Econometric Modelling of the Parameters Actuarial Model of Net Rates for Life Insurance of the Population of Region

Vladimir S. Mkhitarian[1] and Svetlana S. Mikhaylova[2]

[1] National Research University Higher School of Economics, Russia
http://www.hse.ru
[2] East Siberia State University of Technology and Management, Russia
http://www.esstu.ru
Buryat State University, Russia
http://www.bsu.ru

Abstract. The article analysed and solved the problems of constructing mortality tables and determining the amount of payment for the foreseeable future, which are relevant in connection with the development of the classical life insurance in Russia and the growing role of methodological developments aimed at determining the cost of insurance coverage. The algorithm calculation is implemented on the example data of a region of the Russian Federation. The practical significance of the proposed algorithm is that it can be used by insurance companies when calculating the tariff of life insurance in the region.

Keywords: actuarial calculations, life insurance, prediction of mortality, insurance rate

1 Introduction

Life insurance is one of the most effective ways to solve the problem of shortage of regional financial resources through the mobilization and transformation of available funds of the population into investments. The development of life insurance industry, on the one hand, serves as a source of investment funds bearing a principally long-term character, on the other hand, acts as a guarantor of social security of the population, protecting the interests of policyholders [12].

In the life insurance implementation in the region it is necessary to consider its features, as the subjects of the Russian Federation are characterized by different levels of socio-economic development, the diversity of national composition of the population, sharp differences of climatic conditions, which directly leads to the demographic situation, determines the level of mortality in the region [10]. Thus, the reduction of providing insurance protection cost to the population, inadequate assessment of the insurance capacity of the region may lead to financial instability of the insurance company, not profitable insurance operations, and overstatement of tariffs will hinder the development of life insurance, will lead to non-competitiveness of the individual insurer. A reasonable life insurance tariff size from the economic point of view, for the population of the region will provide the required reliability and stability of the insurer. At the same time the insurance companies in the development of regional tariff policy face with the problem of the lack of reliable statistics, a single insurance information base, and methods of calculating insurance rates for the region. Determining the size of the overall life insurance net rates on the basis of regional statistics, as well as the prediction of rate changes caused by socio-economic status, the demographic situation in the future, it is essential for the organization of long-term life insurance in the region [15]. Because insurers are relying on forward-looking information, can adjust the size of the current tariff, as in the upward and downward, which will provide, in the first case, the insurer's stability in the future, in the second case, mass insurance in the current period, which is the main condition for the reliability of the insurance operations [7].

2 The calculation algorithm

1. In the first step of life insurance payment modelling it is necessary to solve the problem of building a regional mortality tables. The table construction includes several stages:

- Calculation of baseline values for all ages on the basis of the mortality statistics (distribution of deaths by age);
- If necessary, processing of this range of values to remove distortion caused by age accumulation;
- Interpolation of row of values to eliminate possible gaps or extrapolation to calculate the values for the older age groups;
- Calculation of the remaining life table functions;

The main methodological problem of constructing mortality tables is associated with the transition from the actual age-specific mortality rates to table probabilities of dying at a given age. Age-specific mortality rates (mx) are calculated separately for men and women as the ratio of the number of deaths in a particular age by the average annual number of men or women in this age group:

$$m_x = \frac{{}_nD_x}{{}_n\tilde{P}_x} \times 1000\%,$$

where ${}_nD_x$ – the number of deaths in the age range $(x+n)$; ${}_n\tilde{P}_x$ – average annual population (men or women) in the age interval $(x+n)$.

To go from the actual levels of age-specific mortality rates m_x to tabular probabilities q_x we can use the analytical laws of mortality. These theoretical models allow us to identify the main patterns of interest to researchers, even a simplified study of reality. At the same time, some real mortality processes are well enough approximated by the considered analytical laws [14].

The most used "law of mortality" is a model Heligmen-Pollard, proposed by the authors in 1980, which describes the change in the mortality rate, represented by the ratio of $(qx/(1-qx))$ the probability of dying at age x to the probability of surviving to age $(x+1)$ year of age x [2].

In the basis of the following law known as the law of Gompertz-Mackhame, lies the assumption that the intensity of mortality increases with age, at a geometric rate, i. e. $\mu_x = A + Bc^x$, where A is the parameter responsible for the deaths that are not associated with health status and age, B is the coefficient of proportionality linking the mortality rate with the weakening of the vitality of the person, C – a parameter characterizing the rate of decrease of a person's ability to resist various health disorders. The law of Gompertz-Mackhame satisfyingly describes mortality only for the older age intervals. This can be explained by the reduced rate of death at an early age.

In 1939, Weibull as a simple approximation of the force of mortality suggested a power function $\mu_x = kx^n$, which will determine the survival function

$$s(x) = \left[-\int_0^x ku^n du\right] = \exp\left[-\frac{k}{n+1}x^{n+1}\right]$$

and the curve of deaths

$$f(x) = -s'\left(x = kx^n \exp\left[-\frac{k}{n+1}x^{n+1}\right]\right).$$

Erlang's model is a function of the form $\mu_x = x/(a(x+a))$. For Erlang's model, 2nd order mortality curve is described by the formula $f(x) = x/a^2 \times e^{-x/a}$, $x \geq 0$, the survival function $s(x) = (x+a)/a \times e^{-x/a}$ [1].

Also to describe mortality at different age intervals you can use the following nonlinear functions: logarithmic, exponential, power, hyperbolic, polynomials of various degrees.

The advantage of the theoretical models is that for them the probability characteristics of demographic indicators can be calculated on a small number of parameters. This fact is particularly important at the limited statistics.

2. To facilitate the calculations of the risk rates in the next step the table of switching functions is built. Standard switching functions are divided into two groups. The basis of the first the number of survivors to a certain age is put, the second – the number of dying. The main in the first group are the functions D_x and N_x:

$$D_x = l_x \times \nu^x, \quad N_x = \sum_{j=x}^{\omega} D_j,$$

where ν – discounting factor in the compound interest rate i, and is determined by the formula $\nu = 1/(1+i)$.

A-priory $N_x = N_{x+1} + D_x$ and $N_\omega = D_\omega$ (ω – the age limit adopted in the mortality table).

Function D_x is used for insurance on survival. D_x is the expected present value of the amount of insurance premiums for insurance on survival initial set of l_0 with the condition of payment of a single sum insured upon reaching the age of x, switching function N_x is used to simplify the calculations of rent insurance. The meaning of this function is the following. At birth, a group of children numbering l_0 is an insurance contract with a condition of payment of a lifelong annuity in size per unit at the beginning of each year, starting at age x, then N_x represents the present value of insurance payments or the total amount of one-time premium. The switching number of the second group is used to simplify calculations in life insurance in case of death:

$$C_x = d_x \times \nu^{x+1}, \quad M_x = \sum_{j=x}^{\omega} C_j.$$

Each insurance company develops its own table of switching functions based on its rates of return, the specifics of the insurance field and existing insurance statistics [3].

3. There is the calculation of the net net-rate on life insurance contracts. When life insurance insured risk is the duration of human life, i. e. the risk is not death itself, but the time of death. Death at a young age causes the need to promote the well-being of the family of the deceased, and survival to deep old age requires obtaining regular income without continuing employment [9].

In accordance with the above-mentioned there are the following types of life insurance:

– insurance in case of death (distinguish between life and term insurance);
– endowment insurance;
– mixed life insurance.

Insurance in case of death. Risk covered by this insurance is the death of the insured person during the period of insurance. The insurer when the insured event guarantees the payment of the insured sum to the beneficiary. Contracts of this type are not associated with the provision of the property interests of the insured but, as a rule, his heirs and creditors.

At *endowment insurance* the risk of survival of the insured is insured before the end of the insurance period. It allows to raise the level of welfare of the insured person by creating savings. endowment insurance is insurance of a certain amount of money for a certain period. In the event of death of the policyholder during the period of validity of the contract the insurance sum is not paid and fees are not refundable.

Mixed insurance covers the risks of survival until the end of the period of insurance, death in action.

4. The size of the relative risk premium is determined. The risk premium is introduced for life insurance in case of death, the life insurance contract; endowment allowance is not entered as the insurance company pays the insurance indemnity only in case if the client survives until the end of the contract, i. e., the size and term of payment is known in advance to the insurer. And the insurance company provided by the correct choice of mortality tables and economically reasonable interest rate compounding means has no risk of exceeding the insurance premiums collected over the net premiums of life insurance on survival. In the contract of the mixed insurance two components are taken into account: life insurance and life insurance on survival for the same period. As the risk premium is calculated only on the first component, the total net rate on the contract of the mixed insurance the risk premium will have little impact.

To determine the size of risk premiums the expectation EX, the variance $D(\xi_i)$, or standard deviation $\sigma(\xi_i)$, indicating the variation from the mean are used. Total net premium P in this case can be calculated as $P = EX + \alpha EX$, $P = EX + \beta\sigma(\xi_i)$ or $P = EX + \gamma D(\xi_i)$, where X is the total amount of claims, EX – its expected value, α, β, γ – ratios, reflecting the sustainability of the portfolio and scatter payouts relative to the average value. You can also enter the risk premium of a mixed type, which when receiving a new risk takes into account its dependence with the already existing portfolio, but also directly determines the degree of accumulation of risks as $P = EX + \alpha EX + \beta\sigma(\xi_i) + \gamma D(\xi_i)$ [4].

In the case of homogeneity of insurance portfolio, all three approaches will give the same results, and the relative increase risky for life insurance in the case of life at age x will be:

$$\Theta_x = t_\gamma \cdot \sqrt{\frac{p_x}{q_x}} \cdot \frac{1}{\sqrt{n}},$$

where t_γ – the reliability coefficient, q_x – the probability of death at age x, p_x – the probability of survival at age x, n – the volume of the insurance portfolio.

It should be noted that the share of risk premiums in the insurance rate with increasing age of the insured is reduced due to the fact that with increasing age the probability of death, respectively, reduces the probability of survival [5]. The increase in the portfolio of the insurer tends to reduce the overall level of risk premium, because the more homogeneous the risks in the portfolio, the smaller the realized total risk is different from the expected. This explains the advantage of large insurance companies. Because of their size they can set lower rates, as a large portfolio allows them to provide a sufficient level of reliability at a lower risk premium than the insurance company, whose portfolio contains fewer contracts.

The insurance company in order to establish competitive rates, while growing the insurance portfolio, may establish a lower risk premium than it allows the portfolio. In this case, a significant factor is the insurance potential of the region, i. e. the number of able-bodied population, living standards, incomes, prospects of socio-economic development of the region, the level of insurance culture, the presence of competitors, etc.

5. The last step is to define the tariff rate life insurance as the sum of the main net rates, risk premiums and load. The load and the brave extra charge are determined by each insurance company on its own, depending on the portfolio, of doing business expenses, profit margins, etc.

3 Results

Let's determine on the basis of regional statistics data the size of net-net rates for life insurance classes for male and female population, promising annual number of concluded life insurance contracts on the territory of the Republic of Buryatia in the period until 2018.

To determine the main net rates types of life insurance for the future period it is necessary to forecast age-specific mortality rates of male and female population of the region. As a method of forecasting age-specific mortality researcher selected the method of "law of mortality" that allows to confine the data of regional statistics and meet the requirements of short-term population projection. Extrapolation method, being less time consuming, does not provide a qualitative forward-looking information, due to the unstable, spasmodic development in the retrospective period [8].

To select analytical functions describing the best way the process of extinction of the region's population ages, the author in the environment of MS Excel has calculated the parameters of analytical models of age specific mortality rates [1]. The main criterion of quality was the coefficient of determination. The study revealed that the distribution of age-specific mortality rates of the population of the Republic of Buryatia corresponds to the Weibull distribution. So a model based on the Weibull distribution and the exponential function, which is a special case of Weibull distribution, allows to describe at least 90.0% of the changes in age-specific mortality of male and female population of the Republic of Buryatia at different age intervals, between 1990 and 2015.

Analytical record for the following models [6]:

- $\mu_x = kx^n$ – a power (Weibull model);
- $\mu_x = ae^{bx}$ – exponential.

The mortality rate among men aged 18–35 is described by the Weibull model and in the older age group of 36–70 the variation of mortality depending on the age and the other is described by an exponential function. The discrepancy of the mortality patterns of young and older ages is due to differences in the increment of the force of mortality at different age intervals, there has been a relatively strong surge in the death rate in the transition from the age of 35 to 36. Mortality among women all over the considered interval is described by an exponential law, but the construction of a model for a single group of 18–70 leads to a deterioration of the quality criterion modelling characteristics, so it seems appropriate to partition the interval 18–70 years, in two age groups: 18–35 and 36–70 years. Table 1 presents the results of modelling age-specific mortality rates for men, similar calculations were performed for the female population of the Republic of Buryatia for the period 1990–2015.

The absolute increasing in mortality by age in the male population of the Republic of Buryatia at the age of 18–35 with increasing age decreases. For example, the mortality rate in 2015 during the transition from the age of 20 to 21 increased by 0.00024‰, from the age of 30 to 31 – 0.00019‰, i. e. in the age group of 20–35, the speed of increment of intensity of mortality is reduced. On the contrary, in the age group of 36–70 years, with increasing age, the rate of increment of the mortality rate increases, averaging 41 years old 0.0049‰, and in 61 years old – 0.0138‰, i. e. at the age of 41 out of 1000 people die at 4.9 man more than at the age of 40, and at the age of 61 there are 13.8 more than at the age of 60.

Changing the age of the female population of the Republic of Buryatia mortality intensity is described by an exponential function and in 2015 at the age of 35 years is equal to the increment of the mortality forces 0.0011‰, in 60 years – 0.0063‰.

Table 1. The results of age-specific mortality levels modeling of the male population of the Republic of Buryatia for 1990–2015

Year	Age groups			
	18–35 years		36–70 years	
	Model #1	The coefficient of determination	Model #2	The coefficient of determination
1990	$\hat{\mu}_E = 0.0005x^{0.7066}$	0.9792	$\hat{\mu}_E = 0.0005e^{0.0684x}$	0.9771
...	...	...	...	...
2000	$\hat{\mu}_E = 0.0003x^{0.9931}$	0.9343	$\hat{\mu}_E = 0.0013e^{0.0578x}$	0.9925
...	...	...	...	...
2010	$\hat{\mu}_E = 0.0003x^{1.0127}$	0.9341	$\hat{\mu}_E = 0.0014e^{0.0554x}$	0.9914
...	...	...	...	...
2014	$\hat{\mu}_E = 0.0001x^{1.7222}$	0.9549	$\hat{\mu}_E = 0.0020e^{0.0515x}$	0.9973
2015	$\hat{\mu}_E = 0.0001x^{1.3799}$	0.9845	$\hat{\mu}_E = 0.0019e^{0.0523x}$	0.9961

Let's examine graphically changes in age-specific mortality rates of male and female population of the Republic of Buryatia for the period of 2009–2015. During this period, values of age-specific mortality rates do not reveal directional changes in time (Fig. 1 for females and Fig. 2 for males).

Let's analyse the changing values of age specific mortality rates, net rates types of life insurance depending on the fluctuations of the parameters of the models of mortality in the value of its standard deviation. Changing model k parameter $\mu_x = kx^n$ level of age-specific mortality of the male population aged 18–35 years, the value of the standard deviation of the standard of the average $k_{average}$ leads to a change in the appropriate direction by an average of 2.38% of your age-specific mortality in this group, which in turn implies to increase or decrease the net rate of life insurance in case of death by an average of 2.2%, on survival – by −0.06%, mixed – 0.06%. if the parameter $n_{average}$, is changed to the value of the standard deviation of the average n average, that the mortality rate at each age is also changed by an average 8%, respectively the net rate of death is changed by 7.87%, endowment – by −0.24%, mixed – 0.24%.

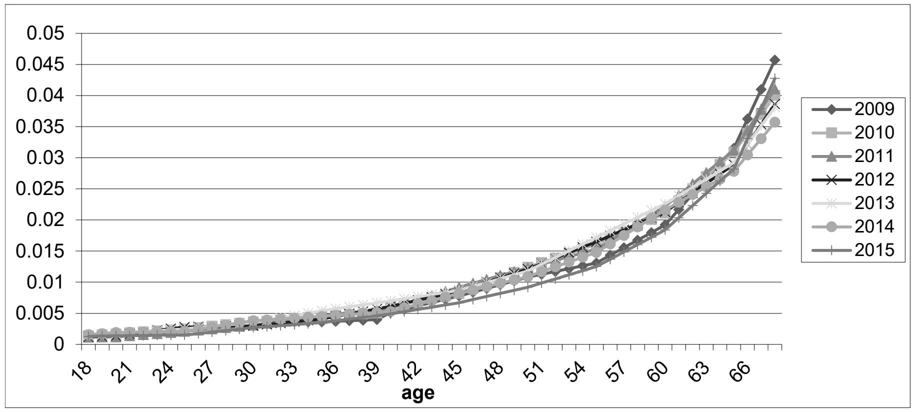

Fig. 1. The curves of mortality of the female population of the Republic of Buryatia for 2009–2015

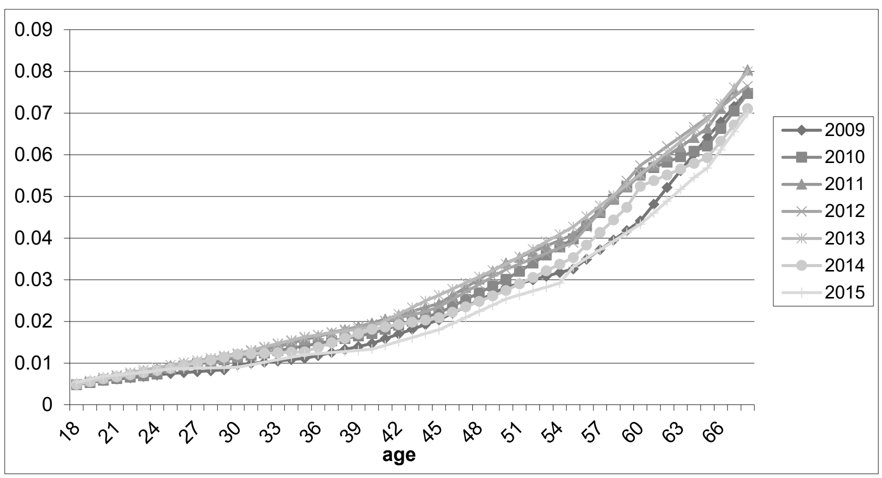

Fig. 2. The curves of mortality of the male population of the Republic of Buryatia for 2009–2015

In the older age group, fluctuations of the model parameters $\mu_x = ae^{bx}$ also not have a significant impact on the net rates. Fluctuations a and b lead to changes in mortality rates at the age of 36–70, an average of 11.8% and 4.6%, respectively, net rates in case of death by 10.9% and 4.33%, – endowment at −1.34% and −0.69%, on the mixed – 0.06% and 0.03%.

For the female population of the Republic of Buryatia age group of 18–35, the deviation of the parameters a and b models $\mu_x = ae^{bx}$ from the mean to the of the standard deviation leads to a change in age-specific mortality rates at 7.14 and 0.6%, respectively, changing the net interest rates on the types of life insurance are as follows: in the event of death, 6.7 and 0.6%, endowment – −0.05 and −0.01%.

In the group of 36–70 deviation a and b leads to changes in mortality rates, average age of 13.5% and 7.55%, respectively, of the net rates on life insurance in case of death by 12.9% and 7.37%, endowment – −0.59%–0.45%, on the mixed – 0.03% and 0.02%.

The changes in the development of demographic processes are long-term, therefore, it's fair to assume that in the absence of crisis in the economy in the short term age-specific mortality rate of the population has the same distribution and is described by a single model. In this case, we can assume that in the short term demographic indicators will be close to a certain constant value, although the study of them in the long run they will definitely show the trend of development. In the absence of trends in mortality at age x the model parameters are also constant.

To define the parameters of prognostic models describing the probability of death at age x of male and female population of the Republic of Buryatia in the planning period of 2016–2018, we will use model of ordinary regression. The application of this model requires the following assumptions: parameters of the incoherent models tend to some constant value; the regression errors are normally distributed.

Thus, it can be assumed that the parameters of the models in the period of 2009–2015 form a sequence of changes in the average expected level which does not depend on the flow of time. To examine the hypothesis let's test the time series of indicators for the presence of trends in their development (Table 2). The most common in the practice of test series for the presence of trends is the use of statistical testing of hypotheses about the immutability of trends in a number, i. e. it is necessary to check the number on the accident distribution. For this purpose T-cumulative criterion method of comparing average levels of time series, Abbe criterion (criterion of consecutive squares of the differences) are most often used [13].

Testing confirmed the hypothesis about the absence of trends in parameters of models of male mortality (Table 2) and female mortality population over the period of 2009–2015, which form a stationary series.

To build predictive models of age-specific mortality of men aged 18–35 data were used from 17 age groups for 7 years (119 cases) and at the age of 35–70, data of 35 age groups for 7 years (245 cases).

The level models of age-specific mortality of the male population of the Republic of Buryatia have the form:

$$\hat{\mu}_E = 0.00003\, x^{1.7441}, \quad R^2 = 0.929, \quad \text{at the age of 18–35}, \tag{1}$$
$$t_{calc} \quad (34.15)$$
$$\hat{\mu}_E = 0.00194\, e^{0.0523x}, \quad R^2 = 0.959, \quad \text{at the age of 35–70}. \tag{2}$$
$$t_{calc} \quad (46.76)$$

Building predictive models for age-specific mortality of women were carried out similarly as for men.

Table 2. Statistical verification of the hypothesis of no trend in the number

Criterion	The table statistics value	The calculated statistics values							
		Model #1		Model #2		Model #3		Model #4	
		k	n	a	b	a	b	a	b
Cumulative T-test	$t_{tab}(\alpha = 0.05; n = 6) = 2.45$	0.369	1.911	1.582	2.233	0.833	0.591	1.763	2.445
Comparison of the average levels	$T_{tab}(\alpha = 0.05; n = 5) = 2.57$	0	0.892	0.604	0.875	0.756	0.583	0.806	1.206
	$F(k_1 = 2; k_2 = 2) = 19$	13	16.8	2.6	16.8	13	7.4	1.3	1.9
Abbe	$\gamma_{0.05}^{\min(7)} = 0.468$	1.55	0.63	0.53	0.49	1.125	1.259	0.746	0.556

Model level of age-specific mortality of the female population of the Republic of Buryatia have the form:

$$\hat{\mu}_E = 0.00031\, e^{0.0734x}, \quad R^2 = 0.962, \quad \text{at the age of 18–35}, \tag{3}$$
$$t_{calc} \quad (37.23)$$
$$\hat{\mu}_E = 0.00037\, e^{0.0658x}, \quad R^2 = 0.977, \quad \text{at the age of 35–70}. \tag{4}$$
$$t_{calc} \quad (54.93)$$

Quality criteria of the models are shown in Table 3.

Table 3. Characteristics of the models adequacy

Statistical characteristics	Model number			
	1	2	3	4
The level of significance (α)	0.05	0.05	0.05	0.05
The coefficient of determination (R^2)	0.929	0.959	0.962	0.977
Average approximation error ($\bar{\delta}$), %	1.626	2.657	1.680	1.768
The observed value of F – Fisher criterion (F_{calc})	1631.4	5645.5	1118.6	10305.0
The tabular value of F – Fisher criterion ($F_{tab}(1; \infty)$)	3.84	3.84	3.84	3.84

Descriptive quality models that characterize the age-specific mortality of the female population of the Republic of Buryatia, are higher than the models that characterize the age-specific mortality of the male population. Men's mortality is more subject to fluctuations depending on the influence of social factors, economic, than female mortality.

4 Discussions

A key indicator of the prospects for the development of life insurance in the region is determining the potential number of concluded life insurance contracts. The number of life insurance contracts is directly proportional to the collected insurance premiums, the ratio is the average insurance premium. Under the influence of various factors, such as inflation, growth of welfare of the population, changing the average term of the life insurance contract, etc., the average insurance premium may be also changed.

5 Conclusions

On the basis of predictive models of age-specific mortality levels the lump-sum net-rates for male and female population of the Republic of Buryatia were built for the future period 2016–2018.

The demographic situation in the region has stabilized and while maintaining the favorable trends of social and economic development, at least not deteriorate. Therefore, insurers, who lead insurance activities on the territory of the Republic of Buryatia, have some possibility to implement one of the basic principles of determining insurance rates in the main part of the net rate – rate stability dimensions. Also the age-specific mortality prediction will help to determine the parameters of the actuarial model of the relative risk premium [11].

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