Proceedings of the VIII International Conference "Distributed Computing and Grid-technologies in Science and Education" (GRID 2018), Dubna, Moscow region, Russia, September 10 - 14, 2018

ORTHOGONALITY-BASED CLASSIFICATION OF DIAGONAL LATIN SQUARES OF ORDER 10

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The article describes combinatorial structures based on diagonal Latin squares (DLS) of order 10 and the orthogonality condition between pairs of such squares. These structures are novel and interesting in the context of the well-known open problem aimed at finding a triple of mutually orthogonal DLSs of order 10. The structures were found in the BOINC-based volunteer computing projects SAT@home and Gerasim@home.

Keywords: combinatorics, diagonal Latin square, orthogonal mate, volunteer computing, BOINC.

Introduction

The search for pairs of orthogonal diagonal Latin squares (ODLS) is a hard combinatorial problem [1]. According to the Euler-Parker approach, a set of diagonal transversals is constructed for a given DLS of order *N*. If a subset of *N* non-overlapping transversals is found, then an orthogonal mate for the DLS can be easily constructed.

Applying volunteer computing to find canonical forms of diagonal Latin squares of order 10 with orthogonal mates

According to some estimations, only 1 DLS of order 10 out of 32 millions has an orthogonal mate. Authors of the volunteer computing projects Gerasim@home[1] and SAT@home[2] maintain a collection of pairs of ODLS of order 10. As of September 2018, the collection contains more than 1 300 000 different canonical forms (CFs) or isotopy classes of DLS of order 10 [2].

DLSs from the collection can be naturally classified by the number of their orthogonal mates. This classification can be expanded [3]. Figure 1 and Table 1 contain examples of DLSs of order 10 that are part of corresponding combinatorial structures. These DLSs were constructed during several computational experiments: random search for DLSs with consequent attempt to construct their orthogonal mates; comprehensive search for DLSs that are symmetric according to one plane (for example, horizontal); comprehensive search for generalized symmetric DLSs for some set of generalized symmetries; random search for partially generalized symmetric DLSs.

The found combinatorial structures (graphs from DLSs on the orthogonality binary relation set) are novel and were not published before. Due to their simplicity they allow a trivial classification based on a vector of degrees of vertices which is sorted in ascending order [4]. In fact, in this case a degree of a vertex is the number of ODLS for the chosen DLS.

[1] http://gerasim.boinc.ru
[2] http://sat.isa.ru

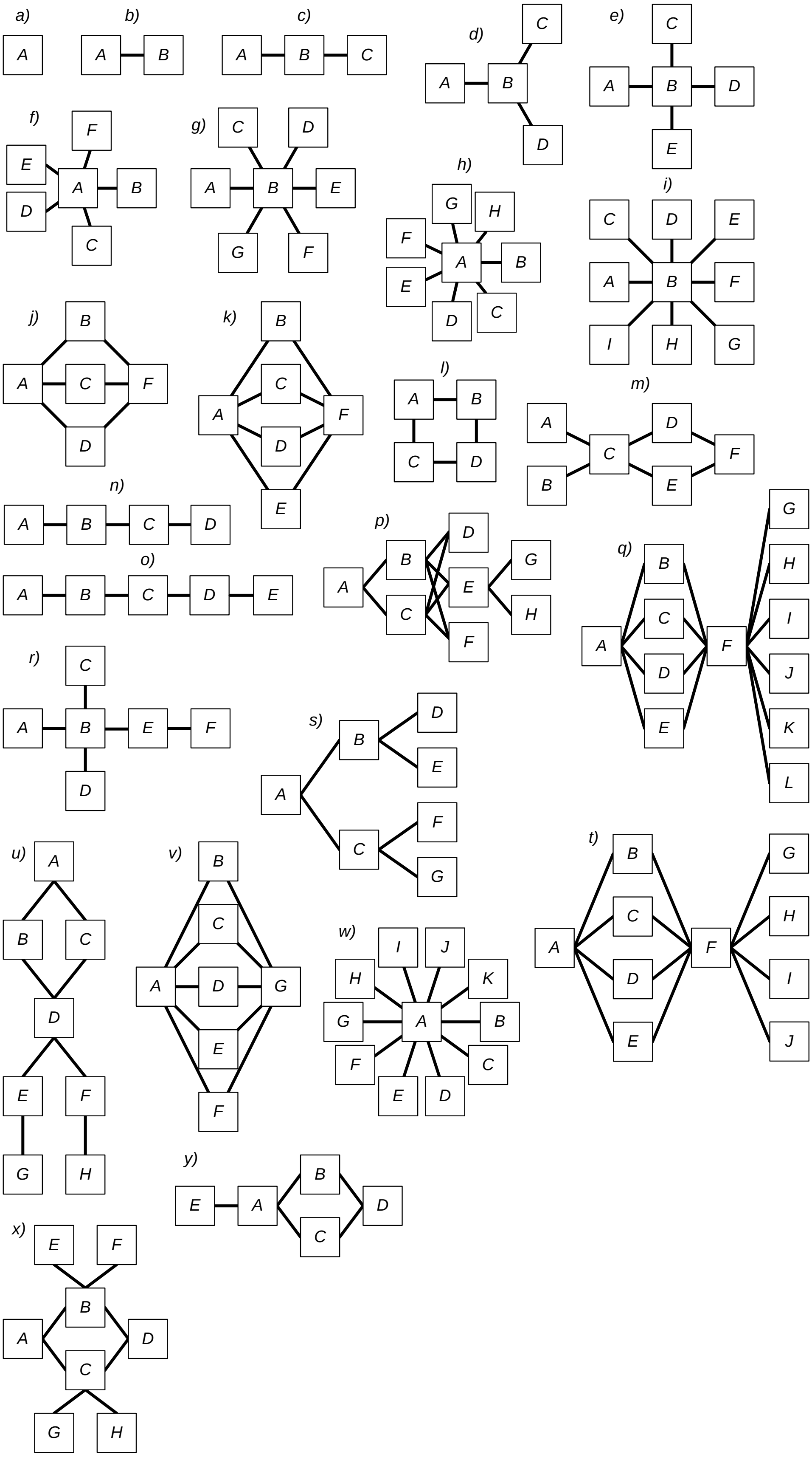

Figure 1. Combinatorial structures based on the orthogonality of DLSs of order 10: a – square with no mate (bachelor); b – line-2 or once; c – line-3; d – triple; e – four; f – five; g – six; h – seven; i – eight; j – rhombus-3; k – rhombus-4; l – loop-4; m – fish; n – line-4; o – line-5; p – flyer; q – daedalus-10; r – cross; s – tree; t – daedalus-8; u – venus; v – rhombus-5; w – ten; x – robot; y – stingray

Table 1. Examples of DLSs from various combinatorial structures. For each structure only one DLS is presented, all other DLSs can be easily constructed too. DLS elements are written row by row

Combinatorial structure type	**Values of DLS elements**	**Number of vertexes and arcs, ascending sorted vector of degrees of vertexes**
a	0123456789120453986735618740929478321605273098514668592473104697013258704619852383156029745982760431	1, 0 [1]
b	0123456789120436597820156348973457981620574980216369812473058376019254963072854175681904324892573016	2, 1 [1, 1]
c	0123456789120436597820358146974697182035978654312034780912566941728503785063941253192708648562907341	3, 2 [1, 1, 2]
d	0123456789123709854640963718259684513072596870243134592806178701635294237596410875128493606840127953	4, 3 [1, 1, 1, 3]
e	0123456789120436597823510984678596723041374918052640682713957480639152691254780396758142305837902614	5, 4 [1, 1, 1, 1, 4]
f	0123456789120487956393176804253795048216258693710479482613508631504972647032589140597126385862193047	6, 5 [1, 1, 1, 1, 1, 5]
g	0123456789123409567823408195679876543210876590432164912780533508721946501763289476591804324982367105	7, 6 [1, 1, 1, 1, 1, 1, 6]
h	0123456789120437956839407158269031847652589762014375680924316475138290261958430783529610744786203915	8, 7 [1, 1, 1, 1, 1, 1, 1, 7]
i	0123456789123054967849682713056874905213735918046294876321503591728046874609352150123678942605814937	9, 8 [1, 1, 1, 1, 1, 1, 1, 1, 8]
j	0123456789123704896560593124788904621537786193520436752840912490867153458617932097485036125312790846	5, 6 [2, 2, 2, 3, 3]
k	0123456789120436597823159048673469780152789154203656801793249742638510493782160585760932416058217493	6, 8 [2, 2, 2, 2, 4, 4]
l	0123456789329601784578306951249347208651268593041719785432064562781093840136957257198243606054172938	4, 4 [2, 2, 2, 2]
m	0123456789120436795887965240316945182370451073982620578136949371608542386927041554380912677682945103	6, 6 [1, 1, 2, 2, 2, 4]
n	0123456789123867094579640835124619702853380251946785973412062450867391908623517457419286306375194028	4, 3 [1, 1, 2, 2]
o	0123456789120436597823491805678765904321948263715058302796147698541032351709284640768132956951728403	5, 4 [1, 1, 2, 2, 2]

p	0123456789120436597823491805678765904321948263715058302796147698541032351709284640768132956951728403	8, 10 [1, 1, 2, 2, 2, 4, 4, 4]
q	0123456789120453796897623851407518692034398674120524598103764830269517867590342163910748525047128693	12, 14 [1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 4, 10]
r	0123456789120479865346598273106548371092901268354757962348013870169425296754013884319052767385012964	6, 5 [1, 1, 1, 1, 2, 4]
s	0123456789120467985384759106327061238594983652140737920841656547893210261034597853897620414958107326	7, 6 [1, 1, 1, 1, 2, 3, 3]
t	0123456789120679345838640251972917384605405283791657912408637580619324637910854296485712308435962071	10, 12 [1, 1, 1, 1, 2, 2, 2, 2, 4, 8]
u	0123456789120478963573856014925916837204365291804790785423164590273168846719052368413259702739064851	8, 8 [1, 1, 2, 2, 2, 2, 2, 4]
v	0123456789123079865468759032415314027896958761432046928310758456379102294816053770692854133701542968	7, 10 [2, 2, 2, 2, 2, 5, 5]
w	0123456789120487956375429183062781035694863950742194582610375097643812631578294038601942754976320158	11, 10 [1, 1, 1, 1, 1, 1, 1, 1, 1, 10]
x	0123456789120478596323816045974539162078746051983296750382145098347621694287315087162903453857921406	8, 8 [1, 1, 1, 1, 2, 2, 4, 4]
y	0123456789120467893558367201943785942610291086534763410975284597281063765831940294625038718079134256	5, 5 [1, 2, 2, 2, 3]

Conclusion

As of September 2018, the collection of different combinatorial structures with more than 1 DLS per structure includes: 1 250 250 different CFs owned by the line-2 structure, 55 293 CFs by line-3, 96 CFs by line-4, 51 CFs by line-5, 1 944 CFs by loop-4, 312 CFs by triples, 1 708 CFs by fours, 12 CFs by fives, 53 CFs by sixes, 8 CFs by seven, 58 CFs by eights, 10 CFs by rhombus-3, 104 CFs by rhombus-4, 16 CFs by fishes, 4 CFs by tree, 24 CFs by crosses, 12 CFs by daudalus-10, 8 CFs by flyer, 5 CFs by venus, 6 CFs by daudalus-8, 5 CFs by rhombus-5, 6 CFs by ten, 10 CFs by robot and 5 CFs by stingray. The structures seven, tree, daedalus-8, daedalus-10, flyer, venus, rhombus-5, ten, robot and stingray are found in a single copy only. In the future, we are planning to find novel combinatorial structures using different partial symmetries neighborhoods. A triple of MODLS of order 10 has not been found so far as a part of any mentioned structures.

Acknowledgement

The research was partially supported by Russian Foundation for Basic Research (grants 16-07-00155-a, 17-07-00317-a, 18-07-00628-a, 18-37-00094-mol-a) and by Council for Grants of the President of the Russian Federation (stipend SP-1829.2016.5). Authors thank citerra [Russia Team] from the internet portal BOINC.ru for his help in the development and implementation of some algorithms. Also, authors thank all the volunteers of SAT@home and Gerasim@home projects for their participation.

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