Control of Chaos in Strongly Nonlinear Dynamic Systems

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Abstract

We consider the dynamic processes models based on strongly nonlinear systems of ordinary differential equations and various stable and unstable solutions of such systems. The behavior and evolution of solutions are studied when the system parameters and external influences change. As a tool for controlling the behavior of solutions is considered a system parameters change. Particular attention is paid to the parameter of dissipation as a tool for controlling chaos. For simple systems, a controlled transition from stable solutions to chaos and back is considered. The generalization of the solutions chaotic behavior for complex systems and the criteria for optimal level of chaos in such systems are discussed.

1 Introduction

For complex systems such as Economics, social relations, communication, chaos is a natural form of behavior. For these systems mathematical models allow qualitatively, but not quantitatively evaluate the system behavior. In real complex systems, both a regular ordered behavior and chaotic states of systems are observed.

Complete elimination of chaos in a complex system, as a rule, leads to system degradation. For the economy this is stagnation, lack of competition, initiatives and new prospective. For social relations, this is the lack of democracy. For information transfer systems, this is regulated and metered information and misinformation, information vacuum, the lack of the possibility of obtaining information from alternative sources.

The lack of complex systems regulation leads to an excessive level of chaos. This usually has negative consequences for the behavior of the real complex system. In the economy, this manifests in the form of chaos, mass evasion from paying taxes, economic crises, excessive volatility of indicators. Excessive chaos in social relations leads to anarchy, the emergence of local groups and leaders who do not submit to control from the center, conflicts between individual groups and the center. The complete lack of regulation in information transmission systems leads to the use of such systems for the exchange of undesirable information. You can observe numerous sad examples of the implementation of such scenarios in real complex systems.

Therefore we can conclude that for complex systems there is an optimal level of chaos. In the economy, not suppressed initiative, originate new business, but complied with the rules of economic behavior. In a society, a

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democratic regime is implemented without elements of anarchy and dictatorship. The information transmission systems are used for free exchange with the blocking of unwanted information. The blocking can be realized by censorship or ignoring of undesirable information by the majority of participants. Unfortunately now only the first steps are being taken to develop mathematical models that allow us to quantify the level of chaos in complex systems.

For simple nonlinear dynamical systems, the situation is radically opposite. For such systems, the absence of the solution chaotic component does not lead to the system degradation, but is one of the main natural forms of behavior. At the same time, for simple nonlinear dynamical systems, chaos is also a natural form of solutions. For such systems, an interactive algorithm for the qualitative and quantitative study of regular and chaotic solutions has been developed [Petrov, 2015] , which allows investigating both types of solutions and transitions between them. For this, periodic solutions are calculated and their stability is analyzed. The periodicity of the solution is determined by the discrepancy between the initial and final points of the trajectory on the solution period. To construct periodic solutions, the concept of minimizing this discrepancy is used. This strategy of finding periodic solutions for essentially nonlinear systems of ordinary differential equations can be interpreted as the search for a global minimum of the distance between the beginning and end of the solution trajectory in one period. Based on the numerical results of essentially nonlinear systems of ordinary differential equations study, we will consider the tools for controlling chaos in simple nonlinear dynamical systems and discuss qualitative analogies with complex nonlinear dynamical systems.

2 Statement of the Problem

As a model problem, consider the essentially nonlinear generalized Duffing equation

$$\ddot{x} + (1-P)\omega_0^2 x(t) + b_1(\dot{x}(t)) + \gamma_1(x(t)) = W\cos(\omega t), \tag{1}$$

where $b_1(\dot{x}(t))$ is, in the general case, a nonlinear dissipative function, $\gamma_1(x(t))$ - function that determines the nonlinear characteristics of the system, $W\cos(\omega t)$ - a periodic external influence. It is only required of functions $b_1(\dot{x}(t))$ and $\gamma_1(x(t))$ that the methods of the Cauchy problem numerical solution for equation (1) used in the algorithm for global optimization of periodic discrepancy allow to construct a solution on one period with a given accuracy. In the classic version of the problem statement $b_1(\dot{x}(t)) = b\dot{x}(t)$ and $\gamma_1(x(t)) = \gamma x^3(t)$. Without limiting the generality, we will take the same form $b_1(\dot{x}(t))$ and $\gamma_1(x(t))$.

We note a fundamentally different character of the system for $P > 1$ and $P < 1$. For the equations of both these classes, the regular solutions, chaotic behavior and strange attractor are known [Kovacic et al., 2011] , [Holmes, 1979] and etc.

For $P > 1$ the simplest prototype corresponding to this equation is the forced oscillations of the ball in the profile with two symmetrical holes (Figure 1). The same equation describes the oscillations of some mechanical systems with a jump, in particular, forced transverse vibrations of the beam, which lost its static stability [Holmes, 1979], [Petrov, 2013]. This is oscillator with negative linear stiffness. The phase space has three singular points. The source of chaos is the presence two (or several in the general case) of the minimum potential energy points and the possibility a solutions jumping to another potential well.

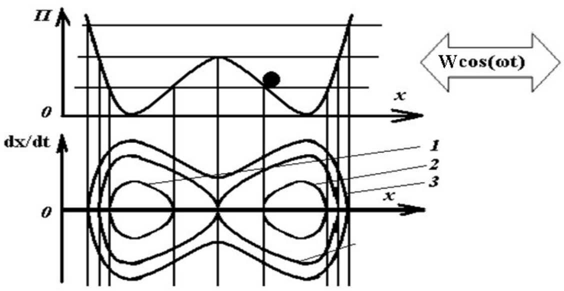

Figure 1: The simplest prototype object and phase trajectories of equation (1) solutions with $P > 1$

For $P < 1$ the classical version of the Duffing equation is realized. The amplitude-frequency characteristic has three lines (Figure 2). The upper and lower lines correspond to stable solutions, the middle line – to unstable solutions. The source of chaos for such system is the possibility of solutions hopping from one line of the amplitude-frequency characteristic to another.

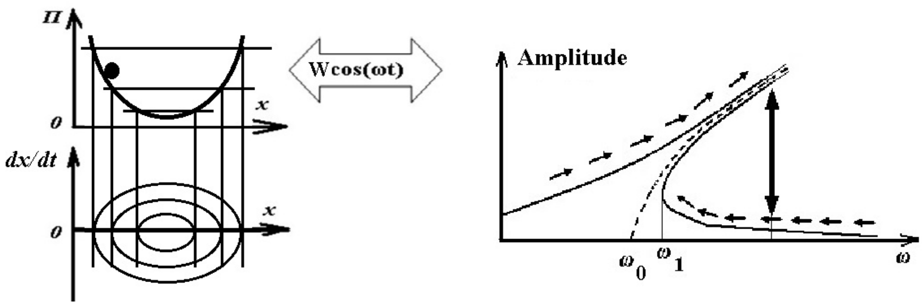

Figure 2: The simplest prototype object, phase trajectories of solutions and amplitude-frequency characteristic of equation (1) with $P < 1$

Based on physical considerations, we will apply the dissipative characteristics of the system, determined by the dissipation coefficient b, the amplitude W and the frequency ω of the periodic external influence as parameters for controlling chaos. The interactive computational numerical-analytical algorithm for ordinary differential equations systems periodic solutions constructing and analyzing the stability [Petrov, 2015] was used for research.

3 Results of Numerical Experiments

As the basic model, we adopted equation (1) with the following values of the parameters [Holmes, 1979] : $P = 2$, $\omega_0^2 = 10$, $gamma = 100$, W -variable parameter (the base value $W = 1.5$), b -variable parameter (the base value $b = 1$), ω -variable parameter (the base value $\omega = 3.76$). At basic values of the parameters, the solution is chaotic (Figure 3).

3.1 Chaos Control Via Amplitude of External Periodic Influence

Study the oscillations dependence on the external influences amplitude allow to detect the range of values at which the solution has the character of a strange attractor [Holmes, 1979]. It is additionally established that the transition from stable periodic solutions to chaos at the boundaries of the strange attractor zone is realized according to the scenario of period doubling bifurcations [Feigenbaum, 1983] , [Petrov, 2010] (Figure 4).

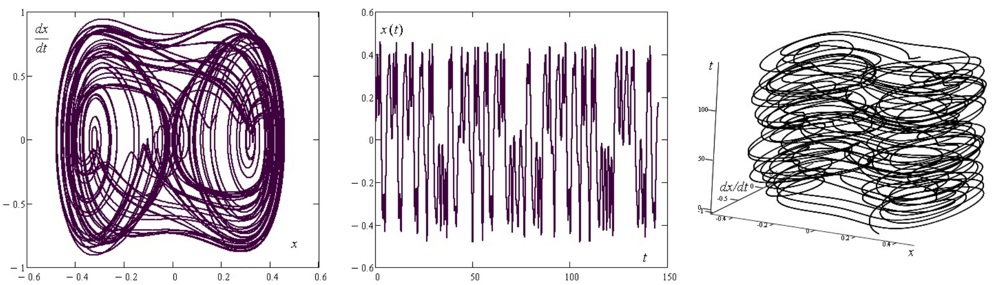

Figure 3: Chaos in the simple system (1)with $P = 2$, $\omega_0^2 = 10$, $\omega = 3.76$, $b = 1$, $gamma = 100$, $W = 1.5$: Phase portrait, the solution and three-dimensional representation

Outside the zone of the strange attractor, stable solutions correspond to external influences – for small W oscillations around one of the two singular points (Figure 1, curves 1 and 2), for large values W oscillations of a larger amplitude, spanning both singular points (curve 3).Chaos is realized when all of these solutions become unstable (Figure 3).

Thus, it can be concluded that the amplitude of external influence W is one of the tools for controlling the system behavior in terms the chaos presence. For large amplitude values W , the external influence determines

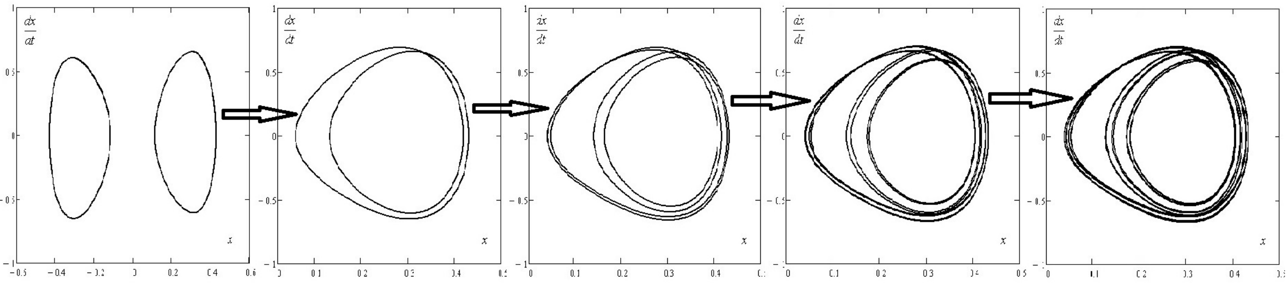

Figure 4: Period doubling bifurcation of the solutions of equation (1) with $P = 2$, $\omega_0^2 = 10$, $\omega = 3.76$, $b = 1$,$gamma = 100$, $W = 0.7$; $W = 0.886$; $W = 0.926$; $W = 0.936$; $W = 0.9379$

the nature of the solution, and chaos does not appear. For small values W, it is not enough of external energy to realize a chaotic state.

We note that within the zone of the strange attractor on the parameter W, anomalous existence of stable periodic ($6\pi/\omega$ and $10\pi/\omega$) solutions detected [Holmes, 1979] , [Petrov, 2010] (Figure 5).

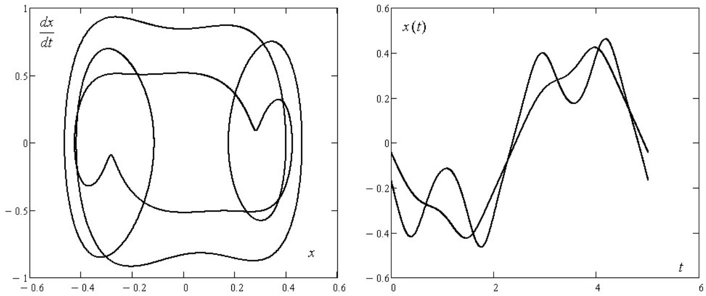

Figure 5: Phase trajectories of pairwise symmetric stable solutions of equation (1) with period $6\pi/\omega$ at $P = 2$, $\omega_0^2 = 10$, $\gamma = 100$, $b = 1$, $\omega = 3.76$, $W = 1.7$

We note the symmetry in the pair stable solutions given in Figure 5.

3.2 Chaos Control Via Dissipation

The influence of dissipation on chaos appears in a different form. In the zone where there was originally the strange attractor, as the dissipation coefficient b increases, stable solutions begin to be determined. At first they have a rather complicated form (Figure 6), but with a further increase in the parameter b the solutions become simpler (Figure 7 , curve 1). In this case, the simplest solutions already exist (Figure 7 , curve 2), but they are still unstable. With further growth of dissipation coefficient b , only the simplest harmonic solutions that are similar to (Figure 7, curve 2) remain stable. Large dissipation leads to the elimination of deterministic chaos. A decrease in the dissipation coefficient b leads to the appearance of stable solutions with a large amplitude (Figure 1 , curve 3). In this case, solutions with small amplitude (Figure 1, curve 1 and 2) are unstable. Thus, both increase and decrease of dissipation is a means for suppressing chaos.

3.3 Chaos Control Via Frequency of External Periodic Influence

The dependence of the solutions form of equation (1) on the external periodic influence frequency ω has not so obvious a physical meaning as on the amplitude W and dissipation b. At low frequencies of the external influence ($0.3 < \omega < 0.6$), stable and unstable solutions have high-frequency components (Figure 8). At high frequencies of external influence, other behavior of solutions is observed (Figure 9).

3.4 Chaos Control in Multidimensional Systems

In addition to the chaos control tools discussed above for dynamical systems with several degrees of freedom the chaotic behavior of solutions may depend on the interaction of oscillations in different degrees of freedom

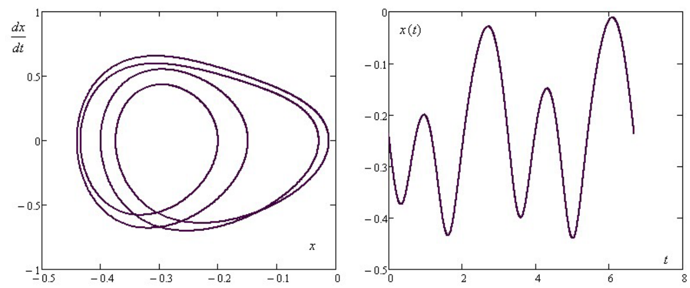

Figure 6: Phase trajectory and one $8\pi/\omega$ period of the equation (1) stable solution with at $P = 2$, $\omega_0^2 = 10$, $\gamma = 100$, $b = 1.81$, $\omega = 3.76$, $W = 1.5$

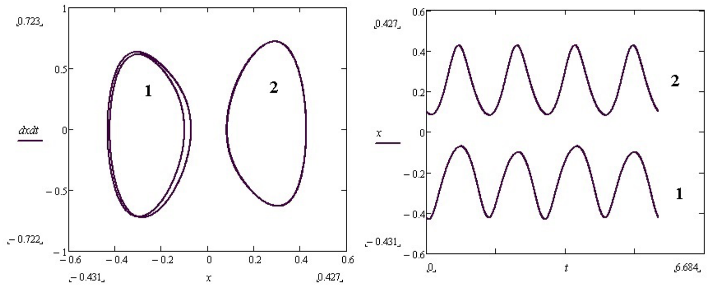

Figure 7: Phase trajectories and solutions of equation (1) with $P = 2$, $\omega_0^2 = 10$, $\gamma = 100$, $b = 2$, $\omega = 3.76$, $W = 1.5$ 1 - stable 2 - unstable

[Petrov, 2013]. It is possible to suppress chaos in one of the oscillation forms due to the influence of regular oscillations in another form. This is a generalization of the linear effect of oscillations dynamic damping on nonlinear multidimensional oscillations with possible chaos.

4 Optimization of Simple and Complex Dynamic Systems in Terms of Chaos

To formulate the problem of optimizing a dynamic system by the chaos criterion, it is necessary to determine the quantitative expression of the objective function. The measure of chaos, including for complex systems, is the fractal dimension of the solution [Peters, 1996] . After determining the chaos control techniques can be formulate the optimization problem of the chaos level with these control tools.

For simple systems the objective function can be formulated on the basis of technical requirements -a minimum value of the solution fractal dimension (when necessary the ordered behavior of the system), a maximum of this dimension (when required the disordered chaotic behavior of the system).

For complex systems the chaos level must be within a certain range, corresponding to optimal or acceptable conditions for the system functioning and development. Qualitatively, these constraints were discussed above. For definiteness, the middle of the range of optimal (acceptable) values of this dimension can be chosen as the value of the objective function (solution fractal dimension) for a complex dynamical system.

For the optimization problem of nonlinear dynamic system by the level of chaos we have formulated the objective function and tools for chaos control. This approach allows us to use optimization methods to find the values of the parameters of a nonlinear dynamic system that provide the target level of chaos in the system.

5 Qualitative Generalizations

The results of our numerical experiments on control the solutions behavior of essentially nonlinear dynamical systems demonstrate the possibility of realizing the transition to chaos and back using the management tools listed above. It is possible as predictable system response to control actions (e.g., bifurcation of period doubling), and unpredictable behavior (for example, the existence of stable solutions in a small region in the area of the strange attractor).

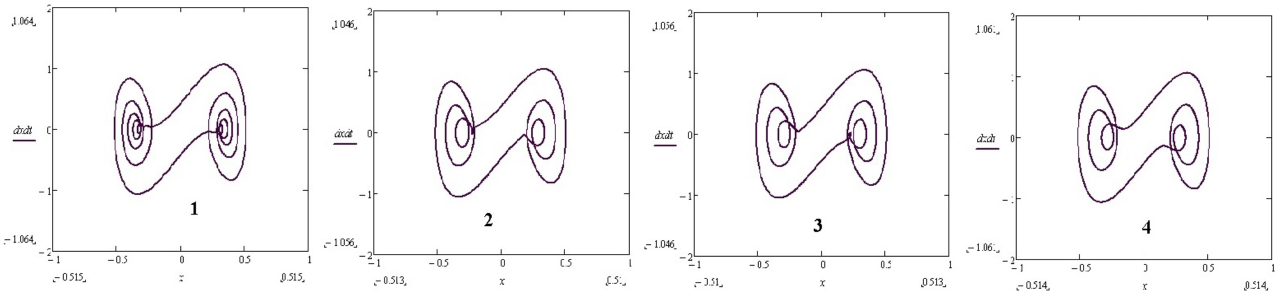

Figure 8: Solutions of the equation(1) with $P = 2$, $\omega_0^2 = 10$, $\gamma = 100$, $b = 1$, $W = 1.5$ at low frequencies of external influence 1 - $\omega = 0.3$ (stable), 2 and 3 - $\omega = 0.5$ (symmetric, unstable) , 4- $\omega = 0.6$ (stable)

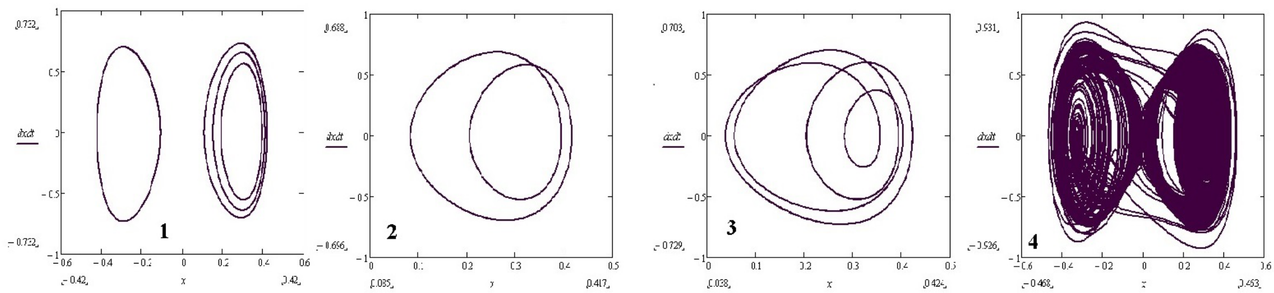

Figure 9: Solutions of the equation(1) with $P = 2$, $\omega_0^2 = 10$, $\gamma = 100$, $b = 1$, $W = 1.5$ at high frequencies of external influence 1 - $\omega = 5.5; 5.0$ (stable), 1 - $\omega = 4.5$ (symmetric, unstable),2 - $\omega = 4.7$ (stable) , 3 - $\omega = 4.6$ (stable) ,4- $\omega = 4.5$ (chaos)

For relatively simple essentially nonlinear dynamical systems, the intensity of external influence, dissipative characteristics, other parameters of the system can serve as an instrument for controlling the chaotic behavior of solutions. At the same time, the results of the investigation are of a quantitative nature with a known error in the calculations and are confirmed by experiments. We note that chaos in simple nonlinear systems is realized when there are several singular points in the phase space. From the point of view of complex systems it is a choice between several approximately equivalent states, realizations, and possibilities.

To qualitatively evaluate the tools for controlling chaos in complex systems, let us assume the following analogies:

Dissipation in the economy – transfer of an economic asset to another form – taxes, losses, suppression of economic initiative, etc.

Dissipation in social relations – suppression of initiative, intimidation, over-regulation, self-restraint, etc. Note that the standard of living can manifest itself as an analog of dissipation – at an extremely low standard of living, the social system tends to go into chaos, with high living standards, chaos in society generally does not manifest itself.

Dissipation in information – censorship and self-censorship, silencing, etc.

External influence in the economy – adding external economic asset.

External influence in social relations – economic influence, agitation, propaganda, fashion, etc.

External influence in information systems – massive information and disinformation with possible blocking of alternative sources of information.

The above analogies are of a debatable nature, but there are already approaches and mathematical models [Milovanov, 2001] and others, which in time will be able to confirm or disprove these assumptions.

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