DIFFERENT APPROACHES FOR ELASTIC IMAGING USING MULTIPROCESSOR COMPUTING SYSTEMS[*]

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One of the important problems of the seismic survey process is the seismic migration. The result provides enough information for the delineation of oil and gas deposits. In this article two independent approaches for solving the migration problem in elastic media are discussed. Main formulas for the Born approximation are provided and advantages of this algorithm are represented. Limitations of the used background model were overcome with the full-wave elastic migration approach. It uses the grid-characteristic method on structured meshes of high-orders. The possibility of the single geological crack identification was tested. A wide range of crack plane orientations was covered.

Keywords: seismic imaging, numerical simulation, Born approximation, grid-characteristic method

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1. Introduction

One of the first paper about the seismic migration was published in 1954 [1]. Later on, Dr. Claerbout postulated the finite-difference algorithm for solving the migration problem as a scalar wave equation [2]. Numerous different algorithms were constructed by now: Kirchhoff integral method [3], the frequency wavenumber migration algorithm [4], the Born approximation [5], the Reverse Time Migration [6]. All of these methods were suffered from the limitations imposed on the background model. A ray-Born method was proposed in [7] to take into account a non-uniform background medium. In most cases authors used the simple acoustic approximation to describe the behavior of the geological medium.

In the paper [8, 9] the Born method and the corresponding migration algorithms were extended to the elastic wavefields as well. Further, the adjoint operator approach [10] and the grid-characteristic method [11] was successfully applied to the elastic migration problem [12].

In this work we initially presented the main formulas of the elastic Born migration algorithm and carried out numerical experiments to highlight advantages of it. Further, we applied the full-wave grid-characteristic migration algorithm to estimate the possibility of the single geological crack identification on the migration image.

2. Elastic migration with Born approximation

The Lame equation for the elastic medium can be written in the operator form as

$$\mathbf{\Lambda u} - \frac{\partial^2 \mathbf{u}}{\partial t^2} = -\frac{1}{\rho}\mathbf{f}^{\mathrm{e}},\ \ \mathbf{\Lambda} = c_p^2 \nabla\nabla\cdot - c_s^2 \nabla\times\nabla\times, \tag{1}$$

where c_p, c_s are the pressure and shear wave velocities; ρ is a mass density of the medium; $\mathbf{f}^{\mathrm{e}}$ is a strength of the external force per unit volume applied to the elastic body; $\mathbf{u}$ is a displacement field.

For the case of the constant background model the integral relationship between the field in the volume and displacements on the day surface is written as

$$\mathbf{u}_\alpha^s(\mathbf{r}', t') = \int_V \int_{-\infty}^{+\infty} \{\Delta\mathbf{\Lambda}(\mathbf{r})[\mathbf{u}^i(\mathbf{r}, t) + \mathbf{u}^s(\mathbf{r}, t)]\} \cdot \mathbf{G}_\alpha^L(\mathbf{r}', t'|\mathbf{r}, t)\,dt\,dV, \tag{2}$$

where $\mathbf{G}_\alpha^L$ is the Green's tensor for the homogeneous space. In the Born approximation we neglect the scattered field with respect to the background field in the volume V and transform the equation as

$$\boldsymbol{u}_\alpha^{s,B}(\boldsymbol{r}', t') = \sum_\beta \int_V \int_{-\infty}^{+\infty} \Delta c_\beta^2(\boldsymbol{r}) \nabla^2 \boldsymbol{u}_\beta^i(\boldsymbol{r}, t) \cdot \boldsymbol{G}_\alpha^L(\boldsymbol{r}', t'|\boldsymbol{r}, t)\,dt\,dV. \tag{3}$$

According to the adjoint operator approach for the inverse problem solution we obtain

$$\Delta c_{\alpha,migr}^2(\boldsymbol{r}) = \int_S \int_T \int_{-\infty}^{+\infty} \{\nabla^2 \boldsymbol{u}_\alpha^i(\boldsymbol{r}, t)\} \cdot \boldsymbol{G}^L(\boldsymbol{r}', t'|\boldsymbol{r}, t) \cdot \boldsymbol{d}(\boldsymbol{r}', t')\,dt\,dt'\,dS. \tag{4}$$

To demonstrate the advantage of the elastic approach over the acoustic approach we used the simple 2D one layered model with the homogeneous spherical inclusion. Parameters of the background model were set as $C_P = 2500\ m/s$, $C_S = 1250\ m/s$, $\rho = 2500\ kg/m^3$ in the rectangle 10 km x 2.5 km. The contrast of the spherical inclusion was only 1% to stay in the Born approximation. Zero-offset seismograms with the 25 Hz peak frequency were used. For comparison we limited the measurements with only the vertical component of displacements for all cases. We parallelized the computational algorithm with the OpenMP standard and ran all calculations on the 12-30 cores system with the shared memory.

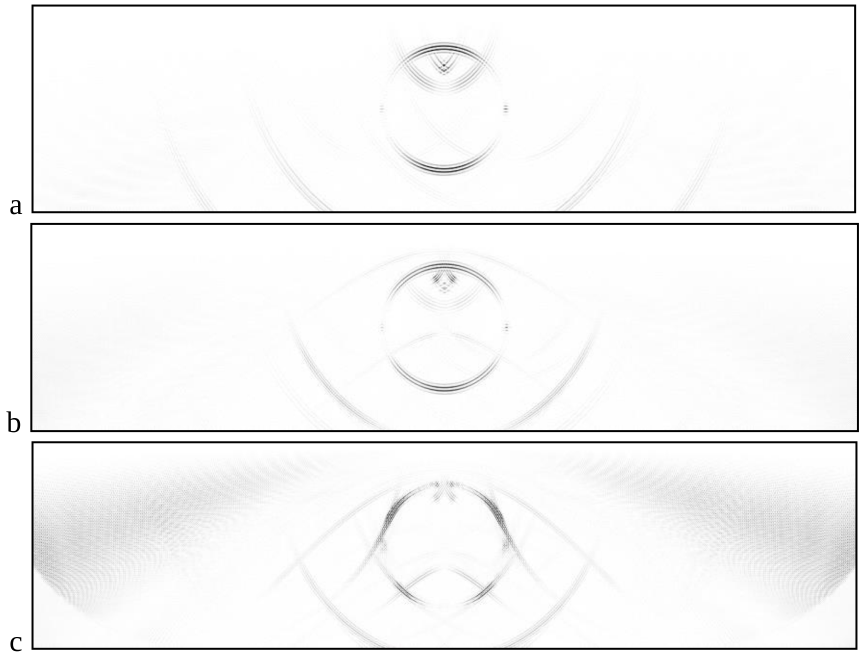

Figure 1. Migration images for the homogeneous spherical inclusion. The acoustic image (a) and elastic images for pressure waves (b) and shear waves (c)

The analysis of these three images shows us that with the acoustic approach we obtain less intensive artefacts on the P-wave image and also reproduce steeper geological boundaries on the S-wave image.

3. Full-wave elastic migration

In general, the dynamic behavior of the geological medium can be precisely described with the elastic medium approach. It consists of the movement equation and the rheological relationship between the stress tensor and the strain tensor. They form the hyperbolic system of equations in partial derivatives:

$$\rho \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ix}}{\partial x} + \frac{\partial \sigma_{iy}}{\partial y} + \frac{\partial \sigma_{iz}}{\partial z}, i = x, y, z \quad (5)$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda \delta_{ij} \delta_{kl} \frac{\partial \varepsilon_{kl}}{\partial t} + \mu \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) \frac{\partial \varepsilon_{kl}}{\partial t}, \quad (6)$$

here σ – the stress tensor, ε – the strain tensor, v_i – the component of the velocity vector.

The solution of this system can be found numerically with different methods: the partial derivatives approximation [13], the continuous or discontinuous Galerkin method [14], the grid-characteristic method [11]. In this paper we used the last one which was successfully applied previously in many direct seismic problems [15-17].

The migration (inverse) problem may be written as a problem of the residual functional minimization [10]. It was formulated mathematically as

$$\chi(\boldsymbol{m}) = \frac{1}{2} \sum_r \int \| \boldsymbol{s}(\boldsymbol{x}_r, t; \boldsymbol{m}) - \boldsymbol{d}(\boldsymbol{x}_r, t) \|^2 dt, \quad (7)$$

here $\boldsymbol{x}_r$ – receiver positions, $\boldsymbol{s}$ – synthetic data, $\boldsymbol{d}$ – field data.

With this approach the sequence algorithm for solving the general migration problem in an arbitrary elastic medium can be proposed as

$$K_Z = K_\rho + K_\kappa + K_\mu, \quad (8)$$

$$K_\rho(\boldsymbol{x}) = \rho(\boldsymbol{x}) \int \boldsymbol{v}^\dagger(\boldsymbol{x}, -t) \boldsymbol{v}(\boldsymbol{x}, t)\, dt, \quad (9)$$

$$K_\kappa(\boldsymbol{x}) = -\kappa(\boldsymbol{x}) \int [\nabla \cdot \boldsymbol{s}^\dagger(\boldsymbol{x}, -t)][\nabla \cdot \boldsymbol{s}(\boldsymbol{x}, t)]\, dt, \quad (10)$$

$$K_\mu(\boldsymbol{x}) = -2\mu(\boldsymbol{x}) \int \mathbf{D}^\dagger(\boldsymbol{x}, -t) \colon \mathbf{D}(\boldsymbol{x}, t)\, dt, \quad (11)$$

$$\mathbf{D} = \frac{\nabla \boldsymbol{s} + (\nabla \boldsymbol{s})^T}{2} - \frac{\nabla \cdot \boldsymbol{s}}{3} \mathbf{I}, \ \mathbf{D}^\dagger \colon \mathbf{D} = \sum_{i,j} D_{ij}^\dagger D_{ij}, \quad (12)$$

$$\boldsymbol{f}^\dagger(\boldsymbol{x}, t) = \sum_r [\boldsymbol{s}(\boldsymbol{x}_r, -t) - \boldsymbol{d}(\boldsymbol{x}, -t)] \delta(\boldsymbol{x} - \boldsymbol{x}_r), \quad (13)$$

here $\boldsymbol{s}$ and $\boldsymbol{v}$ - displacement and velocity vectors and the sign † means the complex conjugation operation.

This algorithm was used in this work to estimate the possibility of the single geological crack identification on the elastic migration image. The homogeneous medium 8 km x 3 km with $C_P = 4000\, m/s$, $C_S = 2300\, m/s$, $\rho = 2500\, kg/m^3$ was used. The fluid-filled crack with the 200 m length was submerged on the 2000 m depth. The set of numerical experiments was carried out with the different orientation of the crack. The angle from the horizontal line was varied in wide range from 10° to 80°. On the Figure 2 two migration images are presented.


a

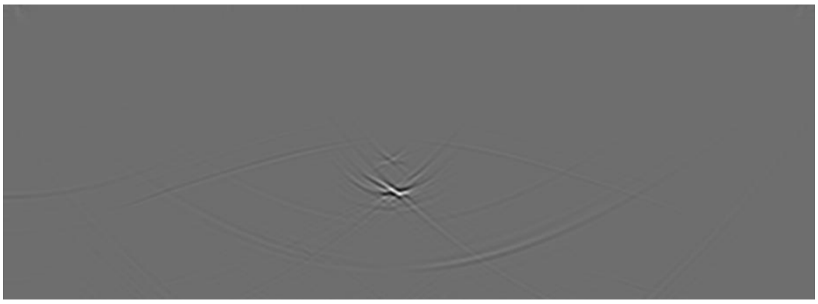
b

Figure 2. Migration images for fractured geological medium. Orientations: 10° (a) and 80° (b)

The analysis of the results of numerical simulations indicates that the visibility degree of the fracture plane is higher for sub-horizontal cracks and is lower for sub-vertical cracks. Also, all images have spherical artefacts and their amplitudes decrease with the increase of the angle of the crack.

4. Conclusion

In this work two different approaches for creating seismic images were considered. As the base of them the system of the linear elasticity was used. For constant background model the formulas for Born approximation were written. Advantages of the described algorithm over the acoustic approximation were shown. To overcome restrictions imposed on the background model the full-wave approach may be applied. We estimated the possibility of the identification of single geological crack inside the homogeneous space. The visibility degree of the fracture plane is higher for sub-horizontal cracks and is lower for sub-vertical cracks. The further direction of the research may be the attempt to remove obtained artefacts from migration images.

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