# Lean Kernels in DLs (Extended Abstract)*

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When trying to understand the axiomatic causes for a consequence to follow from an ontology (e.g. to correct erroneous entailments), it is often convenient to reduce the search space to include only those axioms that play an active role in these entailments. That is, one is interested in finding all the *relevant* axioms: those axioms that appear in at least one MinA (or justification) for a given entailment. Unfortunately, deciding whether an axiom is relevant is a computationally hard problem, even for very weak logics with linear-time entailment relations.

Lean kernels (LKs) were first introduced in the area of propositional logic as a means to approximate the set of all relevant axioms. In a nutshell, the lean kernel is the set of all clauses that appear in a resolution proof for unsatisfiability of a given formula. As such, it is an overapproximation of the set of all relevant clauses for explaining unsatisfiability. Recent work has shown that LKs provide an effective optimization for enumerating the set of all MinAs (or MUSes, as they are called in the propositional satisfiability community). Indeed, LKs are comparatively fast to compute, and typically do not contain many superfluous clauses.

Interestingly, an analogous notion of LKs has never been considered within the DL community. One difficulty when introducing such a notion is that it depends on a specific decision procedure (like resolution). In this work, we consider an abstract notion of decision procedures, called *consequence-based methods*, which include resolution, the completion algorithm for $\mathcal{EL}$, and many other procedures for expressive description logics. We define LKs based on these methods, and show that they can be modified to compute LKs for all consequences with only a linear time overhead w.r.t. standard reasoning. As for propositional formulas, LKs in DLs preserve all relevant axioms. In other words, these sets are small MinA-preserving modules.

We then compare LKs with other known approximations of the set of relevant axioms. Specifically, we show that in $\mathcal{ALC}$ and $\mathcal{EL}^+$, LKs are always strictly contained in locality-based modules. Moreover, an empirical analysis over well-known large $\mathcal{EL}^+$ ontologies shows that the differences are significant: the size of the LK is in many cases one-tenth or less than the locality-based modules. Thus, LKs provide an effective approximation of the set of relevant axioms.

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