target sentences as counterfactual, we added information about a factual case to each task's cover story. The factual cases diverged from the content of the antecedent of the target sentence, e.g. the factual case would state that the side of the die that faces up shows a triangle, and the antecedent would state "If the side were to show a circle."

In addition, we investigated ten tasks involving uncertain argument forms, which—to our knowledge—have not been investigated experimentally with counterfactual conditionals yet. We designed the tasks to investigate inference schemes which are valid/invalid in standard systems of counterfactuals (e.g., Lewis, 1973). The cover story involved the production of toy building blocks in different shapes, colours, and materials. In the Modus Tollens case, an inspector just got a closed box with a toy block in it (i.e., the factual case) and now considers two beliefs (i.e., the premises). She is quite sure that: (A) If the toy block were green, then the toy block would be a cylinder, and she is quite sure that (B) the toy block is not a cylinder. Participants are then asked to judge how sure she can be, based on these two sentences, that the conclusion, (C) the toy block is not green, holds. Participants could respond by either judging that she cannot or that she can conclude (C) based on (A) and (B) (i.e., is the argument probabilistically non-informative or is it informative?). In the latter case, participants additionally gave a response regarding whether she can be quite sure that the sentence (C) holds or whether (C) doesn't hold (i.e., is the degree of belief in the conclusion high or low?).

Procedure The procedure was identical to Experiment 1, except for the addition of argument form tasks, which we handed out as a final pen and paper task booklet. Table 1 lists the investigated argument forms. We also added two questions to the structured interview, to get an insight into the reasoning process during the uncertain argument form tasks.

Results As in Experiment 1, participants in both groups predominantly chose the Conditional Event interpretation (see Figure 2 for details). Also the probability response patterns of the first 19 tasks did not differ significantly between both groups. The probability responses according to the three main interpretations of the conditional for tasks 1-19 (the tasks with counterfactuals as target sentences) were distributed as follows: In the non-causal group ($n_3 = 20$), out of 380 responses, 77% were Conditional Event responses, 13% were Conjunction responses, 1% were Material Conditional responses, and 9% were other responses. In the causal group ($n_4 = 20$), 84% were Conditional Event responses, 8% were Conjunction responses, 2% were Material Conditional responses, and 6% were other responses. Across both groups ($n_3 + n_4 = 40$), 30 participants gave the Conditional Event response for more than 78% of the tasks.

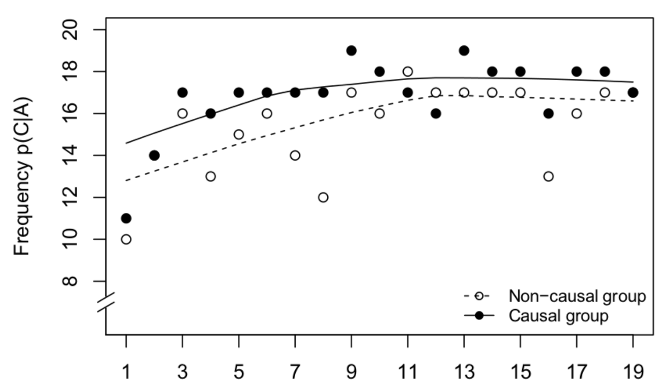

Figure 2: Frequency of participants per task giving the Conditional Event response in the non-causal ($n_3 = 20$) and the causal group ($n_4 = 20$), Experiment 2.

The differences between the non-causal and the causal group with regard to the probability responses for tasks 20 and 21 (the tasks with "not-*A* or *C*" as target sentences) approach significance when the data for task 20 and 21 is pooled for each group (Fisher's Exact test: $p = .07$). In the non-causal group, 13% of responses were consistent with the Conditional Event response, 35% of responses were consistent with the Material Conditional responses, 10% were consistent with the Conjunction response, and 43% were other responses. In the causal group, 35% of responses were consistent with the Conditional Event response, 18% of responses were consistent with the Material Conditional responses, 5% were consistent with the Conjunction response, and 43% were other responses.

The number of shifts of interpretation was similar to Experiment 1. Within the first 19 tasks, 13 participants shifted towards the Conditional Event interpretation and 38% reported higher confidence values within the three tasks after the shift.

In the interview at the end of the experiment, 30 participants explained their solution by appeal to a feature of the Conditional Event interpretation, such as restricting the set of relevant stimuli to those mentioned in the antecedent. Moreover, when participants were asked to construct a consistent premise set based on a given degree of belief in a counterfactual, 26 participants gave a set that corresponds unequivocally to the Conditional Event interpretation.

Like in Experiment 1, we observed that 80% of participants in the causal group judged the symptoms diminish target sentence to be an example of a relation of cause and effect, compared to 60% of participants for the no influence target sentence. By comparison, only 15% of participants in the non-causal group judged the dice target sentences to be examples of a relation of cause and effect.

Table 1: Results ($N = 40$) of the argument form tasks in percentages of responses. Predictions derived from the Conditional Event interpretation are printed in **bold**.

Name	*Argument form*	*T*	*F*	*V*
NMP	$A \Rightarrow B, A \therefore \neg B$	40	**15**	45
PS	$A \Rightarrow B \therefore A \wedge C \Rightarrow B$	50	5	**45**
CM	$A \Rightarrow B, A \Rightarrow C \therefore A \wedge B \Rightarrow C$	**78**	3	20
Cut	$A \Rightarrow B, A \wedge B \Rightarrow C \therefore A \Rightarrow C$	**48**	5	48
HS	$A \Rightarrow B, B \Rightarrow C \therefore A \Rightarrow C$	63	3	**35**
CP	$B \Rightarrow \neg A \therefore A \Rightarrow \neg B$	30	8	**63**
MT	$A \Rightarrow B, \neg B \therefore \neg A$	**55**	3	43
		T	*F*	*CT*
NR	$\neg(A \Rightarrow A)$	10	**78**	13
AT 1	$\neg(A \Rightarrow \neg A)$	**68**	23	10
AT 2	$\neg(\neg A \Rightarrow A)$	**70**	20	10

Note: $\Rightarrow$=counterfactual, $\therefore$=conclusion indicator ("therefore"), $\neg$=negation, $\wedge$=conjunction, *T*=true, *F*=false, *V*=void (i.e., non-informative conclusion probability), *CT*=can't tell, NMP= Negated Modus Ponens, PS=Premise Strengthening, CM=Cautious Monotonicity, HS=Hypothetical Syllogism, CP=Contraposition, MT=Modus Tollens, NR=Negated Reflexivity, AT=Aristotle's Thesis.

Since there are no statistically significant differences between the groups, we pooled the data for the uncertain argument form tasks. The majority of the responses to the uncertain argument forms involving counterfactuals is consistent with indicative versions of these argument forms observed in the literature (Pfeifer, 2012; Pfeifer & Kleiter, 2010)—see Table 1 for detailed results. An exception to this agreement is the large majority of participants who did not assign a low degree of belief in the conclusion of the Negated Modus Ponens (*NMP*). Also the frequency of *true* responses to Cut was lower than expected. While—under the conditional event interpretation—*NMP* is probabilistically informative (i.e., here, the coherent conclusion probability is low), Contraposition (*CP*), Hypothetical Syllogism (*HS*), and Premise Strengthening (*PS*) are probabilistically non-informative (any conclusion probability in the unit interval $[0, 1]$ is coherent; see Pfeifer and Kleiter (2006, 2009)). *CP*, *HS*, *NMP* and *PS* are also invalid in standard systems of counterfactual conditionals (e.g., Lewis, 1973). Pooling the *false* and *void* responses gives an indicator for the participant's evaluation of the validity of the respective argument form. With the exception of Cut, *HS*, and *PS*, the response patterns are also consistent with systems of counterfactual conditionals. Moreover, the clear majority of responses to Negated Reflexivity (*NR*) and both versions of Aristotle's Thesis are consistent with the Conditional Event interpretation (Pfeifer, 2012) and counterfactuals.

Discussion As in Experiment 1, the findings clearly show that the Conditional Event interpretation was the dominant response across both groups. Moreover, more participants in the causal group associated the antecedent and the consequent with cause and effect, respectively, than in the non-causal group. In the uncertain argument form tasks, the majority of responses were consistent with conditional probability and with counterfactuals.

General Discussion

Interpretations of the Conditional Our findings offer a negative reply to our first main question, whether there are any differences in the probabilistic interpretations of indicative and counterfactual conditionals in causal and non-causal settings: In all four conditions, the Conditional Event was the dominant response type.

One main difference between the results of Experiment 1 and Experiment 2 is that the counterfactual conditional tasks in the latter were arguably more difficult for the participants. Moreover, participants in Experiment 1 reported higher confidence in the correctness of their responses across the task set of tasks 1-19 (on a scale from -6 to +6, *M* = 4.04, *SD* = 2.06) than in Experiment 2 (*M* = 2.50, *SD* = 2.47). The reason for the higher difficulty of the counterfactual tasks could stem from the counterfactual conditionals themselves—the surface grammar is more complex than in indicative conditionals and this might be reflected in the reasoning process. Likewise, participants had to evaluate the relevance of the stated factual case (which contradicts the counterfactual antecedent). Across both task types in Experiment 2, 73% of participants (65% in the non-causal group and 80% in the causal group) commented upon the factual case during the experiment or in the interview. In their comments, 43% of participants judged the factual case to be irrelevant for solving the task (25% in the non-causal group, 18% in the causal group).

Our results vindicate the notion that *de Finetti tables* aren't *defective truth tables*, and they thus lend further credence to the main tenets of the New Paradigm Psychology of Reasoning (cf. Pfeifer, 2013). The mental models explanation, appealing to the "implicit" mental model of the conditional as the conjunction of antecedent and consequent or the "explicit" model of the conditional as the material conditional of classical logic (cf. Johnson-Laird & Byrne, 2002), were only used by a small part of all four groups.

Furthermore, the present study contributes to the study of shifts of interpretations of the conditional. However, the effect was weaker than reported in Fugard et al. (2011). Since there was no time pressure during the experiment, it is possible, albeit not verifiable with the data at hand, that some participants mentally shifted towards the Conditional Event while solving task 1, considering the Conjunction interpretation or another interpretation before choosing the Conditional Event response. This idea is supported by the fact that between 69% (Experiment 2) and 75% (Experiment 1) of shifts occurred before task 4, i.e. early on in the experiment.

Causal Conditionals Although our results are in accordance with Over et al. (2007), we observed higher conditional event response frequencies. This could be caused by differences in the experimental material. First, their tasks elicited probabilistic judgements regarding conditional sentences concerning possible states of affairs using background knowledge. Second, more crucially, Over et al. (2007) asked participants to assign probability ratings to the four truth table cases ($T \wedge T, T \wedge F, F \wedge T, F \wedge F$) and then compared these values to the conditional probabilities (the probability of the consequent given the antecedent) that participants had given in addition to the four truth table cases. As pointed out in Fugard et al. (2011), asking for conjunctions could elicit higher frequencies in conjunction responses.

So, while there are some methodological differences between the present study and Over et al. (2007), their results fit well with the results from our experiment: Reasoning with causal conditionals can be best explained by appeal to the probability of the causal conditionals as conditional probability. Our results regarding the similarities between reasoning with counterfactual and indicative conditionals furthermore support their hypothesis that "people [...] make similar probability judgments about [...] indicative and counterfactual conditionals, on the basis of similar psychological processes." (Over et al., 2007, p. 83) We submit that this is due to the central role of probabilistic reasoning for conditional reasoning in all of its modes that we have tested (counterfactual, indicative, causal).

Uncertain Argument Forms Our second main question was: How do people draw inferences from uncertain argument forms involving counterfactual conditionals? The data from the inference tasks suggests the following. As observed by Pfeifer (2012) in the context of indicative conditionals, most participants used the Conditional Event interpretation when reasoning with Aristotle's Thesis (*AT 1* and *AT2*) and Negated Reflexivity (*NR*). This new result for counterfactual conditionals further confirms the results from the probabilistic truth table tasks and the hypothesis that conditional probability is fundamental for reasoning with uncertain conditionals.

The other tasks furthermore provide additional information about inferences from conditionals in more complex cases. One main finding is that only few participants (3–8%) judge the—under the material conditional interpretation—deductively valid (even though, in several cases, probabilistically non-informative) argument forms to be invalid. The responses to the Negated Modus Ponens task are atypical in this regard, also because of the high percentage of "*true*" and "*void*" responses. By comparison, in Pfeifer and Kleiter (2007), the majority of participants gave coherent responses in the Modus Ponens tasks, including Modus Ponens with a negated conclusion. The unusual responses in the present study could be attributed to the task's position in the task set: It was the first task, and the task format was arguably unfamiliar to participants. Also, difficulties in processing negations are a well-known psychological phenomenon (see, e.g., Evans, 1982).

Even the responses that prima facie don't fit with the Conditional Event interpretation don't actually speak against it, but rather highlight a pertinent pragmatic issue in conditional reasoning. The high percentage of participants assigning a high degree of belief to the conclusion of the counterfactual Hypothetical Syllogism can be explained by appeal to the following conversational implicature: When stating $A \Rightarrow B$ as the first premise, one sets a frame of reference for the usage of B in the second premise $B \Rightarrow C$—such that $B \Rightarrow C$ actually means $A \wedge B \Rightarrow C$, as it is formalized in the Cut inference schema (see also Pfeifer & Kleiter, 2010). The slight dominance of the "classical" response in the Premise Strengthening inference can be interpreted analogously; participants might have assumed that the conjunction $A \wedge C$ wouldn't have been introduced without a relevant connection between A and C, such as $A \Rightarrow C$. This explanation also fits with the high percentage of Conditional Event responses for the Cautious Monotonicity (*CM*) task, which mirrors the results of Pfeifer and Kleiter (2010).

Furthermore, as Pfeifer and Kleiter (2010) argue, people's interpretation of Contraposition (*CP*) is an important indicator of how people interpret indicative conditionals. As in their study, we found that the majority (63%) of participants classified the counterfactual *CP* as probabilistically non-informative. Finally, the results for Modus Tollens (*MT*) fit well within the endorsement rates in non-probabilistic indicative versions of MT tasks (see, e.g., Evans, Newstead, & Byrne, 1993).

We conclude from this that the results of our present investigation into counterfactual conditional reasoning underline the importance of conditional probability not only for reasoning about indicative and causal conditionals but also for reasoning about counterfactual conditionals.

Concluding remarks

In both experiments and in all four experimental conditions, the Conditional Event interpretation is the dominant response type. This speaks for the ecological validity of the conditional probability hypothesis and indicates that conditional probabiltity is basic to indicative, counterfactual, and causal conditionals.

Finally, we note that probabilistic approaches where conditional probability ($p(C|A)$) is defined by the fraction of the joint ($p(A \wedge C)$) and the marginal probability ($p(A)$), cannot deal with zero-antecedent probabilities (i.e., $p(C|A)$ is undefined if $p(A) = 0$). However, as pointed out by Pfeifer (2013), zero-antecedent probabilities can be exploited for formalizing the factual falsehood of the antecedents of counterfactual conditionals. Although the coherence approach to probability requires that the antecedent is not logically contradictory, it allows for dealing with zero-antecedent probabilities (see, e.g., Coletti & Scozzafava, 2002; Gilio & Sanfilippo, 2013; Gilio, Pfeifer, & Sanfilippo, 2015). To exploit zero-antecedent probabilities for formalizing counterfactuals requires future research.

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