Consignment Contracting with Inventory Control with Additive Price-Dependent Demand

Milena Bieniek
Maria Curie-Sklodowska University
Plac Marii Curie-Sklodowskiej 5,
20-031 Lublin, Poland.
milena.bieniek@umcs.lublin.pl

Abstract

W investigate a consignment contract in which vendor retains ownership of inventory until the retailer sells the product to the market. At that time the vendor gets paid from the retailer based on actual units sold. We consider a single period supply chain model with uncertain and price-dependent market demand. The vendor decides his consignment price charged to the retailer for each unit sold and the retailer chooses the selling price. Under that framework we consider two consignment arrangements. The first one called retailer managed consignment inventory (RMCI) allows the retailer to choose inventory level together with selling price. In the second one labeled as vendor managed consignment inventory (VMCI) program the vendor decides the inventory level together with consignment price. In our considerations we are taking into account stochastic demand which is linear with respect to the price. We build a game-theoretic model to capture the interactions between vendor and retailer in RMCI or VMCI program. The equilibrium decisions are based on maximization of channel profits.

Key words: supply chain management, games, consignment contract, price and inventory decisions

1 Introduction

Since a market competition becomes more intense then the firms prefer cost reduction instead of focusing only on revenue. Shifting inventory ownership to vendor is one of the top practices for reduction the inventory costs. This activity is equivalent to consignment contract. Consignment is defined as the process vendor placing goods at retailer's location without receiving payment until the goods are used or sold. Under consignment contract the decision how much inventory hold can be operated in two ways. In the traditional way the downstream retailer decides the inventory level and in the new way responsibility for inventory level and stock level decisions are making by the upstream vendor. The first procedure is called Retailer Managed Consignment Inventory (RMCI) and the second one Vendor Managed Consignment Inventory (VMCI). Both arrangements are used in practice but lately VMCI becomes more popular. For instance many big retailers like Wal-Mart, Target or Meijer

In: Yu. G. Evtushenko, M. Yu. Khachay, O. V. Khamisov, Yu. A. Kochetov, V.U. Malkova, M.A. Posypkin (eds.): Proceedings of the OPTIMA-2017 Conference, Petrovac, Montenegro, 02-Oct-2017, published at http://ceur-ws.org

Stores have implemented or planning implementation of VMCI arrangement. Still there are some debates among practitioners who should control the supply chain. [Ru & Wang, 2010] try to give the answer for this question who should be responsible for the level of consigned inventory .

The aim of this paper is to give more light on the consignment contracting. We build the game-theoretic model to investigate the interactions between the vendor and the retailer. We find the equilibrium decisions analytically. Our analysis complement those given in [Ru & Wang, 2010] since they consider only some special form of market demand. They study stochastic price–dependent multiplicative demand in the form $D(p, \epsilon) = ae^{-bp}\epsilon$ with $a, b > 0$, where ϵ is continuous random variable. Using the demand in this special form the authors obtain mathematically tractable results. This let them analyze the properties of equilibrium decisions analytically. We complement the previous results by considering the additive demand in the form $D(p, \epsilon) = a - bp + \epsilon$ with $a, b > 0$. The deterministic part of the demand of this kind is a linear function of price. The case investigated in our study is much more complicated analytically then for multiplicative case. In our considerations we concentrate on VMCI contract since it is much more popular than RMCI and brings more profits. We find the closed form solutions for equilibrium decisions under VMCI contracts and also we obtain some preliminary results for RMCI which can be treated more precisely in future work.

In the literature one of the first authors who consider contracts with inventory ownership are [Wang et al., 2004]. They study a pure consignment contract where vendor retains ownership of inventory. Later [Lee & Chu, 2005] try to answer to the question who should control the supply chain inventory. Other recent papers dealing with production and pricing decisions of decentralized supply chain are among others: [Wang et al., 2004], [Zhao & Atkins, 2008], [Hu et al., 2014], [Hu et al., 2015] or [Wu et al., 2016].

In the following we present the results on centralized and decentralized channels, consecutively. Section 2 is devoted to the centralized channel decisions. In Section 3 we study decentralized decisions for RMCI program and next we give the results for VMCI program. Last section concludes the paper.

2 Centralized Channel Decisions

We consider a single–period supply chain in which a supplier (vendor) produces and sells a product to the retailer. The vendor decides his consignment price charged to the retailer for each unit sold. The retailer chooses retail price w for selling the product to consumers. Denote by: c_s - supplier's unit production cost and c_r - retailer's unit handling cost. Also define $c = c_s + c_r$ as the total unit cost for channel and $\alpha = c_r/c$ as the share of channel cost that is incurred by the retailer. The random demand is defined by

$$D(p, \epsilon) = a - bp + \epsilon, \tag{1}$$

where $a, b > 0$ and p is the selling price. Here ϵ is continuous random variable with expected value μ, cdf $F(.)$ and pdf $f(.)$ with the support $[A, B]$ where $A < 0$ and $B > 0$. For a centralized channel the decision maker has the ability to decide on the quantity to buy and the price to set for the good he sells. Such a decision is based on maximizing the expected channel profit given by:

$$\Pi_c(p, Q) = pE(\min(D(p, \epsilon), Q)) - cQ. \tag{2}$$

Let define $z = Q - a + bp$ and transform $\Pi_c(p, Q)$ to

$$\Pi_c(p, z) = p\mu(z) + p(a - bp) - c(z + a - bp), \tag{3}$$

where $\mu(z) = \mu + \int_z^B (z - u)f(u)du$. As indicated in [Petruzzi & Dada, 1999] the quantity z can be interpreted as a safety stock because for selected value of z we face shortages if $z < \epsilon$ or leftovers if $z > \epsilon$ leftovers. On the other hand z corresponds to a unique customer service level (CSL) which is given by

$$\text{CSL} = P(D(p, \epsilon) \leq Q) = P(\epsilon \leq Q - a + bp = z) = F(z). \tag{4}$$

Indicating the value for z is equivalent to setting up CSL for the system.

Understanding the variability of the function $\mu(z)$ from (3) is crucial for the next analysis. The following statements hold:

1. $\frac{d\mu(z)}{dz} = 1 - F(z)$;
2. $\mu(.)$ is an increasing function of $z \in [A, B]$;

3. $\mu(A) = A < 0$ and $\mu(B) = \mu$.

After some changes the decision maker has the form:

$$\max_{p,z} \Pi_c(p, z) = \max_{p,z}(p(\mu(z) + a + bc) - p^2 b - c(z + a)). \tag{5}$$

To solve this problem we consider the sequential optimization method. This is the way of seeking optimum of a function of several variables be selecting the optimal values of each variable. Finally this method produce the maximum of the function we needed. We use this method and find the optimal solution denoted by (p_c, z_c) to the problem of maximizing the central channel profit. The result needs some new definitions and assumptions. In the next considerations we have to assume that . We need also the following definition.

Definition 1. *([Kocabiyikoglu & Popescu, 2011]) The lost sales rate elasticity for a given price p and service level z is defined as*

$$\kappa(p, Q) := \kappa(p(z), z) = \frac{bp(z)f(z)}{1 - F(z)}.$$

Under the assumption $A + a - bc > 0$ which guarantees the non-negativity of the demand, we get the result.

Theorem 2. *For any given service level $z \in [A, B]$ the unique optimal selling price p_c is given by*

$$p_c(z) = \frac{\mu(z) + a + bc}{2b}, \tag{6}$$

which is increasing and concave with z.

If $\kappa(p_c(z), z) \geq \frac{1}{2}$ then the optimal service level z_c is the unique root of the equation:

$$\frac{\mu(z_c) + a + bc}{2b} = \frac{c}{1 - F(z_c)}. \tag{7}$$

Since the above theorem is equivalent to the statement of [Rubio-Herrero et al., 2015] then the proof is similar and we do not give it here. In [Rubio-Herrero et al., 2015] they consider newsvendor model and in the proof they involve the lost sales rate elasticity. Below in the similar manner we prove the theorems for the decentralized channels.

3 Decentralized Channel Decisions

3.1 RMCI Program

Under RMCI decisions are made in two sequential steps. In step 1 vendor specifies the consignment price to determine the amount of payment he will receive from the retailer for each unit of his product sold. In step 2 the retailer decides the quantity for the vendor to deliver and the retail price for selling the product to the market. By the sequential method we assign the selling price and the service level which maximize the retailer's expected profit given by

$$\Pi_{d,R}(p, Q|w) = (p - w)E(\min\{D, Q\}) - c\alpha Q, \tag{8}$$

or equivalently

$$\Pi_{d,R}(p, z|w) = (p - w)(\mu(z) + a - bp) - c\alpha(z + a - bp). \tag{9}$$

Theorem 3. *Assume that*

$$A + a - b(c\alpha + w) > 0. \tag{10}$$

For any given service level z and consignment price $w > 0$, the unique optimal retail price $p_d(z)$ is given by

$$p_d(z) = \frac{\mu(z) + a + bc\alpha + bw}{2b}. \tag{11}$$

which is increasing and concave function of z. If $\kappa(p(z) - w, z) \geq \frac{1}{2}$ then the optimal service level z_d that maximizes $\Pi_{d,R}(p_d(z), z)$ is uniquely determined by

$$\frac{\mu(z_d) + a + bc\alpha}{2b} - \frac{w}{2} = \frac{c\alpha}{1 - F(z_d)}.$$

Proof. For any given $z \in [A, B]$ and known w we get

$$\frac{\partial \Pi_{d,R}(p, z|w)}{\partial p} = \mu(z) + a + bw + bc\alpha - 2bp \tag{12}$$

and

$$\frac{\partial^2 \Pi_{d,R}(p, z|w)}{\partial p^2} = -2b < 0.$$

Forcing (12) equal to 0 we get (11). Meanwhile for any given z the derivative $\frac{\partial \Pi_{d,R}(p,z|w)}{\partial p} > 0$ for $p < p_d(z)$ and $\frac{\partial \Pi_{d,R}(p,z|w)}{\partial p} < 0$ for $p > p_d(z)$, which assures that $p_d(z)$ is uniquely determined. Furthermore,

$$\frac{dp_d(z)}{dz} = \frac{1 - F(z)}{2b} > 0$$

and

$$\frac{d^2 p_d(z)}{dz^2} = -\frac{f(z)}{2b} < 0,$$

which implies that $p_d(z)$ is increasing and concave.

Now we obtain z_d which maximizes $\Pi_d(p_d(z), z)$. Analyzing the derivatives of the profit at the extreme points we get:

$$\frac{\partial \Pi_{d,R}(p_d(z), z)}{\partial z} = (p_d(z) - w)(1 - F(z)) - c\alpha$$

and

$$\frac{\partial \Pi_{d,R}(p_d(z), z)}{\partial z}|_A = \frac{A + a - b(c\alpha + w)}{2b} > 0,$$

by the assumption (10). Moreover

$$\frac{\partial \Pi_{d,R}(p_d(z), z)}{\partial z}|_B = -c\alpha < 0.$$

This implies that there exist point z_d at which function $\Pi_{d,R}(p_d(z), z)$ attains its maximum. Now we prove the uniqueness by showing that $\Pi_{d,R}(p_d(z), z)$ is concave function. The second derivative

$$\frac{\partial^2 \Pi_{d,R}(p_d(z), z)}{\partial z^2} = \frac{dp_d(z)}{dz}(1 - F(z)) - f(z)(p_d(z) - w) \tag{13}$$

should be negative. The function $\frac{dp_d(z)}{dz}$ is decreasing and attends its maximum at $z = A$ where $\frac{dp_d(z)}{dz}|_A = \frac{1}{2b}$. Then the negativity of (13) implies

$$\frac{dp_d(z)}{dz}(1 - F(z)) - f(z)(p_d(z) - w) \leq \frac{1 - F(z)}{2b} - f(z)(p_d(z) - w) \leq 0.$$

This is equivalent to $\kappa(p_d(z) - w, z) \geq \frac{1}{2}$, which concludes the proof. □

The assumption (10) imposes that regardless of how small A is, the lowest price that can be set guarantees that the realization of the demand $D(p, \epsilon)$ will still be positive.

It should be underlined that in our case of additive demand both the optimal selling price $p_d(z)$ and the optimal service level z_d depend on the consignment price w. It produces many difficulties for obtaining close form solutions. It is worth to note that for multiplicative demand which is the subject of [Ru & Wang, 2010] the optimal service level does not depend on optimal consignment price. The authors adopt a specific demand function form for convenience of getting precise solutions. Using linearly price-dependent demand in RMCI program needs much more attention. We are going to consider it in extended version of the paper.

In the end of this subsection it should be added that at the first stage of RMCI knowing that the retailer chooses (p_d, z_d), the vendor's unique optimal consignment price w_d can be calculated by maximizing the expected vendor's profit. This profit is given by:

$$\Pi_{d,V}(w|p_d, z_d) = w(\mu(z_d) + a - bp_d) - c(1 - \alpha)(z_d + a - bp_d).$$

Since both p_d and z_d depend on w the solution is not obvious and need more attention. We left it for future research. Finally, we can state that in RMCI the total decentralized channel profit is equal to

$$\Pi_d(p_d, z_d) = p_d(\mu(z_d) + a - bp_d) - c(z + a - bp_d). \tag{14}$$

3.2 VMCI Program

Consignment contracting under VMCI program becomes more and more useful in inventory management. In step 1 under VMCI the quantity decision is made by the vendor together with consignment price. In step 2 for a given consignment price and service level, chosen by the vendor at the first stage, the retailer determines the retail price which maximizes his own expected profit. Then the retailer attains the expected profit equal to

$$\Pi_{d,R}(p|w,z) = (p-w)(\mu(z)+a-bp) - c\alpha(z+a-bp).$$

In VMCI program we get the equivalent solution for optimal price p_d as in RMCI.

Theorem 4. *For any given service level $z \in [A,B]$ and consignment price $w > 0$, the retailer's unique optimal retail price $p_d(w,z)$ is given by*

$$p_d(w,z) = \frac{\mu(z)+a+bc\alpha+bw}{2b}. \tag{15}$$

Maximizing the vendor's profit function:

$$\Pi_{d,V}(w,z|p) = w(\mu(z)+a-bp) - c(1-\alpha)(z+a-bp)$$

we get the following theorem.

Theorem 5. *For any given service level z, the supplier's unique consignment price $w_d(z)$ is given by*

$$w_d(z) = \frac{\mu(z)+a+bc(1-2\alpha)}{2b} \tag{16}$$

and if $\kappa(w(z)+c(1-\alpha),z) \geq \frac{1}{2}$ then service level z_d is uniquely determined by

$$\frac{\mu(z)+a-4bc\alpha+3bc}{2b} = \frac{2c(1-\alpha)}{1-F(z)}. \tag{17}$$

Proof. Putting (15) into $\Pi_{d,V}(w,z) = w\mu(z)+w(a-bp)-c(1-\alpha)(z+a-bp)$ we obtain

$$\frac{\partial \Pi_{d,V}(w,z)}{\partial w} = \frac{\mu(z)+a-bc(2\alpha-1)-2bw}{2}.$$

From the equality $\frac{\partial \Pi_{d,V}(w,z)}{\partial w} = 0$ and moreover

$$\frac{\partial^2 \Pi_{d,V}(w,z)}{\partial w^2} = -b < 0$$

we get (16). Additionally for $w < w_d(z)$ the derivative $\frac{\partial \Pi_{d,V}(w,z)}{\partial w} > 0$ and for $w > w_d(z)$ the derivative $\frac{\partial \Pi_{d,V}(w,z)}{\partial w} < 0$, which proves the uniqueness of optimal solution $w_d(z)$.

Now we derive z_d which maximizes $\Pi_{d,V}(p_d(z),z)$. We have

$$\frac{\partial \Pi_{d,V}(p_d(z),z)}{\partial z} = (1-F(z))\frac{\mu(z)+a-bc\alpha}{4b} - c(1-\alpha)\left[1-\frac{3}{4}(1-F(z))\right].$$

From $\frac{\partial \Pi_{d,V}(p_d(z),z)}{\partial z} = 0$ we get that z_d is determined by (17). Now we prove the uniqueness. We have

$$\frac{\partial^2 \Pi_{d,V}(p_d(z),z)}{\partial z^2} = -f(z)\frac{w_d(z)+c(1-\alpha)}{2} + \frac{dw_d(z)}{dz}\frac{1-F(z)}{2}. \tag{18}$$

Moreover $\frac{d^2 w_d(z)}{dz^2} = \frac{f(z)}{2b} < 0$ and $\frac{dw_d(z)}{dz}$ is decreasing . This implies that $\frac{dw_d(z)}{dz}$ attains its maximum at $z = A$

$$\frac{dw_d(z)}{dz} = \frac{1-F(z)}{2b}|_A = \frac{1}{2b}$$

Hence

$$\frac{\partial^2 \Pi_{d,V}(p_d(z),z)}{\partial z^2} \leq -f(z)(w_d(z)+c(1-\alpha)) + \frac{dw_d(z)}{dz}\frac{1-F(z)}{2b} \leq 0$$

if

$$\frac{bf(z)(w_d(z)+c(1-\alpha))}{1-F(z)} \geq \frac{1}{2}.$$

This condition is equivalent to the statement that lost sales rate elasticity $\kappa(w_d(z)+c(1-\alpha), z) \geq \frac{1}{2}$ with price $w_d(z)+c(1-\alpha)$. $\square$

Putting the formula for w_d to (15) we have

$$p_d(w,z) = \frac{3\mu(z)+3a+bc}{4b}. \tag{19}$$

Note the the optimal selling price in VMCI program given by (19) is independent on the share of channel α. Furthermore the consignment price w_d is increasing function of z and concave which is the same as p_d.

Finally, we state that in VMCI the total channel decentralized profit is equal to (14) with p_d and z_d given by (19) and (17), respectively.

4 Conclusions and Future Research

In this paper we use additive demand to investigate a game-theoretic model of consignment contract. We continue the study of [Ru & Wang, 2010] where multiplicative demand form are used. For broader view on this subject we use additive demand which causes much more computational difficulties.

In consignment contract under the demand uncertainty the upstream vendor offers a consignment price charged to the downstream retailer for each unit of the product sold and then the retailer sets a retail price for selling product to the market. After realizing all uncertainties the retailer pays the vendor based on the net selling units. There are two inventory regimes labeled as RMCI and VMCI are studied.

In [Ru & Wang, 2010] the precise solutions are given for RMCI and VMCI programs in case of exponential multiplicative demand function. In our paper we give closed-form solutions for VMCI program with linear additive demand form. It should be noted that VMCI has been started to be a trend in last years. We obtain also some precise results for RMCI program. Rest of them can be given in extended version of this paper as a future research. Moreover, since formulas for equilibrium solutions are mathematically complicated so it is quite hard to examine analytically their sensitivity on parameter's changes. Because of that one can do it numerically for some kind of distribution of random part of demand function. Usually the first choice is normal or uniform distribution. Based on numerical example it is worth to compare the results for RMCI and VMCI programs. The above topics seem to be an interesting subject for future considerations.

References

[Hu et al., 2015] Hu, W., Chen, D., Yu, H. (2015). Benefit and risk analysis of consignment contracts. *Annals of Operations Research, doi:10.1007/s10479-015-1919-0.*

[Hu et al., 2014] Hu, W., Li, Y., Govindan, K. (2014). The impact of consumer returns policies on consignment contracts with inventory control. *European Journal of Operational Research, 233*, 398-407.

[Kocabiyikoglu & Popescu, 2011] Kocabiyikoglu, A., & Popescu, I. (2011). An elasticity approach to the newsvendor with price-sensitivity demand. *Operations Research, 59(2)*, 301-312.

[Lee & Chu, 2005] Lee, C.C., & Chu, W.H.J. (2005). Who should control inventory in a supply chain? *European Journal of Operational Research, 164*, 158-172.

[Petruzzi & Dada, 1999] Petruzzi, N. C., Dada, M. (1999). Pricing and newsvendor problem: a review with extensions.*Operations Research, 47(2)*, 183-194.

[Ru & Wang, 2010] Ru, J., & Wang, Y. (2010). Consignment contracting: Who should control inventory in the supply chain? *European Journal of Operational Research, 201*, 760-769.

[Rubio-Herrero et al., 2015] Rubio-Herrero, J., Baykal-Gursoy, M., Jaskiewicz, A. (2015). A price setting newsvendor problem under mean-veriance criteria. *European Journal of Operational Research, 247*, 575-587.

[Wang et al., 2004] Wang, Y., Jiang, L., Shen, Z. (2004). Channel performance under consignment contract with revenue sharing. *Management Science, 50(1)*, 34-47.

[Wu et al., 2016] Wu, Z., Chen, D., Yu, H. (2016). Coordination of a supply chain with consumer return under vendor-managed consignment inventory and stochastic demand. *International Journal of General Systems, 45(5)*, 502-516.

[Zhao & Atkins, 2008] Zhao, X., & Atkins, D. (2008). Newsvendors under price and inventory competition. *Manufacturing and Service Operations Management, 10(3)*, 539-546.