CHAOTIC DYNAMICS OF INSTANTANEOUS HEART RHYTHM AND ITS PHASE SPACE

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In this paper we propose a new method for calculation of parameters of the fractal dimension D and the fractal phase-space volume Γ of the phase space of instantaneous heart rhythm. The parameters of interest for *D* and Γ were calculated for four patients of the Tver Regional Cardiology Health Center based on the data obtained from 24-hour Holter monitoring. The investigated structure of the phase spaces of instantaneous heart rhythm for these patients is indicative of nondeterministic chaotic behavior of the instantaneous heart rhythm. The degree of similarity of the phase spaces of instantaneous heart rhythm to fractals in patients under examination was evaluated to be not greater than 0.0453 in *C*-metrics, and thus, the instantaneous heart rhythm chaos was identified as a fractal to the same degree of accuracy. Finally, we propose the foundation for identifying the state parameters for the fractal dimension *D* and the *D*-dimensional volume Γ of the phase space of instantaneous heart rhythm.

Keywords: chaos, dynamics, phase space, instantaneous heart rhythm, fractal dimension, phase-space volume

1. Introduction

In earlier papers [1-3] we constructed the phase space (PS) of instantaneous heart rhythm (IHR). It describes the heart rhythm properties more adequately. A PS of IHR is a set of states in the space R^2 defined by the functions $y(t)$ (IHR) and $v(t)$ (IHR change rate) [1]. The function $y(t)$ is a piecewise linear approximation of the $y_i=60/T_{RRi}$ ($i=1,2,...,n$) set, where i is the RR interval number from the time of observation, T_{RRi} is its value in seconds. The function $v(t)$ is a finite-difference derivative of the function $y(t)$.

After processing the RR_i range with the help of the KT-Result program, we construct the IHR functions $y(t)$ and $v(t)$ based on the program system developed and implemented in Ref. [2]. The functions $y(t)$ and $v(t)$ are measured in min^{-1} and $\text{min}^{-1}\text{sec}^{-1}$ respectively. By dividing $y(t)$ by min^{-1} and $v(t)$ by $\text{min}^{-1}\text{sec}^{-1}$ we obtain the dimensionless functions.

Figures 2, 3 (see Ref. [1]) present the PSs of IHR of two patients of the Tver Regional Cardiology Health Center (TvRCHC). These figures demonstrate the chaotic nature of IHR phase point motion, the chaos being nondeterministic.

The indicator of nondeterministic chaos is the presence of points in PS through which two or more phase trajectories pass. The infinitesimal perturbations cause a transition of the phase point from a phase trajectory to other ones.

However, the nondeterministic chaos of IHR can have hidden statistical regularities. The structural features of these regularities are adequate heart rhythm characteristics for the patients under examination.

From Ref. [1], we can see the similarity between the PSs of IHR of the patients under examination and fractals. Consequently, the IHR chaos is a fractal which to the fullest extent reflects such an important heart rhythm property as the heart rhythm self-similarity.

Since the PS of IHR is a fractal, the main IHR state parameters are the following fractal set characteristics: the fractal dimension D and the D-dimensional volume Γ.

The study of dynamic features of $D(t)$ and $\Gamma(t)$ of PS of IHR provides unique information on the patients' cardiovascular system state which cannot be obtained using other approaches.

Studying and predicting borderline states between norm and pathology [4, 5] is a major problem in cardiology. The identification of IHR PS dynamic features in different patients can be helpful to tackle this problem.

2. Dynamics of the IHR: the parameters *D* and Γ

Let us consider the basics of the new method for calculating the parameters D and Γ. First, we cover the IHR phase space with a mesh, the mesh spacing being $\delta=1/B$ and count the number of covering meshes, $N(B,t)$. The IHR phase space is exactly a fractal [1, 2]. Then the function $N(B,t)$ is approximated by the exponential function of B:

$$\overline{N}(B,t)=\Gamma(t)B^{D}. \tag{1}$$

In order to determine the parameters $D(t)$ and $\Gamma(t)$ we take the sequence of B values: $B_k=2^k$, $k=1,2,...,\text{K}$ and define the function $\Lambda(D(t),\Gamma(t))$:

$$\Lambda(\Gamma(t),D(t))=\sum_{k=0}^{K}\left(lnN(B_k,t)-ln\overline{N}(B_k,t)\right)^2. \tag{2}$$

The minimum of $\Lambda(D(t),\Gamma(t))$ corresponds to the minimum of deviation of $N(B,t)$ from $\overline{N}(B,t)$. Then the parameters $D(t)$ and $\Gamma(t)$ are determined from the condition for the minimum of $\Lambda(D(t),\Gamma(t))$:

$$\frac{\partial\Lambda}{\partial D}=0,\ \frac{\partial\Lambda}{\partial\Gamma}=0. \tag{3}$$

Using (3) we calculate the parameters $D(t)$ and $\Gamma(t)$ for four patients of the TvRCHC during two hours of Holter monitoring. We represent the dependences $N^{(p)}(B_k=2^k)$, $K=5$, $p=1,2,3,4$ (where p is the patient number) and their approximations in terms of the exponential functions $\overline{N}^{(p)}(B_k=2^k)=\Gamma^{(p)}2^{kD^{(p)}}$ in Figure 1 as a log-log chart.

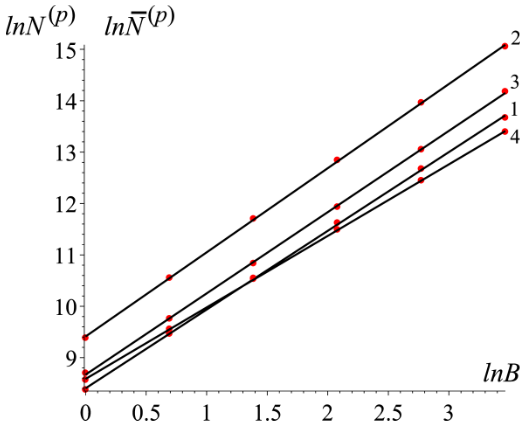

Figure 1. Relationship between the functions $lnN^{(p)}$, $ln\overline{N}^{(p)}$ and lnB, p=1,2,3,4

The similarity of $N^{(p)}(B)$ to fractals is determined in C-metric by the value of $\Delta^{(p)} = \left|\ln\left(N^{(p)}(B)/\overline{N}^{(p)}(B)\right)\right|$. The results of calculations show that $\Delta^{(1)}$=0.0453, $D^{(1)}$=1.531, $\Gamma^{(1)}$=4333; $\Delta^{(2)}$=0.0339, $D^{(2)}$=1.637, $\Gamma^{(2)}$=12257; $\Delta^{(3)}$=0.0310, $D^{(3)}$=1.583, $\Gamma^{(3)}$=6008; $\Delta^{(4)}$=0.0196, $D^{(4)}$=1.398, $\Gamma^{(4)}$=5035.

The dependences $D(t)$ and $\Gamma(t)$ for patients 1-4 obtained from 24-hour Holter monitoring are presented in Figures 2-5.

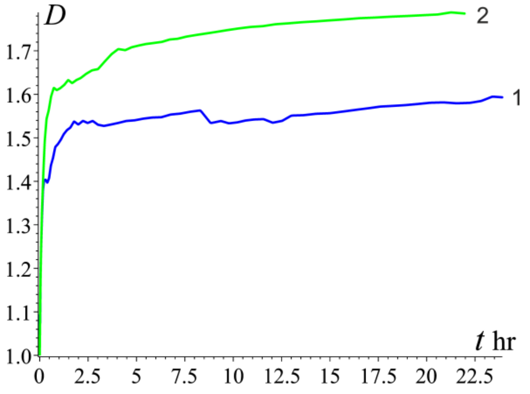

Figure 2. The function $D(t)$ for patients 1 and 2

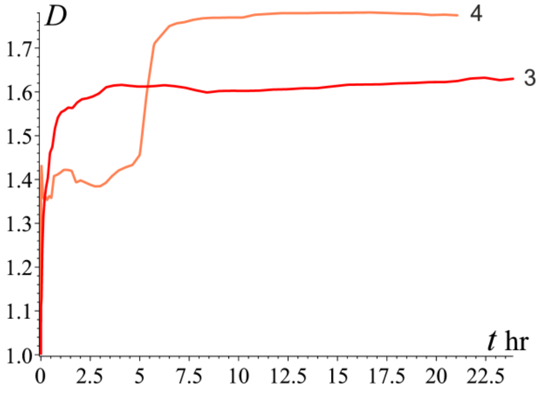

Figure 3. The function $D(t)$ for patients 3 and 4

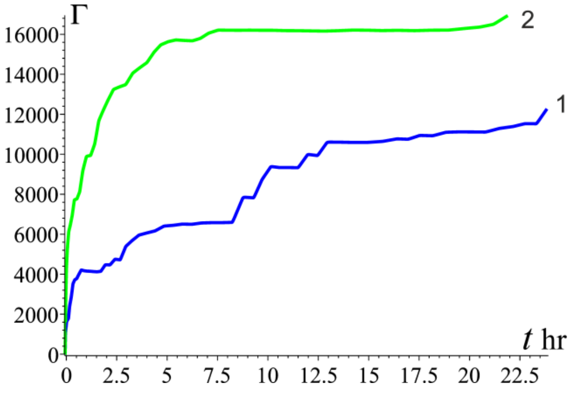

Figure 4. The function $\Gamma(t)$ for patients 1 and 2

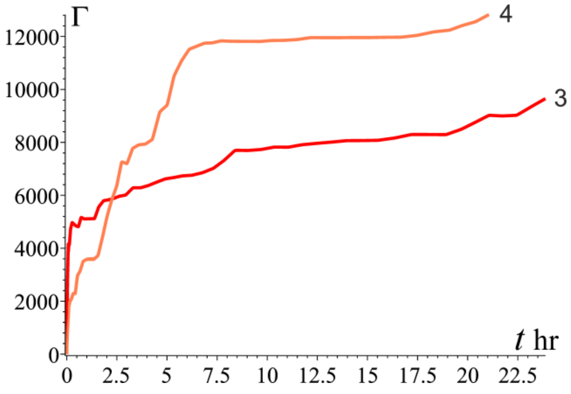

Figure 5. The function $\Gamma(t)$ for patients 3 and 4

The diagnoses of the patients under examination are as foolows: p=1 - norm, p=2 - Ryan's class 5 ventricular arrhythmia, p=3 - Ryan's class 4a ventricular arrhythmia, p=4 - Ryan's class 4a ventricular arrhythmia.

In Ref. [1] the concept of IHR fractal entropy, $S=ln\Gamma$, was introduced. The function $S(D)$ can contain unique information on the IHR pattern. The plots of the function $S(D)$ for patients 1-4 over the total Holter monitoring time are given in Figure 6.

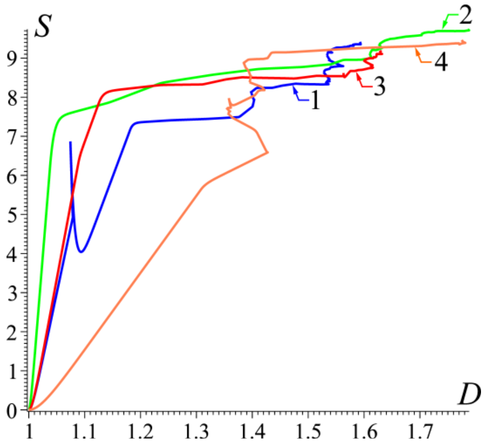

Figure 6. Plots of the function $S(D)$ graphs for patients 1-4

Figure 6 clearly demonstrates significant difference between curves 3 and 4 at the same diagnoses – Ryan's class 4a ventricular arrhythmia. This is a direct manifestation of possible drawbacks in the standard classification of Ryan’s ventricular arrhythmias. So, the question of creation of a new, more exact classification of that sort of heart rhythm disorder is raised.

The plots presented in Figures 2-5 have the pronounced character of tending to constant (asymptotic) values at the final stage of Holter monitoring. The fractal dimension D of IHR PSs demonstrates considerable variability of asymptotic values from 1.553 for the first patient to 1.785 for the second one. This is a sign of high degree of IHR PS randomness, and, thus, of chaotic nature of heart rhythm dynamics. The fractal phase-space volume of IHR of the examined patients increases from 1 to 9003-1650 dimensionless units over the period of 24-hour Holter monitoring. The great variability of fractal phase-space volume values for different patients with different diagnoses permits the use of this parameter as a marker of cardiovascular system condition.

3. Conclusion

A new method for calculating the parameters of the fractal dimension D and the fractal phase-space volume Γ of PS of IHR was offered (Eqs. (1)-(3)). The calculations of D, Γ, S for four patients of the TvRCHC according to the data of 24-hour Holter monitoring were carried out. The computational results are represented graphically in Figures 1-6.

The investigated IHR PS structure for patients 1-4 confirm the nondeterministic nature of IHR chaos.

The degree of similarity between the IHR PSs of the examined patients and fractals was evaluated. Its value appeared to be not greater than 0.0453 in *C*-metrics. Thus, the IHR chaos is a fractal with the same degree of accuracy.

One of the major results of our investigation is justifying the possibility of representation of states of the fractal dimension D and the *D*-dimensional volume Γ of PS of IHR by basic cardiac parameters.

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