Evaluation of Reinvestment Risk for Bond Portfolios

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Abstract. Assessing reinvestment risk in the Russian bond market using computer modeling is extremely important for increasing the efficiency of investing decision-making. The general profit method which is widely used to estimate the reinvestment risk is based on the highly controversial assumption of the reliability of the investor's forecast of the interest rates time structure. We propose the reinvestment risk assessing algorithm which, firstly, interprets the investor's bond market forecast as the basic scenario to form the bond portfolio; secondly, uses the overall time horizon of investment rather than the remaining time to the bond's maturity date; thirdly, calculates the reinvestment risk as the difference between the expected yield of the bond portfolio based on the predicted structure of interest rates and the expected yield of the bond portfolio ignoring the predicted changes in the interest rates structure.

Keywords: investing decision making, risk assessment, bond market, reinvestment risk modeling

1 Introduction and literature review

Bonds are popular instruments in the Russian financial market. According to the Bank of Russia, there three main reasons for the growing demand for bonds in the recent years: evolving market of derivatives, the money market's stability, and the development of investment portfolios of the non-state pension funds and life insurance companies [1, p. 41]. Accordingly, pension funds and insurance companies are interested in development of computer modelling of bonds management in order to determine the expected return, risk, and other important investment characteristics [6]. It is particularly important to develop correct tools for assessing risk associated with bond investment decision making that could be easily incorporated into computer programs used by the financial institutions.

Currently, in the modern specialized scientific literature, there are many different points of view regarding the allocation of risks that a bond holder might face. Some authors, e.g., F.J. Fabozzi, stressed out the reinvestment risk among other risks (the author also uses the term "risk associated with reinvestment").

Fabozzi understands the reinvestment risk as the reinvestment rates' fluctuations caused by changes in interest rates that make the investor to take a risk of reinvesting intermediate cash flow at lower interest rates [3, p. 22]. R. L. Moy and R. Terregrossa points out that the realized compound yield (RCY) is the return that investors earn when all coupons are reinvested; and RCY differs from yield to maturity in a way similar to how modified internal rate of return differs from internal rate of return [5, p. 11–12].

Assessing the reinvestment risk in the bond market is important because, for medium- and long-term investments, the income from reinvesting interim bond payments is the main one for the investor (it may well be about 60-70% of the general revenues or more). For this reason, when forming a bond portfolio, an investor should be able to assess the admissibility (or acceptability) of the facing reinvestment risk, which is reasonable to express in percent per annum. Accordingly, the above suggests that the reinvestment risk should be interpreted as a possible decrease in the return on investment as a result of unfavorable changes in the terms of investment and, above all, the interest rates level.

It is noteworthy that this risk, objectively inherent in any bond market, remains practically unexplored in the academic literature including the publications of both foreign authors and Russian specialists (general problems of the bond market were investigated by G. Markovits, M. Miller, F. Modigliani, Yu. Fama , L.P. Hansen, R. Schiller, etc.). In the overwhelming majority of cases, foreign authors prefer to use the general profit method which is based on the highly controversial assumption that the investor already has a clear (predictive) view of the future structure of interest rates; while Russian researchers are limited to mentioning the existence of the reinvestment risk and a brief description of its essence, without attempting to assess this risk quantitatively and, moreover, not offering specific formalized methods for its evaluation.

For example, V.F. Karbovsky and O.S. Alekseeva, while considering the problems of risk assessment in the bond market, recognize that "there are various risks when investing in bonds." However, the authors note that "now the majority of … risks are difficult to assess due to market underdevelopment; they are less likely and less dangerous as compared to the risk of default". Therefore the authors considered it reasonable to narrow "the concept of risk to the risk of borrower default, failure to fulfill the borrower's obligations", and, respectively, to reduce the task of "the bond loan risk assessment … to the assessment of the financial position of the enterprise that will service the bond loan" [4, p. 10].

In our opinion, the current situation is due to the simultaneous impact of the following factors:

– the orientation of Russian investors on the domestic bond market, where short- and medium-term bond issues are undoubtedly dominant; for such bond issues, the reinvestment risk is relatively small (the main reason for this is the time structure of the cash flow for such bonds);
– the prevailing investment preferences of investors, who often prefer super-short-term bonds, treating them as a kind of analogue of cash in the invest-

ment portfolio structure (for such bonds in the portfolio up to their maturity, the risk of reinvestment is not significant);
- the widespread thesis that the yield on bonds is formed from two main sources: regular coupon payments and changes in market prices of bonds; in fact, this approach ignores the income from reinvestment of coupon payments, which, with a sufficiently long investment horizon, may well exceed (in its absolute value) all other revenues received by the bond holder;
- a long time trend of the high level of interest rates in the Russian bond market (at least in comparison with the level of interest rates prevailing in the biggest local markets of public debt), and that diminishes the degree to which the reinvestment risk affects the overall rate of return of bond portfolios.

Nevertheless, over the past few years (in fact, since 2015), the risk of reinvestment is very relevant for the domestic bond market, and that is particularly testified by the dynamics of the Bank of Russia's key rate for this period: since 2015, its value is systematically decreasing from 17.00% (as of January 1, 2015) to 9.00% (as of July 1, 2017) per annum [2]. For the domestic bond market, this means that at a comparable risk level (taking into account the market liquidity of specific bond issues) each subsequent coupon was reinvested either at a similar or lower interest rate. Hence, almost all participants of the national bond market have faced the risk of reinvestment (except for holders of discount bonds, which, however, are poorly represented on the domestic market).

2 The general profit method: Discussion

In the foreign studies, the traditional method of estimating the reinvestment risk is the method of general profit (also known as the method of profit on a time interval or time horizon). This method uses an assumption that, already at the time of buying bonds, the investor predicts future rates of reinvesting the interim earnings for the whole period of holding the bonds (regardless of whether the investor holds the bonds to maturity or not). In essence, the general profit method is the present value calculation of the of the cash flows from holding the bond using a certain "expected" time structure of interest rates, and the calculation of expected return on investments which can significantly differ from the current yield to maturity.

From the point of view of the mathematical apparatus used, the general profit method is a modification of the traditional algorithm for calculating the rate of return on any investment, including one in the bond market, which postulates the constant interest rate.

In other words, the classical model of estimating the internal rate of return on a bond proposes the following:

$$P = \sum_{i=1}^{n} \frac{C_i}{(1 + r_B)^i}, \tag{1}$$

where P – bond's market price; C – intermediate earnings on bonds, which include both coupon payments and partial redemption of the face value[1]; r_B – bond's internal rate of return. In the framework of the general profit method, the canonical formula takes the following form:

$$P_E = \sum_{i=1}^{n} \frac{C_i}{(1 + r_{BE_i})^i}, \tag{2}$$

where P_E – the expected market price of the bond with the expected time structure of interest rates;

r_{BE_i} – bond's expected internal rate of return for each time interval.

This modification presumes a possible (but not compulsory) inequality of bond's rates of return in each period, i.e., discounting of interim earnings on a bond (interim cash flows) can be carried out at different rates for each time interval (for example, a coupon period or a calendar year).

The logic of applying the general profit method is to choose a bond issue that, with other things being equal, has the highest return with the predicted structure of time interest rates. Accordingly, for the case when interest rates are down, this method allows one to choose a bond issue that has the greatest "immunity" to the reinvestment risk (generating the least losses for the investor), and for the case with the interest rates up – to choose a bond issue which allows to obtain the maximum yield in the case of interest rate growth (generating to the investor the highest return on reinvestment of intermediate income).

The main advantage of the general profit method is its universality: it is applicable to any bond issue (regardless of the structure of its revenues and the revenues' distribution over time) and to any bond portfolio (within the framework of the method's concept, the bond portfolio may well be represented as a set of interim revenues on individual bond issues).

Essential shortcomings of the general profit method should be recognized as follows: firstly, it is based on the assumption of the reliability of the investor's forecast of the interest rates time structure, and secondly, it does not pay attention to the very method of forecasting the future interest rates.

3 Algorithm for assessing the reinvestment risk in the bond market

Obviously, the assumption about the reliability of the forecast of the interest rates time structure is very conditional (the structure can not be determined in advance). Hence, it is more logical to interpret the forecast of the investor in the bond market, not as the "truth in the last resort", but as some basic scenario which is considered to be more likely and on the basis of which the bond portfolio is being formed.

As for the forecasting of the expected rate of return for the bond issue and, accordingly, for the entire bond portfolio as a whole, we propose to use the overall

[1] This indicator can also be interpreted as a cash flow on a bond.

time horizon of investment (M_P) rather than the remaining time to the bond's maturity date (M_B). The fact is that in practice, especially in the Russian bond market, it is almost impossible to form such a portfolio that the maturing periods for all bonds combined in the portfolio are absolutely equivalent to the expected period of investment, i.e. $M_{B_i} = M_P$. In any case, there will be deviations, with the optimal situation being that $M_{B_i} \leqslant M_P$, since in this case the price risk is minimized (the bonds are redeemed at their face value). Accordingly, for each bond issue with $M_{B_i} \leqslant M_P$ a total investment period can be represented as:

$$M_P = M_{B_i} + M_{P-B_i}. \tag{3}$$

From the investor's position, the return on investment for the periods M_{B_i} and M_{P-B_i} will differ. For the period M_{B_i} it will be equal to the bond issue's expected rate of return (r_{E_i}), which we propose to determine by the formula:

$$r_{BE_i} = r_{EG_i} + \overline{S_B}, \tag{4}$$

where r_{EG_i} – expected rate of return of government bonds of comparable duration (for each time interval);

$\overline{S_B}$ – average credit spread which reflects the credit quality of the bonds' issuer.

Using this algorithm to predict the expected yield on a particular bond issue allows us to formalize this process: the expected yield of government bonds of comparable duration can be obtained on the basis of the zero-coupon yield curve for government bonds (the values of the zero-coupon bond yield curve, expressed as percentages per annum, are daily published on the official website of the Bank of Russia [2]), and the size of the average credit spread can be calculated as:

$$\overline{S_B} = \overline{r_{BF_i}} - \overline{r_{FG_i}}, \tag{5}$$

where $\overline{r_{BF_i}}$ – average realized yield of the certain bond issue;

$\overline{r_{FG_i}}$ – average realized yield of government bonds of comparable duration.

The estimation of the expected yield for the period M_{P-B_i} (i.e., after the redemption of the bond issue that was part of the bond portfolio) is determined by the investment strategy selected by the investor:

– for the most conservative strategy which implies the withdrawal of money from the market immediately after each bond issue's maturity:

$$r_{P-B_{E_i}} = 0; \tag{6}$$

– if money is placed in the money market right after the redemption of the bond's face value then:

$$r_{P-B_{E_i}} = r_{EG_i}; \tag{7}$$

– if the amount received upon the redemption of the bond issue is invested into a new bond issue that will mature before the end of the total investment period then:

$$r_{P-B_{E_i}} = r_{EG_i} + \overline{S_{B_j}}, \tag{8}$$

where S_{B_j} – the average credit spread for the new bond issue and $M_j \leqslant M_{P-B_i}$.

Accordingly, the expected yield for a particular bond issue (more precisely, for the amount invested in this issue) ($\overline{r_{BE_i}}$) for the entire period M_P will be:

$$\overline{r_{BE_i}} = r_{BE_i} \times w_{BE_i} + r_{P-B_{E_i}} \times w_{P-B_{E_i}}, \tag{9}$$

where w_{BE_i} – the relative duration (share) of the total investment period, during which the investor was holding the bond issue;

$w_{P-B_{E_i}}$ – the relative duration (share) of the total investment period after the bond issue maturity.

It should be noted that this algorithm for determining the expected yield for the whole period M_P will be relevant also if the investor first acquires one bond issue and holds it until maturity; then the investor spends the money received from the bonds' redemption for a new bond issue with maturity date before time period M_P, etc.

Thus, the expected yield of the entire bond portfolio, taking into account the predicted structure of interest rates ($\overline{r_{PE}}$) for the entire investment period, will be determined by the formula:

$$\overline{r_{PE}} = \sum_{i=1}^{n} \overline{r_{BE_i}} \times f_i, \tag{10}$$

where f_i – the share of a certain bond issue in the investment portfolio.

Wherein, the bond portfolio's expected rate of return without taking into account the change in the structure of interest rates ($\overline{r_{PF}}$) (i.e., based on the classical assumption of reinvestment of interim bonds payments at a rate equal to the current yield to maturity of the bond issue) will be calculated as:

$$\overline{r_{PF}} = \sum_{i=1}^{n} r_{BF_i} \times f_i. \tag{11}$$

When using the method of total profit, we consider it necessary to compare the indicators $\overline{r_{PF}}$ and $\overline{r_{PE}}$:

- if $\overline{r_{PF}} \leqslant \overline{r_{PE}}$, the reinvestment risk is not realized for the investment portfolio; on the contrary, the investor who hold the bonds until maturity (assuming there are no defaults on the securities) gets additional profit from reinvesting the interim proceeds due to the growth of interest rates, i.e., the interim payments are reinvested under the higher interest rate;
- if $\overline{r_{PF}} > \overline{r_{PE}}$, the reinvestment risk is realized for the portfolio, i.e., the realized yield of the bond portfolio will be lower due to reinvestment of interim proceeds at lower interest rates.

We consider it reasonable to measure the risk of reinvestment (R_R) quantitatively as follows:

$$R_r = \overline{r_{PE}} - \overline{r_{PF}}. \tag{12}$$

The negative value of the indicator (R_R) reflects the amount of profit lost by the investor as a result of unfavorable changes in the structure of interest rates (in percentages per annum). The positive value of the above indicator should be interpreted as the additional income received by the investor holding the bond portfolio under a positive change in interest rates (a similar situation can arise if the average level of interest rates increases after the formation of the bond portfolio, i.e., the interim income is reinvested while more favorable terms).

Thus, in general, the proposed algorithm for assessing the reinvestment risk in the bond market can be represented as a sequence of the following stages:

– to calculate expected yield of bonds before maturity (r_{BE_i}) based on the expected yield of government bonds of comparable duration and average credit spread;
– to calculate expected bond yield for the entire investment period ($\overline{r_{BE_i}}$), depending on the investment strategy chosen by the investor;
– to calculate expected yield of the entire bond portfolio taking into account the predicted structure of interest rates ($\overline{r_{PE}}$);
– to calculate expected yield of the bond portfolio without taking into account changes in the structure of interest rates ($\overline{r_{PF}}$);
– to calculate of the reinvestment risk (R_R) and to make the decision if it is reasonable to take the risk.

4 Empirical example

For testing the proposed algorithm of the reinvestment risk estimation, we use the following bond portfolio (see Table 1).

Table 1. Bond Portfolio (formed on 10.07.2017 for the period until 10.07.2018)

	Bond issue	Maturity date	Current yield, in %	The share of the bond issue in the portfolio, in %
1	Chuvashia-10	07.06.2018	8.62	30.0
2	MarEl2014	07.07.2018	8.94	35.1
3	KrasnYarKr8	08.07.2018	9.29	34.9

The presented portfolio is short-term (the maturity of all the bonds is less than one year), while none of the bond's maturity period matches the total investment period of 365 days (for the Chuvashia-10 bond issue the period is equal to 332 days, for MarEl2014 – 362 days, and for KrasnYarKr8 – 363 days), and the acceptable reinvestment risk (the risk that the investor is able / ready to take) is 0.20% per annum

The projected yield on government bonds with maturity in one year is 8.10% per annum, and the nearest current rate of return of government bonds (maturity in one month) being 8.15% per annum, i.e., the investor is forecasting a minor decrease in the level of interest rates.

Table 2 shows the results of the calculation of the expected bond yield before maturity (r_{BE_i}) for each bond issue within the investment portfolio on the basis of the curve of zero-coupon yield of government bonds as at the moment of the bond portfolio formation.

Table 2. Calculation of expected yield of bonds before maturity, in %

	Bond issue	r_{EG_i}	$\overline{S_B}$	r_{BE_i}
1	Chuvashia-10	8.10	0.61	8.71
2	MarEl2014	8.10	0.74	8.84
3	KrasnYarKr8	8.10	0.93	9.03

Calculation of the expected yield of bonds for the entire investment period is carried out on the assumption that, after the bonds are redeemed, the money received is placed in the money market, i.e., for the Chuvashia-10 bonds issue, the period of "not holding bonds" is 33 days; for MarEl2014 – 3 days; for KrasnYarKr8 – 2 days (see Table 3).

Table 3. Calculation of expected bond yield for the entire investment period, in %

	Bond issue	$r_{BE_i} \times w_{BE_i}$	$r_{P-B_{E_i}} \times w_{P-B_{E_i}}$	$\overline{r_{BE_i}}$
1	Chuvashia-10	7.95	0.71	8.66
2	MarEl2014	8.77	0.07	8.83
3	KrasnYarKr8	8.98	0.04	9.02

The expected yield of the entire bond portfolio based on the predicted structure of interest rates ($\overline{r_{PE}}$) is 8.85% per annum, while the expected yield of the bond portfolio without taking into account the change in the structure of interest rates ($\overline{r_{PF}}$) is 8.97%.

The risk of reinvestment is −0.12% (8.85%–8.97%), that is, because of the change in the time structure of interest rates (in the decreasing direction), the holder of the bond portfolio will "lose" 0.12% per annum. Hence, given the accepted value of the reinvestment risk (−0.20% per annum), the investor should decide that forming such an investment portfolio is admissible (acceptable).

5 Conclusion

The reinvestment risk assessment algorithm developed for bond portfolios held until maturity allows us to take into account two factors that are extremely important for the bond market participants: first, the predicted change in the interest rates time structure, and, second, the need to invest funds from repaying bonds at interest rates lower than the yield of bonds originally included in the portfolio. To significantly increase the efficiency of investing decision-making, the proposed algorithm can be embedded in a computer decision support system designed for selection bonds by the risk criteria. The computer support is very important for technical simplification and improvement of assessment reliability.

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