

Methods for comparing numbers in non-positional notation of residual classes

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Abstract. Methods for optimizing computations in computer systems and components that are based on the use of arithmetic transformations in the system of residual classes are considered. Methods of comparing numbers in the system of residual classes based on the representation and processing of data without directly converting the compared numbers from a modular code to a positional code and back are investigated. Some methods are based on the principle of obtaining and comparing a unitary single-row code. Based on the proposed methods, algorithms for their implementation have been developed, in accordance with which a class of patentable devices has been developed for performing arithmetic and algebraic comparison of numbers in the system of residual classes.

Keywords. Non-positional number system, class of residues, arithmetic non-positional coding, computer systems and components.

1 Introduction

It is known that the use of non-positional number system in residual classes (RCS) significantly increases the reliability and performance of the computer system (CS) [1-8]. However, the need to determine the positional characteristics of numbers in the RCS reduces the overall effectiveness of the use of modular codes. Existing methods for processing positional data, in particular, methods for comparing numbers into RCS, have significant drawbacks, the main of which is the need to convert numbers from RCS to positional number system and vice versa, which reduces user productivity and reliability of CS [9-12].

In the article there are four methods for comparing numbers into RCS, based on the presentation and processing of data without directly converting the compared numbers from a modular code (an RCS code) into a positional code and backward.

2 Methods for arithmetic comparison of numbers in RCS

Let the RCS be given by ordered $(m_i < m_{i+1})$ mutually n pairs by simple natural numbers (bases) m_1, m_2, \dots, m_n , d let the compared operands be represented as:

$$A = (a_1, a_2, \dots, a_n), B = (b_1, b_2, \dots, b_n).$$

In the case, it is assumed that the source operands lie in the appropriate intervals:

$$\left[\frac{j_1 M}{m_n}, \frac{(j_1 + 1)M}{m_n} \right) \text{ and } \left[\frac{j_2 M}{m_n}, \frac{(j_2 + 1)M}{m_n} \right),$$

where $M = \prod_{i=1}^n m_i$, and the interval's number $j_k + 1$ is determined by the well-known expression $j_k = \gamma_n \bar{m}_n \pmod{m_n}$, where the value of m_n is determined from the comparison solution $\bar{m}_n M / m_n \equiv l \pmod{m_n}$. When $j_1 \neq j_2$ the operation of arithmetic comparison can be implemented by comparing the number of intervals, namely: if $j_1 < j_2$, then $A < B$, if $j_1 > j_2$, then $A > B$. When $j_1 = j_2$ is determined by the number $j_2 + 1$ of the interval $\left[\frac{j_3 M}{m_n}, \frac{(j_3 + 1)M}{m_n} \right)$, in which the number $A - B$ is located. If

$0 \leq j_3 < (m_n + 1) / 2$, then $A < B$, and if $\frac{m_n + 1}{2} \leq j_3 < m$, then $A > B$.

The well-known [1] method of arithmetic comparison of numbers with RCS involves the conversion of numbers and type $A^{(H)} = (0, 0, \dots, \gamma_n)$, which requires $n - 1$ clock cycles of the null operation. In addition, it is necessary to make a positional comparison of the $(j_1 + 1)$ and $(j_2 + 1)$ intervals of the source operands A and B . All of this complicates the comparison algorithm and increases the comparison time of numbers, which leads to the need to develop a comparison method for RCS, which do not require determination of positional characteristics. Consider each of these methods.

2.1 The method of arithmetic comparison of numbers in the RCS (the method of arithmetic parallel subtraction)

We will consider comparable numbers in arbitrary intervals: $[jm_i, (j + 1)m_i)$, where

$$\overline{j = 1, N - 1} \left(N = \prod_{\substack{k=1 \\ k \neq i}}^n m_k \right).$$

In the case, the source operands A, B are reduced to numbers that are multiples to m_i , by modular subtraction of the following form:

$$A_{m_i} = A - a_i = (a_1^{(i)}, a_2^{(i)}, \dots, a_{i-1}^{(i)}, 0, a_{i+1}^{(i)}, \dots, a_n^{(i)}),$$

$$B_{m_i} = B - b_i = (b_1^{(i)}, b_2^{(i)}, \dots, b_{i-1}^{(i)}, 0, b_{i+1}^{(i)}, \dots, b_n^{(i)}),$$

where

$$a_i = (a'_1, a'_2, \dots, a_i, a'_n), \quad b_i = (b'_1, b'_2, \dots, b_i, b'_n).$$

Further, by means of a set of constants $0, m_i, 2m_i, \dots, (N-1)m_i$, represented by $(n-1)$ -y base of the RCS $m_1, m_2, \dots, m_{i-1}, m_{i+1}, \dots, m_n$, the construction of the so-called single-row code, respectively, in the form

$$K_N^{(n_A)} = \{z_N z_{N-1} \dots z_2 z_1\}, \quad z_{n_A} = 0 \quad (z_1 = 1; 1 = \overline{1, N}, 1 \neq n_A),$$

$$K_N^{(n_B)} = \{z'_N z'_{N-1} \dots z'_2 z'_1\}, \quad z'_{n_B} = 0 \quad (z'_1 = 1; 1 = \overline{1, N}, 1 \neq n_B).$$

The algorithm for constructing a single-row code in RCS can be represented as follows (1):

$$\left\{ \begin{array}{l} A_{m_i} - 0 = z_1, \\ A_{m_i} - m_i = z_2, \\ A_{m_i} - 2m_i = z_3, \\ \dots \\ A_{m_i} - \left(\prod_{\substack{\rho=1 \\ \rho \neq 1}}^{n-1} m_\rho - 1 \right) m_i = z_N, \end{array} \right. \quad \left\{ \begin{array}{l} B_{m_i} - 0 = z'_1, \\ B_{m_i} - m_i = z'_2, \\ B_{m_i} - 2m_i = z'_3, \\ \dots \\ B_{m_i} - \left(\prod_{\substack{\rho=1 \\ \rho \neq 1}}^{n-1} m_\rho - 1 \right) m_i = z'_N. \end{array} \right. \quad (1)$$

In the case, we have:

$$z_{n_A} = 0, \text{ at } A_{m_i} - n_A \cdot m_i = 0; \quad z_{n_A} = 1, \text{ at } A_{m_i} \neq n_A \cdot m_i;$$

$$z'_{n_B} = 0, \text{ at } B_{m_i} - n_B \cdot m_i = 0; \quad z'_{n_B} = 1, \text{ at } B_{m_i} \neq n_B \cdot m_i.$$

Geometrically, this number comparison method can be explained as follows. The interval $\left[0, \prod_{i=1}^n m_i\right)$ is divided into segments. The source operands A и B , by subtract-

ing the constants of the form $\prod_{\substack{\rho=1 \\ \rho \neq i}}^{n-1} m_\rho$, c and shift to the left edge of their hit interval.

This is the equivalent to bringing the compared numbers to numbers, which are compile to comparison m_i of the RCS. On this basis, the accuracy W c and comparison of the operands depends on the size of the base m_i , that is $W = W(m_i)$. However, with the maximum accuracy of the comparison $W_{max} = W(m_{min})$ (for an ordered of the RCS $W_{max} = W(m_1)$) the number of equipment for technical devices realizing the comparison operation in the RCS increases dramatically. Indeed, the number of equipment N_0 of the comparison device in the RCS significantly depends on the number of address N_i , performing the operation of parallel subtraction:

In this way, necessity of ensure a high degree of accuracy of comparison requires a significant amount of equipment, which reduces the efficiency of using existing methods of comparing numbers in the RCS. This situation determines the relevance and importance of finding more effective methods of comparing numbers in the RCS, ensuring a high accuracy of W comparison with a minimum numbers N_0 of equipment comparing devices.

In general, the task is interpreted as follows. It is necessary to find $N_0 = min$ at W_{max} , i.e. $N_0(W_{max}) = min$. As shown above $W_{max} = W(m_1)$. In this case, a change in the base m_i ($i = \overline{1, n}$) affects only the number N_i of the equipment of the group of adders. In this case, the problem is correctly formulated as a definition (2)

$$N_1(W_{max}) = min . \quad (2)$$

Obviously, with high accuracy equal to the unit length of the interval, $W_{max} = W(m_i = 2)$. However, in this case $N_1(W_{max}) = max$, и this result does not satisfy the condition (2). On the other hand – $N_1(W_{min}) = min$. Thus, it is necessary to develop such a method of comparison (2). This problem is solved by the method described below.

2.2 The method of arithmetic comparison of numbers in the RCS (the method of parallel subtraction with the comparison of residues)

We introduce an additional operation of comparing the residues a_n and b_n the magnitude of the bases m_n of the RCS. In this case, the result of the comparison of the residuals simultaneously with the result of the comparison of the single-row code $K_N^{(nA)}$ and $K_N^{(nB)}$, is determined by the solution of the problem (2), i.e. W_{max} and with the minimum amount of equipment N_{min} is the result of solving the operation of

comparing two numbers in RCS. The algorithm for determining the result of an arithmetic comparison operation can be represented as follows:

$$\left\{ \begin{array}{l} \text{if } n_A > n_B, \text{ then } A > B; \\ \text{if } n_A < n_B, \text{ then } A < B; \\ \text{if } n_A = n_B, \text{ and at the same time} \\ \left\{ \begin{array}{l} a_n = b_n, \text{ at } A = B, \\ a_n > b_n, \text{ at } A > B, \\ a_n < b_n, \text{ at } A < B. \end{array} \right. \end{array} \right. \quad (3)$$

The set of relations (3) represents the general algorithm for the implementation of the operation of arithmetic comparison of numbers in the RCS. It is advisable to consider examples of specific performance of the operation of arithmetic comparison of numbers in the RCS. Let the RCS be given by bases, $m_1 = 2$, $m_2 = 3$ and $m_3 = 5$. Code words are given in Table 1. In Table 2 given constants $a_n(b_n)$ presented in a given RCS, and in Table 3, constants of a single-row code are $\Delta \cdot m_n$

$$\left(\Delta = 0, \prod_{i=1}^{n-1} m_i \right) m_i \text{ on the basis of the RCS } (i = \overline{1, n-1}).$$

Table 1. Table of code words in RCS

| A | A in RCS | | | A | A in RCS | | |
|-----|------------|-----------|-----------|-----|------------|-----------|-----------|
| | $m_1 = 2$ | $m_2 = 3$ | $m_3 = 5$ | | $m_1 = 2$ | $m_2 = 3$ | $m_3 = 5$ |
| 0 | 0 | 00 | 000 | 15 | 1 | 00 | 000 |
| 1 | 1 | 01 | 001 | 16 | 0 | 01 | 001 |
| 2 | 0 | 10 | 010 | 17 | 1 | 10 | 010 |
| 3 | 1 | 00 | 011 | 18 | 0 | 00 | 011 |
| 4 | 0 | 01 | 100 | 19 | 1 | 01 | 100 |
| 5 | 1 | 10 | 001 | 20 | 0 | 10 | 000 |
| 6 | 0 | 00 | 001 | 21 | 1 | 00 | 001 |
| 7 | 1 | 01 | 010 | 22 | 0 | 01 | 010 |
| 8 | 0 | 10 | 011 | 23 | 1 | 10 | 011 |
| 9 | 1 | 00 | 100 | 24 | 0 | 00 | 100 |
| 10 | 0 | 01 | 000 | 25 | 1 | 01 | 000 |
| 11 | 1 | 10 | 001 | 26 | 0 | 10 | 001 |
| 12 | 0 | 00 | 010 | 27 | 1 | 00 | 010 |
| 13 | 1 | 01 | 011 | 28 | 0 | 01 | 011 |
| 14 | 0 | 10 | 100 | 29 | 1 | 10 | 100 |

Table 2. Table of constants

| γ_3 | Constants |
|------------|--------------|
| 000 | (0, 00, 000) |
| 001 | (1, 01, 001) |
| 010 | (0, 10, 010) |
| 011 | (1, 00, 011) |
| 100 | (0, 01, 100) |

Example 1. Let the compared numbers be represented as $A_{23} = (1,10,011)$ and $B_{21} = (1,00,001)$. In this case, the values of the constants (Table 2) determine the values $A_{m_n} = A_{23} - a_n = (0,10,000)$, $B_{m_n} = B_{21} - b_n = (01,00,000)$, which corresponds to the shift of the operands A and B to the left edge of the interval $[20, 25)$. Then we use the constants of the single-row code (Table 3), we determine the single-row code for the input numbers in the form:

$$K_N^{(n_A)} = K_6^{(4)} = \{110111\}; K_N^{(n_B)} = K_6^{(n_B)} = K_6^{(4)} = \{110111\},$$

where

$$N_1 = \prod_{i=1}^{n-1} m_i = 6.$$

At the same time the comparison result $a_n = 011 > b_n = 001$ is determined in parallel with the $(n = \lceil \log_2(m_n - 1) \rceil + 1)$ -th bit comparison circuit in time. So, if $n_A = n_B = 4$, in accordance with the above algorithm, we determine that $A_{23} > B_{21}$.

Table 3. Table of constants

| $(0 \div N - 1) \cdot m_n$ | RCS | | Number of zero position |
|----------------------------|-----------|-----------|-------------------------|
| | $m_1 = 2$ | $m_2 = 3$ | |
| $0 \cdot m_3 = 0$ | 0 | 00 | 1 |
| $1 \cdot m_3 = 5$ | 1 | 10 | 2 |
| $2 \cdot m_3 = 10$ | 0 | 01 | 3 |
| $3 \cdot m_3 = 15$ | 1 | 00 | 4 |
| $4 \cdot m_3 = 20$ | 0 | 10 | 5 |
| $5 \cdot m_3 = 25$ | 1 | 01 | 6 |

Example 2. Let the compared numbers be represented as $A_{23} = (1,10,011)$, $B_3 = (1,00,011)$. In this case, the following differences are determined from the values of the constants (Table 2):

$$A_{m_n} = A_{23} - a_n = (0,10,00) \text{ and } B_{m_n} = B_3 - b_n = (1,00,000),$$

which corresponds to the shift of the operand A_{23} to the left edge of the interval $[20, 25)$, and the operand B_3 to the left edge of the interval $[0, 5)$. Next, using the constants of the single-row code (Table 3), we determine the single-row code for the considered input operands A_{23} , and B_3 :

$$K_N^{(n_A)} = K_6^{(5)} = \{1011111\}, K_N^{(n_B)} = K_6^{(1)} = \{1111110\}.$$

So, if $n_A = 5 > n_B = 1$, in accordance with the above algorithm, we determine that $A_{23} > B_3$.

2.3 The arithmetical method of comparison of numbers in the RCS (the method of comparison with a constant)

Consider the method of implementing the operation of arithmetic comparison of numbers in the RCS. The essence of this method is that not the operands A and B , are directly compared, but the quantities $\chi = (A - B) \bmod M = (\gamma_1, \gamma_2, \dots, \gamma_n)$ and m_1 .

In this case, the value is determined:

$$\chi_{m_1} = \chi - \gamma_1 = (0, \gamma'_2, \gamma'_3, \dots, \gamma'_n),$$

where constants $\gamma_1 = (\gamma_1, \gamma'_2, \gamma'_3, \dots, \gamma'_n)$ and $\gamma_1 = (a_1 - b_1) \bmod m_1$ are represented in a given RCS.

Then the general algorithm for comparison of the operands is presented in the form:

$$\begin{cases} A > B, \text{ at } \chi_{m_1} \leq m_1; \\ A < B, \text{ at } \chi_{m_1} > m_1; \\ A = B, \text{ at } \chi_{m_1} = 0. \end{cases} \quad (4)$$

By dialing constants

$$0, m_1, 2m_1, \dots, (N-1) \cdot m_1 \left(N = \prod_{i=1}^n m_i \right),$$

which were represented in the RCS with bases m_2, m_3, \dots, m_n , the construction of a single-row code in the form of:

$$K_n^{(n_\chi)} = \{z_N z_{N-1} \dots z_2 z_1\},$$

where

$$\begin{cases} \chi_{m_1} - 0 = z_1, \\ \chi_{m_1} - m_1 = z_2, \\ \chi_{m_1} - 2m_1 = z_3, \\ \dots \dots \dots \\ \chi_{m_1} - (N-1)m_1 = z_n. \end{cases}$$

In addition

$$z_{n_\chi} = 0, \text{ for } \chi_{m_1} - n_\chi m_1 = 0,$$

$$z_{n_\chi} = 1, \text{ for } \chi_{m_1} - n_\chi m_1 \neq 0.$$

The first module m_1 RCS is also represented by a single-row code of length N binary digits, in which the second place on the right will be zero ($m_1 - 1 \cdot m_1 = 0$), and on the rest - one, i.e. the single-row unitary code will be represented as: $K_n^{(2)} = \{11\dots 101\}$.

Further, by known methods, in accordance with algorithm (4), operands A and B , represented by a single-row code, are compared.

Example 3. For the above RCS we consider an example of the implementation of this method. Let the numbers be equal $A = (0,01,010)$ and $B = (1,10,011)$. We define the value:

$$\begin{aligned} \chi &= (A - B) \text{ mod } M = [(0-1) \text{ mod } 2, (01-10) \text{ mod } 3, (010-011) \text{ mod } 5] = \\ &= (1,10,100). \end{aligned}$$

By value $\gamma_1 = 1$ we define a constant in the form $\gamma_1 = (1,01,001)$ (Table 4). Then we perform the operation:

$$\chi_{m_1} = \chi - \gamma = (00,01,011).$$

The operand χ_{m_1} , which is a multiple of the module $m_1 = 2$ value, goes to the first inputs of the corresponding adders, the second inputs of which receive the corresponding constants (Table 5).

Table 4. Constants

| γ_1 | Constants |
|------------|--------------|
| 00 | (0, 00, 000) |
| 01 | (1, 01, 001) |

Table 5. Table of constants number of zero position

| $(0 \div N - 1) \cdot m_1$ $\Delta \cdot 2$ | RCS | | Number of zero position |
|--|-----------|-----------|-------------------------|
| | $m_2 = 3$ | $m_3 = 5$ | |
| 0 $m_1 = 0$ | 00 | 000 | 1 |
| 1 $m_1 = 2$ | 10 | 010 | 2 |
| 2 $m_1 = 4$ | 01 | 100 | 3 |
| 3 $m_1 = 6$ | 00 | 001 | 4 |
| 4 $m_1 = 8$ | 10 | 011 | 5 |
| 5 $m_1 = 10$ | 01 | 000 | 6 |
| 6 $m_1 = 12$ | 00 | 010 | 7 |
| 7 $m_1 = 14$ | 10 | 100 | 8 |
| 8 $m_1 = 16$ | 01 | 001 | 9 |
| 9 $m_1 = 18$ | 00 | 011 | 10 |
| 10 $m_1 = 20$ | 10 | 000 | 11 |
| 11 $m_1 = 22$ | 01 | 010 | 12 |
| 12 $m_1 = 24$ | 00 | 100 | 13 |
| 13 $m_1 = 26$ | 10 | 001 | 14 |
| 14 $m_1 = 28$ | 01 | 011 | 15 |

Since $\chi_{m_1 - 14_{m_i}} = 0$, then the single-row code will take the form:

$$K_n^{(n, \chi)} = K_{15}^{(14)} = \{0111111111111111\}.$$

In accordance with algorithm (4), we determine that $A < B$.

The advantage of the considered method is to ensure maximum accuracy of comparison with an acceptable amount of equipment for its implementation.

2.4 The method of algebraic comparison of numbers in the RCS

It is easy to go from the implementation of the operation of arithmetic comparison of numbers to the algebraic comparison of numbers. In this case, the compared numbers A and B have one additional significant digit, i.e. the number is accompanied by a indication $\Omega_A(\Omega_B)$ of the sign $signA(signB)$, where:

$$\Omega_A(\Omega_B) = \begin{cases} 0, & \text{if } A(B) \geq 0, \\ 1, & \text{if } A(B) < 0. \end{cases}$$

In this way, the compared numbers are presented in the form:

$$A' = (\Omega_A; A) = \{\Omega_A; (a_1, a_2, \dots, a_n)\}, \text{ and } B' = (\Omega_B; B) = \{\Omega_B; (b_1, b_2, \dots, b_n)\},$$

and the method of comparing numbers A' and B' is determined by the set of operations (5).

We will conduct a comparative analysis of the implementation time of the comparison operation of two numbers A and B or the proposed method and the most well-known. The essence of the known method is to convert the numbers A and B from the system of residual classes to the positional number system A_{PNS}, B_{PNS} and the further comparison of the operands of A_{PNS} и B_{PNS} . Moreover, the transfer of numbers from RCS to PNS is made in accordance with the expression:

$$A_{PNS} = \left| \sum_{i=1}^n a_i B_i \right|_M,$$

where $a_i = [A_{PNS} / m_i] m_i$, where B_i – is the orthogonal basis over the m_i base of the RCS.

$$\left\{ \begin{array}{l} \text{if } n_A > n_B \left\{ \begin{array}{l} \Omega_A = 0, \Omega_B = 0, \text{ then } A' > B', \\ \Omega_A = 0, \Omega_B = 1, \text{ then } A' > B, \\ \Omega_A = 1, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 1, \Omega_B = 1, \text{ then } A' > B'; \end{array} \right. \\ \\ \text{if } n_A < n_B \left\{ \begin{array}{l} \Omega_A = 0, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 0, \Omega_B = 1, \text{ then } A' > B', \\ \Omega_A = 1, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 1, \Omega_B = 1, \text{ then } A' > B'; \end{array} \right. \\ \\ \text{if } n_A = n_B \left\{ \begin{array}{l} a_n = b_n \left\{ \begin{array}{l} \Omega_A = 0, \Omega_B = 0, \text{ then } A' = B', \\ \Omega_A = 0, \Omega_B = 1, \text{ then } A' > B', \\ \Omega_A = 1, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 1, \Omega_B = 1, \text{ then } A' = B'; \end{array} \right. \\ \\ a_n > b_n \left\{ \begin{array}{l} \Omega_A = 0, \Omega_B = 0, \text{ then } A' > B', \\ \Omega_A = 0, \Omega_B = 1, \text{ then } A' > B', \\ \Omega_A = 1, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 1, \Omega_B = 1, \text{ then } A' < B'; \end{array} \right. \\ \\ a_n < b_n \left\{ \begin{array}{l} \Omega_A = 0, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 0, \Omega_B = 1, \text{ then } A' > B', \\ \Omega_A = 1, \Omega_B = 0, \text{ then } A' < B', \\ \Omega_A = 1, \Omega_B = 1, \text{ then } A' > B'. \end{array} \right. \end{array} \right. \quad (5)$$

For the known and proposed methods of comparing numbers, the implementation time is determined by the corresponding mathematical relations

$$T_{RCS}^{(1)} = t^{(11)} + t^{(12)} + t^{(13)} + t^{(14)}, \quad (6)$$

$$T_{RCS}^{(2)} = t^{(21)} + t^{(22)} + t^{(23)}, \quad (7)$$

where

- $t^{(11)}$ – is the implementation time of the multiplication operation for the maximum m_n by the magnitude of the RCS module (the implementation time of the multiplication operation of two-digit binary numbers $K = \lceil \log_2(m_n - 1) \rceil + 1$);
- $t^{(12)}$ – is the time $(n-1)$ of the i th addition of two numbers of the type $a_i B_i + a_{i+1} B_{i+1} (i = \overline{1, n-1})$;
- $t^{(13)}$ – is the time for determining the deduction of the number of $A_{PNS} (B_{PNS})$;
- $t^{(14)}$ – is the comparison time of the positional operands of the A_{PNS} and B_{PNS} ;
- $t^{(21)}$ – is the time of implementation of the operation of subtraction in the RCS $A_{COK} - a_i$;
- $t^{(22)}$ – is the time of implementation of the operation of subtraction in the RCS $A_{m_i} - k \cdot m_i$;
- $t^{(23)}$ – is the time of comparison of two positional N -bit single-row unitary codes of two corresponding numbers.

It is known that the time of addition of t_c and t_y multiplication of two operands in the PNS is determined by the following relations:

$$t_c = \tau(2\rho - 1) \text{ and } t_y = 2\tau\rho^2, 0$$

where ρ is the bit width of the processed operands; τ – is the time of "shift" of one binary digit. In this case, the time of "triggering" of the logical element AND (OR) is determined by the expression: $t_{AND} \approx t_{OR} \approx \tau / 2$, and $t^{(23)} \approx 6t_n$ and expression

$$t^{(13)} \approx 2\tau \left\{ \left[\log_2 \left(\prod_{i=1}^n m_i - 1 \right) \right] + 1 \right\}^2.$$

Taking into account the above, relations (6) and (7), respectively, are presented in the form:

$$T_{RCS}^{(1)} = 2\tau k^2 + (n-1)\tau(2k-1) + 2\tau \left\{ \left[\log_2 \left(\prod_{i=1}^n m_i - 1 \right) \right] + 1 \right\}^2 + 3\tau; \quad (8)$$

$$T_{RCS}^{(2)} = 2\tau + 2\tau + 3\tau. \quad (9)$$

According to with expressions (8) and (9), values are calculated $T_{RCS}^{(1)}$, $T_{RCS}^{(2)}$ (table 6) for various l – byte bit CS grids ($l = \overline{1,4}$). From Table 6 it can be seen that with increasing length of the CS discharge grid, which is typical of the current trend in the development of systems and tools for processing digital information, the effectiveness of applying the proposed methods for comparing numbers, as compared to existing ones, increases.

Table 6. Comparative analysis of the implementation time of the comparison operation

| T | l | | | |
|-----------------|-----|-----|------|------|
| | 1 | 2 | 3 | 4 |
| $T_{RCS}^{(1)}$ | 324 | 870 | 1916 | 3334 |
| $T_{RCS}^{(2)}$ | 6 | 6 | 6 | 6 |

On the basis of the proposed methods, algorithms for their implementation have been developed, in accordance with which a class of patentable devices has been developed for performing arithmetic and algebraic comparisons of numbers in an RCS [13-15]. Prospective direction of a further research is the argumentation of practical recommendations concerning a realization of the introduced method and the ways of its use in different mechanisms of an information security of telecommunications networks and systems [16-31].

3 Conclusion

In this paper, a method is proposed for comparing numbers in a non-positional number system of remainder classes, which is based on the principle of obtaining and comparing a unitary single-row code.

The developed method can be used to improve advanced computer systems and their components. In particular, its practical use allows to increase the performance of computer calculations and the reliability of information systems.

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