Shape in Mathematical Discourse, As Observed by an Artist Explorer

Katie MCCALLUM[a]
[a] *University of Brighton, UK*

Abstract. An analysis of the forms that appear in mathematical discourse and the structures through which mathematical spaces are shared. A presentation of work from a new research project centred around the blog infiltratemathematics.wordpress.com. Spatial and physical language allude to the conceptual spaces of mathematical thought, captured by notation with iconic and symbolic properties.

Keywords. Linguistics, Mathematical speech, Outreach

1. Introduction

The language developed to express mathematics has made it possible to talk about strange and complex subjects using hyper-refined symbolic expressions. It is very difficult to access mathematics without knowledge of the conventions that give its notations their meaning. As an artist with a sincere fascination with mathematics and the ontology of mathematical objects I am investigating ways for outsiders to gain access to some of the ideas and culture of the discipline, and embodied and extended mind theories suggest that observable, physical aspects of mathematics can be understood as integral to the discipline.

I have been carrying out observations of the ways in which experts communicate through writing, drawing, speech and gesture, in order to learn about mathematics from the outside. As Latour and Woolgar aimed to do in their study of scientific culture(1), I am maintaining an outsider's position to remain free to consider what I see in a wide cultural context rather than interpreting it completely according to the conventions of the discipline, whilst also examining the limitations on my engagement. I have been attending mathematics conferences, writing and publishing observations and responses in my sculptural practice on the public blog infiltratemathematics.wordpress.com.

Mathematics is given a physical form in various aspects of a presentation, and careful analysis of the gestures and metaphorical language used by experts to guide one another through mathematical spaces can help the outsider to learn something about the shape of those spaces. Here I present some observations and responses from a lecture given by André Neves on Min-max theory and its applications(2), and a sketch for a research project in its infancy that investigates some of the forms and structures present in mathematical discourse both through written analysis and sculptural practice.

2. The Metaphor: Selection and Extraction

Lakoff and Nuñez' *Where Mathematics Comes From* describes mathematics as arising from cognitive metaphorical mappings that extend the structures of bodily experience into conceptual domains(3). Their work has drawn criticism from various angles but has opened up a debate around the bearing that embodied and extended mind theories can have on mathematics. These ideas present a framework from which to approach learning about mathematics through observation and sculptural practice, understanding physicality not just as auxiliary to the 'real' doing of mathematics, but as an essential part.

To an outsider, some of the gestures used in a mathematics presentation seem very surprising; theoretical, high-dimensional 'objects' are described with movements that trace out a physical shape or manipulate them in space.

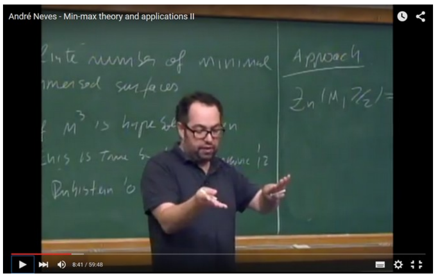

Figure 1. Flipping gesture

"*...a hypersurface or the same hypersurface with different orientation, they are the same object.*"(2)

As Neves speaks this sentence, he accompanies it with a flipping of the hand from palm up to down and back. This suggests flipping around a particular axis, the hand either becoming a representation of the hypersurface or supposed to be manipulating it in space, drawing out the interesting assertion that "they are the same object", rather than two similar descriptions of theoretical surfaces. This description of the two hypersurfaces as *being the same object* is not reflected in the formal notation of this piece of mathematics; rather it gives descriptions for two manifolds which have certain characteristics in common but others that differ. As with much of Lakoff's Cognitive Linguistics, Lakoff and Núñez' model rests on cognitive metaphorical mappings that set up a statement that connects two domains; in this instance the cognitive metaphor might be something like HYPERSURFACES ARE PHYSICAL OBJECTS IN SPACE. What is missing, though, is an account of how certain characteristics are chosen to be mapped and others are not; we might understand the hypersurfaces as maintaining aspects of their form like objects when flipped, but we aren't given the impression that it would be possible to pick one up and put it in one's pocket.

Attempts have been made to reconcile Sperber and Wilson's Relevance Theory(4) with some of Cognitive Linguistics' model of metaphor(5), and have had some success. Relevance Theory rests on a communicative principle of relevance, which suggests that by making an utterance or 'ostensive act', a speaker is conveying that what they are saying will provide cognitive effects that will justify the effort needed to process them; ostensive acts are therefore assumed to have been chosen for optimal relevance, and metaphors are understood by “using linguistic and contextual clues to create new 'ad-hoc' (occasion-specific) concepts”(6) using the features that give the greatest cognitive benefit for the least mental work. The most relevant features of the gesture are picked out – the fact that the hand used is a part of a person, for example, is not of interest, nor the blood pumping through it, nor the size of the movement that it makes. What is important is that the gesture involves a thing being in one orientation and then another, but maintaining a set of characteristics, be they fingers or singularities. The gesture then can be understood as altering the audience's cognitive environment, using motion to spell out a sameness that suggests the kind of relationship that can be seen between the manifolds.

It is useful to think about sculpture as an ostensive act made with physical material. Which aspects of a physical form or a material promise the greatest cognitive effect in return for attention might depend on the way they are shaped, how they are combined, or how they have been experienced previously in the wider world. Taking an approach to the physical communication of mathematics that focuses on these processes of selection presents one framework through which mathematics and sculpture might relate.

3. The Concrete: Written Traces and Notation

Among the many alternatives to Lakoff and Núñez' book, De Freitas and Sinclair's 2014 book *Mathematics and the Body*(6) offers an inclusive materialist view of mathematics pedagogy that rejects the mind-body duality still somewhat present in *Where Mathematics Comes From*, framing mathematics instead in terms of extended bodies that extend beyond the (very much physical) mind to include the body, chalk, blackboard, lecture hall, audience and institution in their view of the substance of mathematics. Presented mathematics is normally focused around an oral performance and a temporary physical mapping. The board and chalk lose their 3-dimensionality and become a surface, their value purely visual, but more than that they become a set of signposts that can be read. The board is divided up and elements laid out in structural relationship to each other, and the writing is the subject of animated discussion for a short while, but once the argument has moved on the chalk is rubbed off and left to settle on the floor. As such chalk has become a central element of my art practice, as a particular crystallisation of an ephemeral element of the physicality of mathematics.

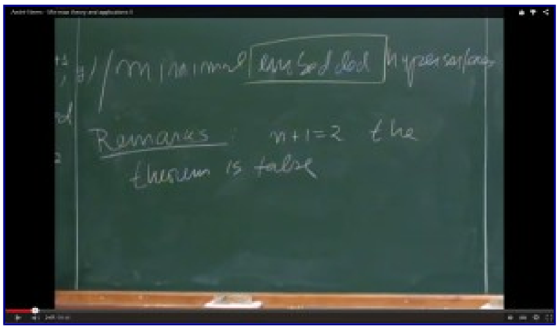

Figure 2. “Embedded” highlighted

“*...if n plus 2 equals 1, the theorem is false. And the reason is because of uh one word I forgot to write which is. Then there’s an infinite number of minim- oh yeah I wrote it. OK. Then the reason is because of this word. (draws a square around 'embedded')*”(2)

Neves focuses on the word 'embedded' on the blackboard, drawing a line around the area on which it is written as he asserts that *this* is the reason that the theorem is false. By highlighting a particular part of the written statement the speaker alters the audience's perception of the problem, drawing additional attention to an element that may not have been prominent in their initial reading. The location of that problem, though, is an interesting question. What is sure is that the problem is not a purely visual thing; it isn't something that the mathematician would normally carry around written down, to look at; the speaker doesn't copy the words or the shapes from a visual representation up on to the board, and the audience doesn't look to the shapes of the letters to provide the explanation.

Why, then, does Neves emphasise the writing on the board using a drawn shape, though the 'embedded' condition could just as well be emphasised using speech alone? The expectation must be that the speaker has chosen this as the optimal, most efficient way to highlight the appropriate idea. The problem was written at the top of the board, fixing initial conditions, their spatial position emphasising their priority. By returning to this writing, the speaker emphasises that the reason why the theorem in this particular case is false is something that requires nothing more than the conditions set out in the original problem to deduce; the audience doesn't have far to go. Writing is also static and visible. It is present on the board while the speaker is writing something else, and it can be referred back to in a concrete way. For people unfamiliar with the problem this property allows them to 'see' and have easy access to the elements of the problem in a way that perhaps the speaker can have without reference to written forms after a long period of familiarity. Material aspects of mathematics offer certain affordances that are invaluable to its communication.

4. The Narrative: Constructions and Ghosts

“*...we want to make sure that we capture these k-projective planes...”*(2)

Brian Rotman has written brilliantly about the different entities invoked in a mathematical paper: the person reading it is echoed in a Subject implied by the language of direction who must surely carry these commands out, and a yet more abstracted Agent is supposed to execute the more unrealistic commands, such as taking the sum of an infinite series(6). The act of presenting a paper sees a mathematician go beyond this to place themselves in structural relationship to other groups.

The story of the original discovery by the mathematician, often full of happenstance, collaboration and mistakes, is transformed into an explanatory narrative through a highly selective retelling that gives particular positions and relationships to the author, audience and mathematical community. Though the work presented is already proved and published, presentations are littered with the language of collective endeavour: *we want* to do something, *we'll try* this. The mathematician becomes a performer, a storyteller, showing the audience the footholds to create the mathematical meaning in their own minds. The dynamic of the retelling shows where the important connections are made, and which developments should be given the most attention.

The use of the collective “we” also places this as part of the work of the 'entity' that is the mathematical community, which must, because of extreme specialism within fields, be relied upon to validate it. Once accepted, a proof shifts from a story to an object and is tagged with a name and a date, and used as a point from which to build. Frequent references to names and dates pepper mathematical speech – Cauchy data, Riesz potential, “by Khan-Markovic”. This reference points to a certain person at a point in history, their life's work and history turned into an object to be moved around like a building block in a proof. It also obscures the complex socio-cultural factors that have decided which name is attached to a concept, particularly given that mathematics is subject to Stigler's law: the law that no scientific discovery is named after its original discoverer.

Figure 3. “Three Narrators of mathematical discourse: Maker, Storyteller, Historian”

Figure 3 shows an example from the studio practice that will run alongside these written reflections, using physical elements to explore themes of interpretation, representation and abstraction. This is a model of these three threads running through mathematical discourse, referencing the shapes seen in the storyteller ceramics made by the Pueblo people in New Mexico(7), and the famous cuneiform tablet *Plimpton 322* from about the 18th Century BC, which gives a list of Pythagorean triples(8), here rendered in wet clay. The movement from Maker to Storyteller involves a flattening, a discarding of information; chalk is seen both as a raw material and formed for the purposes of institutional use.

Creative practice in this study will be used as a demonstrative exploratory medium, making use of non-verbal experimentation concerned with methods of representation and abstraction. Using it in combination with observation can engage with mathematics on different terms to those commonly used in art-mathematics work, which tends to focus on artefacts from mathematics more than the processes that create them. Rather than creating an attractive illustration I want to present insights into the culture of the discipline, and use a visual, spatial language to make them accessible to an audience whose knowledge of the mathematical world is normally severely restricted.

5. Conclusion

This paper is a sketch for an ongoing exploratory research project whose aim is to bring back interesting observations from the world of mathematics and present them in a way that is tangible and engaging even to the non-expert. The first result of this research is a blog on which sculpture made in response to these observations is being published. Though the difficulties are significant, I believe it important to investigate ways to access and discuss cultural and conceptual structures in this fascinating realm.

The incompleteness of this experience is key. It calls to mind Susan Gerofsky's desire to find a place in mathematics education to “dwell with ambiguity”(9), so that students can learn to spend time getting the feel of concepts rather than simply looking to find the correct answer. This, perhaps, is where art can have something useful to say about, or even to, mathematics, sidestepping the rigour that has left most of the population feeling excluded and disinterested, and creating a space for a concept to exist in ambiguity.

In this paper I have discussed some of the ways that shape and our experience of physical reality are manifested in mathematics, and how this can be fed into sculptural work. Going forward I will be carrying out more observations at conferences and will be exploring the role of movement and gesture and what it can communicate about the shape of mathematical reality. I will also be investigating the metaphors that are evoked when a mathematician is asked to explain their work to a layperson and how their form expresses aspects of the work done, as well as their inherent limitations, in a set of humorous sculptural diagrams. Though inherently challenging, I believe that this interdisciplinary project will have many surprising perspectives to offer, to mathematicians and outsiders alike.

References

[1] B. Latour, S. Woolgar, *Laboratory Life: The Construction of Scientific Facts*, Princeton University Press Princeton, 1979

[2] A. Neves. *Min-max theory and applications II, Recorded lecture at the Instituto Nacional de Matemática Pura e Aplicada*, 4-10 January 2015 https://www.youtube.com/watch?v=PWcSh5z0ldw

[3] G. Lakoff and R.E. Núñez. *Where Mathematics Comes from: How the Embodied Mind Brings Mathematics Into Being*, Basic Books, California 2000

[4] D. Sperber, and Deirdre Wilson. *Relevance: Communication and Cognition*.,Wiley 1996

[5] D. Wilson. *Parallels and differences in the treatment of metaphor in relevance theory and cognitive linguistics.* Intercultural Pragmatics. Volume 8, Issue 2, Pages 177–196, ISSN (Online) 1613-365X, ISSN (Print) 1612-295X, DOI: 10.1515/iprg.2011.009, May 2011

[6] E. de Freitas and N. Sinclair. *Mathematics and the Body: Material Entanglements in the Classroom*, Cambridge University Press, Cambridge 2014

[7] B. Rotman. *Mathematics as Sign*, Stanford University Press, 2000

[8] B. A. Babcock. *The Pueblo Storyteller: Development of a Figurative Ceramic Tradition,* Edited by Guy Monthan and Doris Monthan, University of Arizona Press, Tucson 1991

[9] *‘Before Pythagoras: The Culture of Old Babylonian Mathematics’,* Accessed 2 January 2016. http://isaw.nyu.edu/exhibitions/before-pythagoras/items/plimpton-322/.

[10] S. Gerofsky. *Why does a letter always arrive at its destination? Opening up living space between problem and solutions in math education,* Chreods, 12, 1997