# Predications, fast and slow

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## Abstract

Notions of predication based on extensional and intensional subsumption, as described by Woods, are related to Kahneman's systems of thinking fast and slow. Path-based reasoning with links is applied to predication over not only individuals but (following Carlson) kinds and stages/time. Predications, fast and slow, are formulated in monadic second-order logic over strings, analyzed in Goguen and Burstall's institutions.

## 1 Introduction

In his ACL Lifetime Achievement Award lecture, William Woods contrasts two competing traditions in Knowledge Representation for Natural Language Understanding

1. logical reasoning, which is rigorous and formal, but often counterintuitive, and which has algorithms that match expressions, substitute values for variables, and invoke rules, and
2. associative networks, which are structured and intuitive, but typically informal; however, they support efficient algorithms that follow paths through links to draw conclusions

[Woods, 2010, page 625]. These traditions offer different perspectives on concepts and *subsumption* $\sqsubseteq$ between concepts. The custom in logic is to interpret a concept $C$ as a set $[\![C]\!]$ of $C$-instances, called its *extension* (under $[\![\cdot]\!]$), with $C$ subsumed by a more general concept $C'$, $C \sqsubseteq C'$, when every $C$-instance is a $C'$-instance

$$C \sqsubseteq C' \text{ under } [\![\cdot]\!] \iff [\![C]\!] \subseteq [\![C']\!] \qquad (1)$$

[Baader *et al.*, 2003]. Rejecting the reduction of a concept to its extension, Woods advocates a notion of *intension* that "is as much a psychological issue as a logical issue" [Woods, 2007, page 83]. Woods steps away from arbitrary instances given by some interpretation $[\![\cdot]\!]$ to a carefully crafted conceptual taxonomy, for an intensional subsumption quicker to compute than the extensional notion specified by (1).[1] Woods hypothesizes the bulk of reasoning is of the quick "recognize and react" variety, implicating the efficient algorithms of associative networks, as opposed to the ponderous mechanisms of logic. In a similar vein, the psychologist Daniel Kahneman argues that thinking operates largely under a fast system, breakdowns in which trigger a second slower system that would otherwise lie dormant [Kahneman, 2011]. Woods and Kahneman independently suggest commonsense reasoning is often easy but at times hard.

The fast/slow intensional/extensional contrasts are taken up together below, starting in §2 with *Formal Concept Analysis* [Ganter and Wille, 1999] for a tight conceptual pairing of extension and intension (shortened there to extent and intent). Well-known complications in predication point to a loosening of that pairing, paving paths in §3 to logical systems in §4 that Goguen and Burstall [1992] call *institutions*. Institutions based on monadic second-order logic over strings are defined, providing a uniform path-based account of predication over kinds and stages in the sense of Carlson [1977], with monadic second-order variables ranging over paths, and many models reduced to finite ones, amenable to finite-state methods. The bias *What you see is all there is* (WYSIATI) from Kahneman [2011] is formulated as a *satisfaction condition* characteristic of institutions. Reasoning may slow down due to adjustments within an institution or perhaps worse, changes of institutions. The former explores unknowns that are known to an institution, while the latter arises from unknown unknowns (to borrow Donald Rumsfeld's words).

## 2 Formal Concept Analysis and partiality

To bring out a notion of intension buried in purely extensional accounts, FCA defines a *context* to be a triple $\langle \mathcal{O}, \mathcal{A}, H \rangle$ consisting of

(i) a set $\mathcal{O}$ of *objects* $d, d', \ldots$,

(ii) a set $\mathcal{A}$ of *attributes* $a, a', \ldots$, and

(iii) a binary relation $H \subseteq \mathcal{O} \times \mathcal{A}$ specifying the attributes $a$ in $\mathcal{A}$ that an object $d$ in $\mathcal{O}$ *h*as (so $dHa$ can be read: object $d$ *has* attribute $a$).

Now fix a context $\langle \mathcal{O}, \mathcal{A}, H \rangle$. The *extent of* a set $A \subseteq \mathcal{A}$ of attributes is the set

$$A_H \ := \ \{d \in \mathcal{O} \mid (\forall a \in A)\ dHa\}$$

[1] Analyzing intension as a function from indices (or points of reference) to extensions [Carnap, 1947; Montague, 1974] arguably only compounds (1), multiplying notions of instance by indices.

of objects that have every attribute in $A$, while the *intent of* a set $D \subseteq \mathcal{O}$ of objects is the set

$$D^H \ := \ \{a \in \mathcal{A} \mid (\forall d \in D)\ dHa\}$$

of attributes that every object in $D$ has. The notions of extent and intent constitute an antitone Galois connection inasmuch as for every $D \subseteq \mathcal{O}$ and $A \subseteq \mathcal{A}$,

$$D \subseteq A_H \iff A \subseteq D^H. \tag{2}$$

The inclusions $\subseteq$ in (2) are strengthened into equalities in defining a concept to be a pair $(D, A)$ such that $D = A_H$ and $A = D^H$. Equivalently, a concept is a subset $A$ of $\mathcal{A}$ such that $A = (A_H)^H$ (replacing $D$ by $A_H$, to focus on attributes). Reducing extension to extent, extensional subsumption (as described in §1) between concepts $A$ and $A'$ is just inclusion of extents

$$A \sqsubseteq_H A' \iff A_H \subseteq A'_H$$

which, in view of (2), is the converse of inclusion of intents

$$A \sqsubseteq_H A' \iff A' \subseteq A.$$

We can turn an attribute or object $x \in \mathcal{A} \cup \mathcal{O}$ unambiguously into a concept

$$\text{concept}(x) \ := \ \begin{cases} \{x\}^H & \text{if } x \in \mathcal{O} \\ (\{x\}_H)^H & \text{if } x \in \mathcal{A} \end{cases}$$

assuming $\mathcal{A} \cap \mathcal{O} = \emptyset$, and then for $x' \in \mathcal{A} \cup \mathcal{O}$, define

$$x \ \textit{is\_a}^H\ x' \iff \text{concept}(x) \sqsubseteq_H \text{concept}(x')$$

so that for $d \in \mathcal{O}$ and $a \in \mathcal{A}$,

$$d \ \textit{is\_a}^H\ a \iff dHa$$

and for $d' \in \mathcal{O}$,

$$d \ \textit{is\_a}^H\ d' \iff \{d'\}^H \subseteq \{d\}^H.$$

The $\Longrightarrow$-half of the last biconditional is the inference rule

$$\frac{d'Ha \qquad d \ \textit{is\_a}^H\ d'}{dHa} \tag{3}$$

for property inheritance. Shortcomings of (3) are longstanding concerns in Knowledge Representation, a common example being that (a1) to (a3) leave out penguins.

(a1) *bird H fly* (i.e., birds fly)

(a2) *tweety is_a*$^H$ *bird* (i.e., Tweety is a bird)

(a3) *tweety H fly* (i.e., Tweety flies)

To accommodate exceptions, let us replace *is_a*$^H$ by a binary relation IS (left unspecified for the moment) and, following Reiter [1980], add an assumption M(dHa) to (3) for

$$\frac{d'Ha \qquad d \ \text{IS}\ d' \qquad \mathsf{M}(dHa)}{dHa} \tag{4}$$

with M pronounced "it is consistent to assume" (left implicit in the :-notation for justifications in *Default Logic*). Under (4), (a3) follows from (a1) and *tweety* IS *bird* only with M(a3), which fails when there is information to the contrary of (a3). Now, assuming $a$ comes with a contrary attribute $\overline{a} \in \mathcal{A}$, a plausible candidate for $\mathsf{M}(dHa)$ is the negation $\neg(dH\overline{a})$ expressing the absence of contrary information. Were it the case that

$$dH\overline{a} \iff \neg(dHa) \tag{5}$$

(4) would be vacuous, as $\neg(dH\overline{a})$ reduces to the conclusion $dHa$ of (4). But (5) cannot hold in a context $\langle \mathcal{O}, \mathcal{A}, H\rangle$ with a bird that flies and another that doesn't,

$$\text{neither } \textit{bird}\ H\ \overline{\textit{fly}} \text{ nor } \textit{bird}\ H\ \textit{fly}$$

exposing a sense in which $H$ is partial, and pushing us beyond $H$ if, as commonsense demands, we are to make anything of (4).

## 3 Varieties of predication and causal paths

What could *birds fly* mean when plainly some birds don't? The linguist Greg Carlson contrasts two views of generic sentences, an inductive approach based on observed instances, and a "rules and regulations" view emphasizing not so much their "episodic instances but rather the causal forces behind those instances" [Carlson, 1995, page 225]. The latter causal approach (which Carlson favors) is broadly in line with the proposal made in Steedman [2005] that causality and goal-directed action lie at the heart of temporal semantics. A simple example of goal-directed action is expressed by the predication *die*(*tweety*), which overturns the temporal proposition *alive*(*tweety*). The proposition *alive*(*tweety*) is inertial inasmuch as it persists in the absence of a force overturning it — or to bring out the similarity with (4),

$$\frac{\textit{alive}(\textit{tweety})@t \qquad t \ \text{S}\ t' \qquad \neg\text{opp}(\textit{alive}(\textit{tweety})@t)}{\textit{alive}(\textit{tweety})@t'} \tag{6}$$

where

(i) *alive*(*tweety*)@$t$ says "*alive*(*tweety*) holds at time $t$"

(ii) $t$ S $t'$ says "$t$ is succeeded (temporally) by $t'$"

(iii) opp($\psi$) says "some force opposes $\psi$"

whence $\neg$opp($\psi$) says "no force opposes $\psi$."

For rigour, let us associate with every attribute $a \in \mathcal{A}$, a distinct unary relation symbol $P_a$, allowing us to encode an FCA-relation $H \subseteq \mathcal{O} \times \mathcal{A}$ as a $\{P_a\}_{a \in \mathcal{A}}$-model $[\![\cdot]\!]$ over the universe/domain $\mathcal{O}$ interpreting $P_a$ as the subset

$$[\![P_a]\!] \ = \ \{d \in \mathcal{O} \mid dHa\}$$

of $\mathcal{O}$, so that

$$d \in [\![P_a]\!] \iff dHa$$

for all $d \in \mathcal{O}$. But the point of the relation symbols $P_a$ is not to recreate a particular relation $H \subseteq \mathcal{O} \times \mathcal{A}$ but to move beyond such relations, as described by (4) and (6) above. And central to these moves are the relations IS in (4) and S in (6), for which we introduce a binary relation symbol $S$. We can then reformulate (4) as the sentence

$$\text{ih}[a] \ := \ \forall x \forall y((P_a(y) \wedge ySx \wedge \neg P_{\overline{a}}(x)) \supset P_a(x))$$

saying $a$ is inherited through $S$ in the absence of information $\overline{a}$ to the contrary (assuming $\overline{a} \in \mathcal{A}$). $S$ inverts IS in ih[$a$]

for uniformity with S in (6), which we generalize to $a$ as the inertial requirement

$$\text{ir}[a] \;:=\; \forall x \forall y((P_a(y) \wedge ySx \wedge \neg P_{o(a)}(y)) \supset P_a(x))$$

assuming an attribute $o(a) \in \mathcal{A}$ for an $a$-opposing force. An obvious choice for $o(\textit{alive}(\textit{tweety}))$ is *die*(*tweety*), which describes an event that terminates the state *alive*(*tweety*). The distinction between events and states is critical to temporal semantics [Kamp and Reyle, 1993; Allen and Ferguson, 1994], with ir$[a]$ suited as a requirement for $a$ equal to *alive*(*tweety*) but not for *die*(*tweety*). Further evidence for the significance of the event/state divide is the $\exists/\forall$-contrast illustrated by (a4) and (a5).

(a4) Tweety flew in his first year.

(a5) Tweety was flightless his first five weeks.

While *some* flight by Tweety in his first year is enough to make (a4) true, (a5) specifies flightlessness at *every* instant of his first five weeks. That is, (a4) claims of an interval that a certain event happens within it, while (a5) claims of an interval that a certain state holds at each instant in it. Inasmuch as states holds at instants, while events happen over intervals, an analogy can be drawn with individuals/kinds

$$\frac{\text{state}}{\text{event}} \approx \frac{\text{instant}}{\text{interval}} \approx \frac{\text{individual}}{\text{kind}}.$$

Carlson [1977] describes *kind-level* predicates such as *widespread* that range over kinds, but not over individuals such as Tweety, the failure of (a7) being, as (a8) sugggests, a sortal error.

(a6) Birds are widespread.

(a7) $^?$Tweety is widespread.

(a8) $^?$A typical bird is widespread.

As presupposition failures, sortal errors are commonly held to apply equally to negations, expressed in (5) and ih$[a]$ by $\overline{a}$, and not to be confused with $\neg$. To block the leap from (a6) to (a7) on the basis of ih(*widespread*), we must refrain from requiring ih$[a]$ of kind-level predicates, just as we refrain from requiring ir$[a]$ of predicates describing events (for which $o(a)$ may not even be defined). That said, the present paper focuses on attributes $a$ suited to ih$[a]$ or to ir$[a]$. Let us agree to call the former *inheritable*, and the latter *inertial*.

Ensuring an inheritable attribute $a$ satisfies ih$[a]$ or an inertial attribute satisfies ir$[a]$ may call for some repairs on $[\![P_a]\!]$. The repair for ih$[a]$ can be described through a fresh attribute $a_+$ with $P_{a_+}$ given by two rules

$$\frac{P_a(x)}{P_{a_+}(x)} \qquad\qquad \frac{P_{a_+}(y) \qquad ySx \qquad \neg P_{\overline{a}}(x)}{P_{a_+}(x)} \tag{7}$$

extending $P_a$ so that $P_{a_+}(x)$ can be read

> there is an $S$-path to $x$ avoiding $P_{\overline{a}}$ from some $y$ such that $P_a(y)$.

That is, $P_{a_+}(x)$ can be formulated as

$$\exists y(P_a(y) \wedge yS_a^* x)$$

where $S_a^*$ expresses the reflexive transitive closure of the restriction $S_a$ of $S$ to pairs $(x_1, x_2)$ such that not $P_{\overline{a}}(x_2)$

$$S_a \;:=\; \lambda x_1 \lambda x_2(x_1 S x_2 \wedge \neg P_{\overline{a}}(x_2)).$$

Now, whether or not ih$[a]$ is true at a model $[\![\cdot]\!]$, the aforementioned assumptions about $P_{a_+}$ make ih$[a_+]$ true at $[\![\cdot]\!]$ with $\overline{a_+} = \overline{a}$. Put another way, ih$[a]$ is satisfied by a model $[\![\cdot]\!]_+$ identical to $[\![\cdot]\!]$ except possibly at $P_a$, where

$$[\![P_a]\!]_+ \;:=\; [\![P_{a_+}]\!].$$

As for the inertial sentence ir$[a]$, we introduce, in place of $a_+$ for ih$[a]$, an attribute $a_o$ with $P_{a_o}$ given by

$$\frac{P_a(x)}{P_{a_o}(x)} \qquad\qquad \frac{P_{a_o}(y) \qquad ySx \qquad \neg P_{o(a)}(y)}{P_{a_o}(x)} \tag{8}$$

so that $P_{a_o}(x)$ says

> an $S_a^o$-path to $x$ exists from some $y$ such that $P_a(y)$

where $S_a^o$ is the counterpart in ir$[a]$ of $S_a$

$$S_a^o \;:=\; \lambda x_1 \lambda x_2(x_1 S x_2 \wedge \neg P_{o(a)}(x_1)).$$

Then $[\![\cdot]\!]_o$ satisfies ir$[a]$ where $[\![P_a]\!]_o := [\![P_{a_o}]\!]$. $S_a$-paths and $S_a^o$-paths alike are causal, implementing the inheritance and inertial laws ih$[a]$ and ir$[a]$, respectively.

Looking back at the previous section, it is doubtful FCA-intents are what Woods has in mind by intension (as marvellous as Galois connections are). Accordingly, we trade subsumption $\sqsubseteq_H$ between intents of FCA-concepts for a predicate symbol $S$ that has many interpretations, in line with Woods contention that intension is as much psychological as logical. Insofar as no single interpretation of $S$ on its own will do, it is not unreasonable to keep these interpretations as simple as possible. Suppose, for example, $[\![S]\!]$ were given by some finite string $d_1 \cdots d_n$ of $n$ distinct objects $d_i$

$$[\![S]\!] = \{(d_i, d_{i+1}) \mid 1 \leq i \leq n\}$$

and $\mathcal{A}$ were finite. Then the construction of $[\![\cdot]\!]_+$ to ensure ih$[a]$ for $a$ in some set $\mathit{Ih} \subseteq \mathcal{A}$ of inheritable attributes can be implemented by a finite-state transducer transforming a string $A_1 \cdots A_n$ with

$$A_i = \{a \in \mathcal{A} \mid d_i \in [\![P_a]\!]\}$$

to $A'_1 \cdots A'_n$ with

$$A'_i = \{a \in \mathcal{A} \mid d_i \in [\![P_a]\!]_+\}.$$

The transducer has subsets of $\mathit{Ih}$ as states $q$, the empty set as the initial state, all states final, and transitions

$$q \overset{A:A'}{\to} A' \cap \mathit{Ih} \;\text{ where }\; A' := A \cup \{a \in q \mid \overline{a} \notin A\}$$

so that the current state consists of the inheritable attributes in the last set $A'$ of attributes returned. Moreover, for $a \in \mathit{Ih}$ with $\overline{a} \in \mathit{Ih}$ and $\overline{\overline{a}} = a$, we have

$$[\![P_a]\!]_+ \cap [\![P_{\overline{a}}]\!]_+ = \emptyset \iff [\![P_a]\!] \cap [\![P_{\overline{a}}]\!] = \emptyset$$

allowing us to set $\overline{a_+}$ equal to $\overline{a}_+$. As for ir$[a]$ where $a$ belongs to some set $\mathit{Ir} \subseteq \mathcal{A}$ of inertial attributes, we can build a finite-state transducer taking $A_1 \cdots A_n$ to $A_1^o \cdots A_n^o$ where

$$A_i^o = \{a \in \mathcal{A} \mid d_i \in [\![P_a]\!]_o\}$$

using subsets of $\mathit{Ir}$ as states $q$, and transitions $q \overset{A:A^o}{\to} q'$ where $A^o := A \cup q$ and $q' := \{a \in A^o \cap \mathit{Ir} \mid o(a) \notin A\}$.

## 4 Predications in flux: an MSO institution

We step in this section from an FCA context $\langle \mathcal{O}, \mathcal{A}, H\rangle$ up to an *institution* $\langle Sign, sen, Mod, \models\rangle$ in the sense of [Goguen and Burstall, 1992], where contexts can be varied systematically via *signatures*. The definition of an institution uses a modicum of category theory, and is, on first exposure, a mouthful:

(i) *Sign* is a category, with objects $\Sigma$ called *signatures*

(ii) *sen* is a functor from *Sign* to the category of sets and functions, with elements of the set $sen(\Sigma)$ called $\Sigma$-*sentences*

(iii) *Mod* is a contravariant functor from *Sign* to the category of (small) categories and functors, with $Mod(\Sigma)$-objects called $\Sigma$-*models*

(iv) $\models$ is an indexed family $\{\models_\Sigma\}_{\Sigma\in|Sign|}$ of binary relations

$$\models_\Sigma \;\subseteq\; |Mod(\Sigma)| \times sen(\Sigma)$$

between $\Sigma$-models and $\Sigma$-sentences, indexed by signatures $\Sigma$ such that for every *Sign*-morphism $\Sigma \overset{\sigma}{\to} \Sigma'$, $\Sigma'$-model $M$ and $\Sigma$-sentence $\varphi$,

$$M' \models_{\Sigma'} sen(\sigma)(\varphi) \iff Mod(\sigma)(M') \models_\Sigma \varphi. \quad (9)$$

A context $\langle \mathcal{O}, \mathcal{A}, H\rangle$ can be packaged as an institution with exactly one signature $\Sigma$, trivializing the category-theoretic requirements above so that the functor *sen* amounts to

$$sen(\Sigma) = \mathcal{A},$$

the functor *Mod* to

$$Mod(\Sigma) = \mathcal{O} \quad \text{(understood as a discrete category)}$$

and the indexed family $\models$ to

$$\models_\Sigma = H.$$

A far more interesting institution associated with $\langle \mathcal{O}, \mathcal{A}, H\rangle$ pieces together finite fragments of $\langle \mathcal{O}, \mathcal{A}, H\rangle$ on which the $S$-paths behind $a_+$ and $a_o$ in §3 tread. Some preliminary definitions help make this precise. Given a finite subset $A$ of $\mathcal{A}$,

(i) the fragment $\text{MSO}_A$ of *Monadic Second-Order logic over strings* [e.g., Libkin, 2004] is given by a binary predicate symbol $S$ (expressing succession) and a unary predicate symbol $P_a$ for each $a \in A$

(ii) an $MSO_A$-*model* $\langle [n], \mathsf{S}_n, \{[\![P_a]\!]\}_{a\in A}\rangle$ with domain

$$[n] := \{1, \ldots, n\},$$

interpreting $S$ as the successor (plus 1) relation

$$\mathsf{S}_n := \{(i, i+1) \mid 1 \leq i < n\}$$

on $[n]$, and each $P_a$ as a subset $[\![P_a]\!]$ of $[n]$ can be identified with the string $A_1 \cdots A_n$ of subsets

$$A_i \;:=\; \{a \in A \mid i \in [\![P_a]\!]\}$$

of $A$ consisting of attributes $a$ in $A$ with $i$ in the interpretation $[\![P_a]\!]$ of the unary predicate for $a$.[2]

[2] Proceeding from the string $A_1 \cdots A_n$, the equations

$$[\![P_a]\!] = \{i \in [n] \mid a \in A_i\} \qquad (a \in A)$$

take us back to $\langle [n], \mathsf{S}_n, \{[\![P_a]\!]\}_{a\in A}\rangle$, justifying the identification of $\text{MSO}_A$-models with strings of subsets of $A$.

$\text{MSO}_A$-sentences and $\text{MSO}_A$-satisfaction $\models^A$ are defined as usual in classical predicate logic so that, for example,

$$A_1 \cdots A_n \models^A \exists x(P_a(x) \wedge \forall y \neg ySx) \iff a \in A_1$$

for all strings $A_1 \cdots A_n$ of subsets of $A$. A fundamental result is the Büchi-Elgot-Trakhtenbrot theorem [Libkin, 2004, page 124]: the set

$$\text{MSO}_A(\varphi) \;:=\; \{s \in (2^A)^* \mid s \models^A \varphi\}$$

of strings satisfying an $\text{MSO}_A$-sentence $\varphi$ is regular, and conversely, every regular language over the alphabet $2^A$ (of subsets of $A$) can be obtained this way from some $\text{MSO}_A$-sentence $\varphi$. The use of the powerset $2^A$ as the alphabet of strings constituting $\text{MSO}_A$-models departs from the custom of $A$ in statements of the Büchi-Elgot-Trakhtenbrot theorem but is crucial when forming an institution where $A$ varies [Fernando, 2016].[3]

Let $\mathbb{I}_\mathcal{A}$ be the institution where *Sign* is the set $Fin(\mathcal{A})$ of finite subsets $A$ of $\mathcal{A}$ partially ordered by $\subseteq$ (for a category), and for every $A \in Fin(\mathcal{A})$,

(i) an $A$-sentence is an $\text{MSO}_A$-sentence, an $A$-model is a string over the alphabet $2^A$ and $\models_A$ is $\text{MSO}_A$-satisfaction

(ii) for $A \subseteq A' \in Fin(\mathcal{A})$, $sen(A, A')$ is the inclusion of $\text{MSO}_A$-sentences in $\text{MSO}_{A'}$-sentences, while $Mod(A', A)$ intersects a string $A'_1 \cdots A'_n \in (2^{A'})^*$ componentwise with $A$

$$Mod(A', A)(A'_1 \cdots A'_n) \;:=\; (A'_1 \cap A) \cdots (A'_n \cap A).$$

Next, we bring in an FCA-context $\langle \mathcal{O}, \mathcal{A}, H\rangle$ to define for every $d \in \mathcal{O}$ and $A \in Fin(\mathcal{A})$, the set

$$A[d] \;:=\; \{a \in A \mid dHa\}$$

of attributes in $A$ that $d$ has, which we then use to map a string $d_1 \cdots d_n \in \mathcal{O}^*$ to the string

$$A[d_1 \cdots d_n] \;:=\; A[d_1] \cdots A[d_n]$$

over the alphabet $2^A$. Let $\mathbb{I}(\mathcal{O}, \mathcal{A}, H)$ be the institution with the same signature category and sentence functor as $\mathbb{I}_\mathcal{A}$, but with a string $d_1 \cdots d_n \in \mathcal{O}^n$ as an $A$-model that $\models_A$-satisfies an $\text{MSO}_A$-sentence $\varphi$ precisely if $A[d_1 \cdots d_n] \models^A \varphi$ (in $\text{MSO}_A$). That is, $\mathbb{I}(\mathcal{O}, \mathcal{A}, H)$ picks out the $A$-models $A_1 \cdots A_n$ in $I_\mathcal{A}$ given by strings $d_1 \cdots d_n$ and $H$

$$A_i = \{a \in A \mid d_i H a\}.$$

But $H$ might be transformed to a different FCA-context, as we saw in the transformations in §3 of $[\![\cdot]\!]$ to $[\![\cdot]\!]_+$ and $[\![\cdot]\!]_o$ to satisfy the MSO-sentences

$$\text{ih}[a] \;:=\; \forall x \forall y((P_a(y) \wedge ySx \wedge \neg P_{\overline{a}}(x)) \supset P_a(x))$$

saying $a$ is inherited unless there is information $\overline{a}$ to the contrary, and

$$\text{ir}[a] \;:=\; \forall x \forall y((P_a(y) \wedge ySx \wedge \neg P_{o(a)}(x)) \supset P_a(x))$$

saying $a$ persists unless opposed by some force $o(a)$. The requirement $\text{ih}[a]$ is suited to attributes $a$ belonging to a set *Ih* such that

[3] The reason is that the structure $\langle [n], \mathsf{S}_n, [\![P_a]\!]_{a\in\mathcal{A}}\rangle$ encoded by a string $A_1 \cdots A_n$ over $2^\mathcal{A}$ has $A$-reduct $\langle [n], \mathsf{S}_n, [\![P_a]\!]_{a\in A}\rangle$, encoded by the string $(A_1 \cap A) \cdots (A_n \cap A)$ over $2^A$.

(Inh) $Ih \subseteq \mathcal{A}$ and for each $a \in Ih$, $\overline{a} \in Ih$ and $\overline{\overline{a}} = a$ with

$$ySx \text{ saying: } \quad y \text{ subsumes } x.$$

Inertia ir$[a]$ is suited to attributes $a$ belonging to a set $Ir$ such that

(Inr) $Ir \subseteq \mathcal{A}$ and for each $a \in Ir$, $o(a) \in \mathcal{A} - Ir$, $\overline{a} \in Ir$ and $\overline{\overline{a}} = a$ with

$$ySx \text{ saying: } \quad y \text{ is followed by } x.$$

We can use a monadic second-order variable $X$ to pick out a path, defining the MSO$_{\{a,\overline{a}\}}$-formula $path_a(X)$

$$path_a(X) \ := \ \forall x(X(x) \supset (P_a(x) \vee \exists y(X(y) \wedge ySx \wedge \neg P_{\overline{a}}(x))))$$

by inverting the rules (7) for $P_{a_+}$ with $X$ in place of $P_{a_+}$. Then ih$[a_+]$ follows from the MSO$_{\{a,\overline{a}\}}$-formula

$$\forall x(P_{a_+}(x) \equiv \ \exists X(X(x) \wedge path_a(X))) \qquad (10)$$

reducing $P_{a_+}(x)$ to the possibility of putting $x$ in a set $X$ such that $path_a(X)$. Similarly, for inertia ir$[a]$ and $a_o$, we invert the rules (8) for $P_{a_o}$ with $X$ in place of $P_{a_o}$ for

$$path^o_a(X) \ := \ \forall x(X(x) \supset (P_a(x) \vee \exists y(X(y) \wedge ySx \wedge \neg P_{o(a)}(x))))$$

to derive ir$[a_o]$ from the reduction

$$\forall x(P_{a_o}(x) \equiv \ \exists X(X(x) \wedge path^o_a(X))) \qquad (11)$$

of $P_{a_o}$ to $path^o_a$. The finiteness of $[\![S]\!]$ (or more specifically $[\![S]\!]$-chains) is crucial to push through the arguments for ih$[a_+]$ and ir$[a_o]$ above (extracting $S_a$- and $S^o_a$-paths that reach $P_a$ from $path_a(X)$ and $path^o_a(X)$).

For $a$ in $Ih$ or $Ir$, it is natural to assume $\overline{a}$ does not co-occur with $a$

$$\text{nc}[a] \ := \ \forall x \neg(P_a(x) \wedge P_{\overline{a}}(x))$$

banning contradictions in $[\![P_a]\!] \cap [\![P_{\overline{a}}]\!]$. The reduction (10) above yields the equivalence of nc$[a]$ and nc$[a_+]$

$$\text{nc}[a] \equiv \text{nc}[a_+]$$

provided $P_a$ does not discriminate between $S$-predecessors of the same object

$$\forall x \forall y \forall y'(ySx \wedge y'Sx \wedge P_a(y) \supset P_a(y'))$$

which follows from the uniqueness of $S$-predecessors

$$\forall x \forall y \forall y'(ySx \wedge y'Sx \supset y = y').$$

Otherwise, nc$[a]$ may hold while nc$[a_+]$ fails because an object in $[\![P_a]\!]$ has the same $S$-successor as another in $[\![P_{\overline{a}}]\!]$

$$d_1[\![S]\!]d \text{ and } d_2[\![S]\!]d \text{ with } d_1 \in [\![P_a]\!] \text{ and } d_2 \in [\![P_{\overline{a}}]\!].$$

The same applies to inertia ir$[a]$ and $a_0$; nc$[a_0]$ follows from nc$[a]$ and the reduction (11), under the interpretation above of $S$ as

$$\mathsf{S}_n := \{(i, i+1) \mid 1 \leq i < n\}$$

for some positive integer $n$. As observed at the end of §3, this interpretation of $S$ allows us to build a finite-state transducer that computes $[\![P_{a_+}]\!]$ for finitely many inheritable $a$ in one pass from $A_1$ down to $A_n$. Similarly for inertia and $a_o$.

The institution $\mathbb{I}_{\mathcal{A}}$ accommodates different FCA-contexts with attribute set $\mathcal{A}$, providing accounts at bounded granularities $A \in Fin(\mathcal{A})$ that can be refined or coarsened, as required. Assuming $\mathcal{A}$ is infinite, we can always extend $A$ to a larger finite $\mathcal{A}$-subset $A' \supseteq A$, leading, after repeated extensions, to infinite models at the limit (or, as detailed below, inverse limit). How smoothly can these extensions be carried out? As a partial answer, clause (iv) in the definition above of an institution provides the biconditional

$$M' \models_{\Sigma'} sen(\sigma)(\varphi) \iff Mod(\sigma)(M') \models_{\Sigma} \varphi \qquad (9)$$

known as the *Satisfaction condition* [Goguen and Burstall, 1992]. In the present institution $\mathbb{I}_{\mathcal{A}}$, (9) unwinds to

$$A'_1 \cdots A'_n \models_{A'} \varphi \iff (A'_1 \cap A) \cdots (A'_n \cap A) \models_A \varphi$$

whenever $A \subseteq A' \in Fin(\mathcal{A})$, and for every MSO$_A$-sentence $\varphi$ and string $A'_1 \cdots A'_n$ of subsets of $A'$. In particular, for a fixed sentence $\varphi$, we can set $A$ to the *vocabulary of* $\varphi$, $voc(\varphi)$, defined to be the set of all attributes mentioned in $\varphi$ (making $voc(\varphi)$ the smallest subset $A$ of $\mathcal{A}$ for which $\varphi$ is an MSO$_A$-sentence). Satisfaction of $\varphi$ by a string $A'_1 \cdots A'_n$ of subsets of $A' \in Fin(\mathcal{A})$ with $voc(\varphi) \subseteq A'$ then reduces to satisfaction by the string

$$(A'_1 \cap voc(\varphi)) \cdots (A'_n \cap voc(\varphi))$$

of subsets of $voc(\varphi)$. In other words, the attributes we see in $\varphi$ can — as far as the issue of satisfying $\varphi$ (or not) is concerned — be assumed to be all there is, in accordance with the bias *What you see is all there is* (WYSIATI) from Kahneman [2011]. Of course, "what you see" in $\varphi$ depends on what we put in $\varphi$. And while, for example, ih$[a]$ mentions only the attributes $a$ and $\overline{a}$, many more attributes may appear once we try to spell out what information to the contrary is buried in $\overline{a}$ (not to mention $a$). In the simplest case, $\overline{a}$ might be false (i.e., $\forall x \neg P_{\overline{a}}(x)$) turning ih$[a]$ into

$$\forall x \forall y((P_a(y) \wedge ySx) \supset P_a(x))$$

but the point of ih$[a]$ is to deal with more interesting cases. As Reiter [1980] notes, the list of birds that do not fly is open-ended, expressed below as $\cdots$ in the *non*-well-formed MSO-formula (12).

$$P_{bird}(x) \supset \ (P_{\overline{fly}}(x) \equiv (P_{penguin}(x) \vee P_{ostrich}(x) \vee \cdots)) \qquad (12)$$

The same open-endedness applies to the forces overturning an inertial attribute $a$, glossed over by the non-inertial attribute $o(a)$ in ir$[a]$. In practice, simplifying assumptions are adopted (dropping, for example, $\cdots$ in (12)) to facilitate reasoning. In situations where these assumptions fail, reasoning may break down. As Shanahan [2016] points out,

> Because it sometimes jumps to premature conclusions, bounded rationality is logically flawed, but no more so than human thinking.

Elaborating on the attributes $a, \overline{a}, o(a)$ is an art in managing flaws, and recovering from missteps (slowing reasoning down).

Apart from the attributes in $\mathcal{A}$, there is also the binary predicate symbol $S$, interpreted as the successor relation $\mathsf{S}_n$ on $\{1, 2, \ldots, n\}$ for some integer $n > 0$ for either subsumption (Inh) or temporal precedence (Inr). Widespread views that time branches and is infinite, and similar claims about conceptual taxonomies raise the question:

*is it not problematic to assume that*
$[\![S]\!]$ *is* $\mathsf{S}_n$ *for some integer* $n > 0$*?*

It is, if an $A$-model that interprets $S$ as a finite successor relation is asked to carry on its own the burden of representing an infinite branching structure. But once we adjust our sights from a single context $\langle \mathcal{O}, \mathcal{A}, H\rangle$ for defining concepts to a multitude of satisfaction relations $\models_A$ capturing finite fragments of a multitude of contexts, the problem arguably dissolves. If time branches, it is because there is more than one $A$-model in *Mod*$(A)$ describing a branch up to $A$. And if time is infinite, it is because no single signature $A$ can represent the totality of signatures in *Sign* $=$ *Fin*$(\mathcal{A})$.

And exactly how might infinite branching structures emerge from these finite approximations? Very briefly, through an inverse limit construction linking vocabulary ($A \in$ *Fin*$(\mathcal{A})$) with ontology (*Mod*$(A)$). More precisely, we work with strings $A_1 \cdots A_n$ of subsets of $\mathcal{A}$ that are *stutterless* in that

$$A_i \neq A_{i+1} \text{ for all } 1 \leq i < n$$

(the intuition being that a stutter is some substring $A_i A_{i+1}$ such that $A_i = A_{i+1}$). Clearly, a string $s$ of subsets of $\mathcal{A}$ is stutterless iff $s = \mathit{bc}(s)$ where the *block compression* $\mathit{bc}(s)$ *of* $s$ is defined by

$$\mathit{bc}(s) := \begin{cases} s & \text{if length}(s) \leq 1 \\ \mathit{bc}(As') & \text{if } s = AAs' \\ A\,\mathit{bc}(A's') & \text{if } s = AA's' \text{ where } A \neq A' \end{cases}$$

(implementing a form of the Aristotelian claim, no time without change). Next, for any $A \in$ *Fin*$(\mathcal{A})$, let $\mathit{bc}_A : (2^{\mathcal{A}})^* \rightarrow (2^A)^*$ be the function intersecting a string $A_1 \cdots A_n \in (2^{\mathcal{A}})^*$ componentwise with $A$ before destuttering

$$\mathit{bc}_A(A_1 \cdots A_n) := \mathit{bc}((A_1 \cap A) \cdots (A_n \cap A))$$

(recalling from the definition of $\mathbb{I}_{\mathcal{A}}$ that whenever $A \subseteq A' \in$ *Fin*$(\mathcal{A})$, *Mod*$(A', A)$ intersects a string in $(2^{A'})^*$ componentwise with $A$). Now, the *inverse limit*

$$\varprojlim\{\mathit{bc}_A\}$$

*of* the indexed family $\{\mathit{bc}_A\}_{A \in Fin(\mathcal{A})}$ of functions is the set of functions $f :$ *Fin*$(\mathcal{A}) \rightarrow (2^{\mathcal{A}})^*$ such that

$$f(A) = \mathit{bc}_A(f(A')) \text{ whenever } A \subseteq A' \in Fin(\mathcal{A}).$$

This equality ensures that $f$ provides a consistent system of $A$-approximations $f(A)$ of a structure that is infinite insofar as for any positive integer $n$, there is an $A \in$ *Fin*$(\mathcal{A})$ such that $f(A)$ is a string of length $> n$. For branching in $\varprojlim\{\mathit{bc}_A\}$, we lift the prefix relation $\preceq$ on strings

$$s \preceq s' \iff s' = ss'' \text{ for some string } s''$$

to a relation $\prec_{\mathcal{A}}$ on $\varprojlim\{\mathit{bc}_A\}$ by universal quantification

$$f \preceq_{\mathcal{A}} f' \iff (\forall A \in Fin(\mathcal{A}))\ f(A) \preceq f'(A).$$

For $\mathcal{A}$ equal to the set of rational numbers, we can express the Dedekind cut construction of a real number $r \in \mathbb{R}$ as a function $f_r \in \varprojlim\{\mathit{bc}_A\}$ to get a copy of the real line $\mathbb{R}$ from $\preceq_{\mathcal{A}}$ restricted to the functions $f_r$ (for $r \in \mathbb{R}$).[4]

Compression $\mathit{bc}_A$ conditioned by $A$ links the ontology $[n]$ of an $A$-model $\langle [n], \mathsf{S}_n, \{[\![P_a]\!]\}_{a \in A}\rangle$ to the granularity $A$. Consider, for instance, the finite-state transducer above for inheritance, enforcing ih$[a]$ for $a \in$ *Ih* (assumed finite, for convenience). Suppose on input $A_1 \cdots A_n$, this transducer outputs the string $T(A_1 \cdots A_n)$. Applying $\mathit{bc}_{Ih}$ to this output yields the block compression of $T$'s output on input $\mathit{bc}_{Ih}(A_1 \cdots A_n)$

$$\mathit{bc}_{Ih}(T(A_1 \cdots A_n)) = \mathit{bc}(T(\mathit{bc}_{Ih}(A_1 \cdots A_n))).$$

Similarly for inertia and its transducer $T_o$,

$$\mathit{bc}_A(T_o(A_1 \cdots A_n)) = \mathit{bc}(T_o(\mathit{bc}_A(A_1 \cdots A_n)))$$

where $A = Ir \cup \{o(a) \mid a \in Ir\}$. Attention to granularity $A$ pays off in bounding the search space of candidate $A$-models. But before requiring an $A$-model $A_1 \cdots A_n$ is equal to $\mathit{bc}_A(A_1 \cdots A_n)$, we should be clearer about what the strings $A_1 \cdots A_n$ represent. Common to ih$[a]$ and ir$[a]$ is some form of Leibniz' Principle of Sufficient Reason, no change (in $P_a$ over $S$) without a reason,

$$\text{PSR}[a] := \forall x \forall y((P_a(y) \wedge ySx \wedge \neg P_a(x)) \supset xR_a y)$$

where the reason $R_a$ is reduced to a unary relation

$$xR_a y := \begin{cases} P_{\overline{a}}(x) & \text{for inheritance (Inh)} \\ P_{\mathsf{o}(a)}(y) & \text{for inertia (Inr).} \end{cases}$$

PSR$[a]$ with $xR_a y$ as $P_{\mathsf{o}(a)}(y)$ is essentially an *explanation closure* axiom [Lifschitz, 2015]. The non-inertial attribute $o(a)$ designates any force opposing $a$, while $\overline{a}$ serves in ih$[a]$ as a differentia in a taxonomy, a path in which is picked out by an interpretation of $S$. By placing demands on the signature $A$ of MSO$_A$, these attributes provide a causal ontology for change. It is instructive to eliminate the negation $\neg$ in PSR$[a]$ and ih/ir$[a]$ by moving $P_a(x)$ to the right of the implication $\supset$ for a choice

$$(P_a(y) \wedge ySx) \supset (P_a(x) \vee xR_a y)$$

between $P_a(x)$ and $xR_a y$ given $P_a(y)$ and $ySx$. A bias towards $P_a(x)$ leading to $S$-paths in §3 amounts to a minimisation assumption on reasons and on a domain $[n]$ subject to $\mathit{bc}_A$. Of course, we can eliminate a stutter in a string not just through $\mathit{bc}$ but by adding an attribute $n$ to $A$ that names a unique position

$$\forall x \forall y(P_n(x) \wedge P_n(y) \supset x = y)$$

(corresponding to a nominal in Description Logic). But this goes against the custom of using attributes for universals,

[4]Inverse limits aside, the focus of the present work is very much on the approximations from finite strings, which, under the prefix relation $\preceq$, do not form a complete partial order — a basic requirement on domains over which default reasoning is investigated in [Hitzler, 2004]. These finite strings enjoy a special status as compact elements in algebraic cpo's. (I am indebted to a referee for bringing this paper to my attention.)

rather than particulars, and the anti-nominalist thrust of an attribute-centered institution $\mathbb{I}_{\mathcal{A}}$ that constructs individuals and temporal instants through inverse limits over kinds and temporal intervals (conceived as basic, rather than as sets of instances and instants).

## 5 Conclusion

The account of predication above proceeds from

(a) an extensional conception of predication as instantiation, analyzed in section 2 as extensional subsumption $is\_a^H$, given a fixed FCA-context $\langle \mathcal{O}, \mathcal{A}, H \rangle$

to

(b) an institution $\mathbb{I}_{\mathcal{A}}$ in section 4 within which finite fragments of various FCA-contexts are represented by strings $A_1 \cdots A_n$ of finite subsets of $\mathcal{A}$, the successor relation in which, $S$, is an intensional alternative to (the converse of) $is\_a^H$, along which fast path-based reasoning advocated by Woods [2007] can be carried out.

*So what?* The move from (a) to (b) challenges the primacy accorded to the set of instances of a predicate by the traditional set-theoretic analysis of predication, proposing a shift in focus away from instances towards strings that track mechanisms for predication, including inheritance (between instances and kinds) and inertia (over time). This is a significant shift, support for which can be found in

(i) the *rules-and-regulations* view defended in Carlson [1995] that generic statements are about causal forces,

(ii) the proposal from Steedman [2005] that temporality in natural language primiarily concerns the "representation of causality and goal-directed action,"

(iii) the pluralistic perspective promoted in Goguen [2004] away from any one isolated context, towards a space of contexts subject to a *Satisfaction Condition* characteristic of institutions, and

(iv) the notion of "semantics in flux" challenging "the impression" from Montague [1974] "of natural languages as being regimented with meanings determined once and for all by an interpretation" [Cooper, 2012, page 271].

An institution $\mathbb{I}_{\mathcal{A}}$ is presented above where the Satisfaction Condition amounts to *What you see is all there is* [Kahneman, 2011], and notions of inheritance and inertia can be established through paths described by monadic second-order variables within a logic which (by a fundamental theorem due to Büchi, Elgot and Trakhtenbrot) represents finite-state mechanisms (linked above to fast predications).

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