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Method for Recognizing and Processing Complex Signals

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Abstract. The methods of processing digital samples of complex structure signals with unknown parameters are considered. With the use of algebraic methods, the following tasks are sequentially solved: clock synchronization, determining the range of carrier frequencies, the multiplicity of phase modulation and obtaining a stream of information bits. The methods for improving the quality of processing digital samples of signals based on solving special overdetermined systems of linear equations are proposed. The estimation of efficiency of the offered method is carried out by an imitation statistical modeling. The advantages of the proposed methods of signal processing for the telecommunications and radio monitoring systems are shown.

Keywords. Orthogonal Frequency Division Multiplexing, digital sampling, linear algebraic equations, correlation convolutions.

1 Introduction

The construction of effective information transmission systems is inextricably linked with the problem of intensifying the usage of the time and frequency-energy resource of communication channels. One of the ways to solve this problem is using the complex signals with combined types of modulation in combination with the methods of spectrum narrowing and noise-resistant coding [1-5]. In this connection, the structure of the signals used to transmit information is becoming complicated, and, consequently, the algorithms of their processing are becoming complicated as well [6-11].

The most promising type of signal code constructions in wireless networks is OFDM (Orthogonal Frequency Division with Multiplexing) [2-6, 12]. The basic idea of building such signals is arranging a set of mutually orthogonal frequency subchannels so that, on the one hand, one subchannel does not interfere with the other, and on the other hand, the spectra of the subchannels overlap. Due to the orthogonality of the linear subchannels, each of them can be considered independently of the others. Errors caused by the interference in one of the subchannels do not lead to errors in the other. As a result, only a small part of the transmitted information is distorted. Error-correcting coding being used the errors can be corrected [13-18]. The structure of signals with multiple simultaneously operating subcarrier frequencies has well established itself in conditions of heterogeneity of the propagation medium. In recent years, the capabilities of systems with OFDM signals have evolved significantly. Such signals began to be used in a wide variety of telecommunication systems operating in different radio frequency bands.

The complex structure of such signals, the a priori uncertainty of the channel properties cause significant difficulties in solving the problems of radio control and radio monitoring. A distinctive feature of such tasks is the absence of data on the structure and informative parameters of the measured signals. This information should be obtained from the results of the study, with high accuracy and as soon as possible. Therefore, the tasks of developing mathematical methods for analyzing complex signals based on digital measurement sequences are highly relevant.

2 Mathematical model of OFDM signals

For correct choice of the methods for digital analysis of the primary parameters of OFDM signals, a brief description of their basic properties is necessary. Arbitrary OFDM signal $S_j(t)$ on j-th modulation interval T_p is formed by algebraic summation of the several harmonic oscillations of the same amplitude. Each of the oscillations has m options of modulation phase shift. The value m determines the multiplicity of the used phase (PM) modulation and corresponds to the base of the numerical source code. Commonly, $m = 2^k$ where k is the number of binary symbols (bits) represented by the elementary signal on one modulation interval. When using relative phase coding and a unit value of the amplitude of the oscillation subcarriers, the mathematical model of the signal can be represented as the following sequence:

$$S_j(t) = \sum_{i=0}^{n_f-1} \sin\left\{ 2\pi\left(f_0 + \frac{i}{T} \right) \cdot \left(t - T_p \left\lfloor \frac{t}{T_p} \right\rfloor \right) + \varphi_{j,i} \right\}, \quad (1)$$

where t – current time; f_0 – the lowest subcarrier frequency in the signal spectrum; $T = 1/\Delta f$ – inverse of the minimum subcarrier spacing Δf ; n_f – the number of frequencies used ; $\varphi_{j,i}$ – the value of the manipulation angle of i-th fluctuation on j-th modulation interval. This angle can take one of m values depending on the manipulation code used. The informative features in the signal described by model (1) are relative phase jumps in carrier frequencies. These jumps are measured for each of the frequency subcarriers separately: $\varphi_{j,i} - \varphi_{j-1,i}$, $i = 0,\ldots,n_f - 1$. The time parameters of the modulation interval used in model (1) are tied by the relation:

$$T_p = T + \Delta T = \frac{1}{\Delta f} + \Delta T, \quad (2)$$

where ΔT - the duration of the prefix part of the signal. The prefix part (hereinafter – the prefix) is a repeating (to the exact sign) initial part of the signal added at the end of the modulation interval T_p. Prefix structure is used to facilitate synchronization in the presence of channel irregularities. For OFDM synchronization violations the signal can be correctly received and processed for any segment with the duration of T within the full modulation interval .. T_P ... Commonly, the choice of the prefix duration on the modulation interval corresponds to the ratio $\Delta T = (0,1 \div 0,5)T$. The sign of the prefix depends on the value of the following parameter:

$$P = f_0 \bmod (\Delta f) . \tag{3}$$

For existing OFDM standards, parameter (3) can take two values that determine the sign of the cyclic prefix: when $P = 0$ the prefix is positive and when $P = \Delta f/2$ – inverse. Fig. 1 gives a qualitative idea of the form of the signal envelope constructed in accordance with model (1) on two adjacent modulation intervals with $n_f = 16$ and $T_p = 1,47 \cdot T$. At the end of each of the intervals T_p the inverse cyclic continuation of the signal with duration ΔT is located, which repeats, up to a sign, the shape of the initial segment of the signal on the modulation interval. For the example in question $P = \Delta f/2$ therefore the prefix part is the inverse of the initial part of the signal.

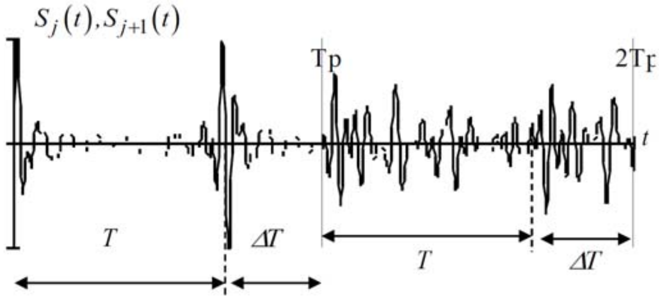

Fig. 1. Example of OFDM signal

3 The main stages of the structural analysis of OFDM signals

A comprehensive analysis of the properties of complex signals is advisable to implement on the basis of phased processing. At each stage, only a part of the signal parameters is determined. Given the fact that OFDM signals contain a prefix, it is advisable to use the correlation method for determining structural time parameters T_p and T at the first stage. This technique is based on the principle of "sliding" time window. This makes it possible to determine the following parameters of an OFDM signal: the value of the orthogonality interval, the duration of the modulation interval, and the value of the frequency spacing between the channels.

At the second stage of the analysis, the tasks of determining the number and the values of service and information channel frequencies, as well as, the signal phase demodulation.

The two-stage processing results in the possibility to extract an information flow from signals of an a priori unknown structure without using traditional fast Fourier transform algorithms (FFT – Fast Fourier Transformation).

3.1 Correlation method for determining the time parameters of OFDM signals

We propose a correlation method for determining structural time parameters T_p and T. The basis is the "sliding time window" principle. The most probably value of the time interval between the most correlated segments (with the "+" or "-" sign) of the segments from the digital sample of signal measurements is determined. The assessment of T_p – the most likely period of the emergence of "bursts" of correlation in the process of moving the viewing window on the samples of the array of measurements $Q = \{q_0, q_1, \ldots\}$ is determined as well. The scheme of the calculation procedure is presented in Fig. 2. For the correlation analysis the two vectors, each containing K elements of array Q in two non-overlapping time observation windows of the signal are formed,

$$\begin{aligned} Y0 &= \{q_j, q_{j+1}, \ldots q_{j+K-1}\}, \\ Y1 &= \{q_{j+i+K}, q_{j+i+K+1}, \ldots q_{j+i+2K-1}\}, \end{aligned} \tag{4}$$

removed from each other by i, $i = 0, \ldots, M$. The position of the second time window corresponding to the vector $Y1$ is determined by the successive change in the offset index $i = 0, \ldots, M$ that ensures its "slip" along the signal sample Q at each of the values $j = 0 \ldots L$.

For a wide range of analyzed signals, for example, for the 0.3-3.4 KHz frequency band with minimum quality ADC, the most universal limits of the values of these parameters, resulting in a quick and accurate assessment, are $K = 10 \div 30$, $M = 200 \div 300$ and $L = 1000$.

At each value of index j (moving the window slip area) the $M+1$ dimensional vector is being formed

$$V_j = \{v_0^j, v_1^j, \ldots, v_M^j\}, \tag{5}$$

the elements of which are the coefficients of mutual correlation of vectors $\mathbf{Y}0$ and $\mathbf{Y}1$. The calculations (according to the Fig. 2) are performed after centering and normalizing the vectors by the formulas:

$$Y0_N = \left[Y0_j - \frac{1}{K}\sum_{i=0}^{K-1} Y0_i \right] \cdot \left[\frac{1}{K}\sum_{i=0}^{K-1} Y0_i \right]^{-1};$$
$$Y1_N = \left[Y1_j - \frac{1}{K}\sum_{i=0}^{K-1} Y1_i \right] \cdot \left[\frac{1}{K}\sum_{i=0}^{K-1} Y1_i \right]^{-1}. \qquad (6)$$

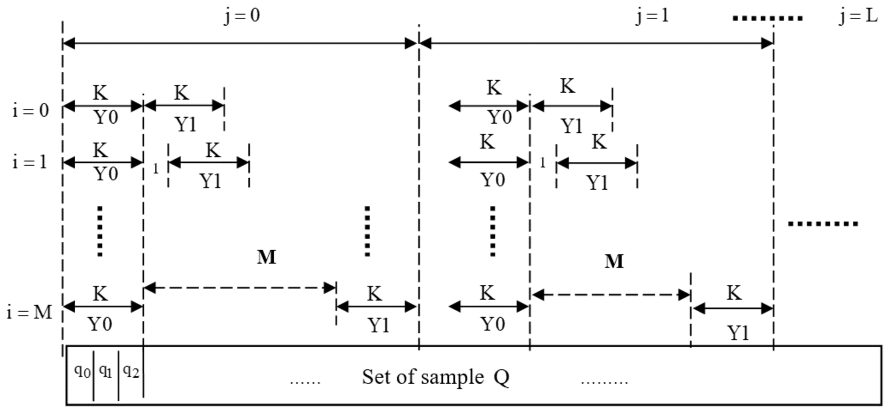

Fig. 2. Calculation scheme

The resulting vectors $\mathbf{Y}0_N$ and $\mathbf{Y}1_N$ in normalized space have the same length, equal to $\sqrt{K}$, and the cosine of the angle between these vectors is equal to the cross-correlation coefficient:

$$cos(\mathbf{Y}0_н, \mathbf{Y}1_н) = \sum_{i=0}^{K-1} Y0_{нi} \cdot Y1_{нi} = r(\mathbf{Y}0_н, \mathbf{Y}1_н). \qquad (7)$$

Since the prefix is a repetitive (up to sign) part of the OFDM signal, ideally the correlation coefficient between these parts is ± 1.

Fig. 3 shows the distribution of the values of the elements of the vector $\mathbf{V}_j$ calculated according to a specific implementation OFDM signal (16 carrier frequencies, a modulation rate – 75 bauds) at $M = 200$. The presence of pronounced extreme values which are close in magnitude to unity is obvious. According to the results of calculations when $j = 0 \ldots L$ the two new vector $\mathbf{V}1 = \{v1_0, v1_1, \ldots, v1_L\}$ and $\mathbf{V}2 = \{v2_0, v2_1, \ldots, v2_L\}$ are formed. Their elements are calculated according to the rules:

$$\begin{matrix} v1_j = match\left[\min(\mathbf{V}_j), \mathbf{V}_j\right]_0 + K; \\ v2_j = match\left[\max(\mathbf{V}_j), \mathbf{V}_j\right]_0 + K; \end{matrix} \quad j = 0 \ldots L. \qquad (8)$$

Here the function $match[x, \mathbf{X}]$ calculates the indices of the elements of the vector $\mathbf{X}$ equal to x, where the index 0 in the function (8) indicates a selection of the element with a minimum sequence number, if there are several such elements in the vector. The elements of the vector $\mathbf{V}1$ represent the number of sampling intervals that fit between the initial elements $\mathbf{Y}0$ and $\mathbf{Y}1$ with minimal (negative) correlation on j-th step of moving the observation window. Accordingly, the elements of the vector $\mathbf{V}2$ are calculated for the maximum (positive) correlation of the vectors $\mathbf{Y}0$ and $\mathbf{Y}1$. Simultaneous determination of the maximum and minimum is necessary to reveal the value of function (3). It is obvious that the elements of the vectors $\mathbf{V}1$ and $\mathbf{V}2$ defined by expression (8), can take values only in the range $K \dots K+M$. To study the statistical distribution of the values of the elements the histograms for the elements of the vectors $\mathbf{V}1$ and $\mathbf{V}2$ are formed:

$$\begin{aligned} \mathbf{H}1 &= \{h1_0, h1_1, \dots, h1_{K+M}\}, \\ \mathbf{H}2 &= \{h2_0, h2_1, \dots, h2_{K+M}\}. \end{aligned} \tag{9}$$

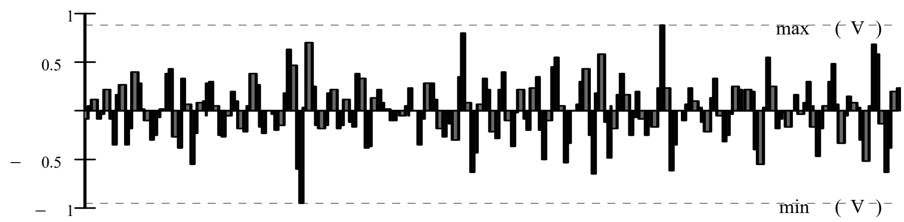

Fig. 3. Cross-correlation coefficients

Obtaining distributions (9) gives an opportunity to estimate the most likely value Num – the number of sampling intervals between the initial measurements of the segments of the digital sample $\mathbf{Q}$ with maximal (positive or negative) correlation:

$$Num = \begin{cases} \underset{\langle \max(\mathbf{H}1) \geq \max(\mathbf{H}2) \rangle}{match\left[\max(\mathbf{H}1), \mathbf{H}1\right]_0}; \\ \underset{\langle \max(\mathbf{H}1) < \max(\mathbf{H}2) \rangle}{match\left[\max(\mathbf{H}2), \mathbf{H}2\right]_0}. \end{cases} \tag{10}$$

The value Num determines the number of sampling intervals that fit on the orthogonality interval of the signal T. It gives an opportunity to find two interrelated OFDM signal parameters: the orthogonality interval and the minimum carrier frequency spacing:

$$T = \frac{Num}{f_\partial} \text{ and } \Delta f = T^{-1}. \tag{11}$$

Besides, choosing the corresponding method of detecting the Num value according to the condition specified in (10) automatically determines the value of the function (3), and, consequently, the ratio between the frequency parameters f_0 and Δf. If the value Num is determined by the first line of expression (10), then $f_0 = \left(k + \frac{1}{2}\right) \cdot \Delta f$. Otherwise the minimal carrier frequency is multiple to the spacing of the carrier frequencies: $f_0 = k \cdot \Delta f$ where k – any positive integer.

As a result of processing histograms (9) according to (10) based on the values of function (3), only one of the vectors $\mathbf{V}1$ or $\mathbf{V}2$ is left for the further analysis, hereinafter denoted $\mathbf{V}^*$. This is possible because the prefix repetition sign is defined. Based on the elements of the vector $\mathbf{V}^*$ another vector $\mathbf{V}3$ is formed for the analysis:

$$\mathbf{V}3 = match\left[Num, \mathbf{V}^*\right]. \tag{12}$$

The elements of this vector are equal to the numbers of the elements of the vector $\mathbf{V}^*$ in which the numbers Num are located. The feature of the vector $\mathbf{V}^*$ provided that the analyzed signal belongs to the OFDM class, is that it contains a sequence of periodic series of numbers which are close or equal to Num. Therefore, the values of the elements of $\mathbf{V}3$ in order of increasing their indices will be the segments (series) of an ordinary positive integer sequence with some gaps in the sequence. Small gaps can be observed inside the series too. A possible approximation of the sequence of the elements of $\mathbf{V}3$ is illustrated by the following expression:

$$\mathbf{V}3 = \left\{ \underbrace{11, 12, 14, 15, 16}_{\text{series №1}}, \underbrace{105, 107, 108, 110}_{\text{series №2}}, \ldots, \underbrace{620, 621, 622, 623, 624}_{\text{series №}\, n_c} \right\}.$$

The length of series of consecutive numbers (position numbers) may differ due to measurement errors, features of the signal envelope and rounding during calculations. However, when the vector length is sufficient, averaging results converges to the true estimation in accordance with the law of large numbers. To exclude "fragmentation" of the series, small gaps between adjacent numbers of the series must be ignored. It has been empirically found that in most cases the number N_i should be considered to belong to the current series of numbers if $N_i - N_{i-1} \le \varepsilon$ where $\varepsilon = 3 \div 5$. In general, the structure of the vector $\mathbf{V}3$ can be depicted as shown in Fig. 4.

$$\left\{ \underbrace{\boxed{v3_0, \ldots, v3_{k_1}}}_{\text{series № 1}}, \underbrace{\boxed{v3_{n_2}, \ldots, v3_{k_2}}}_{\text{series № 2}}, \ldots, \underbrace{\boxed{v3_{n(n_c-1)}, \ldots, v3_{k(n_c-1)}}}_{\text{series № }(n_c-1)}, \underbrace{\boxed{v3_{n(n_c)}, \ldots, v3_{k(n_c)}}}_{\text{series № } n_c} \right\}$$

Fig. 4. Example of the $V3$ series structure

In Fig. 4: $v3_{n_j}$, $v3_{k_j}$ – initial and final elements of the j-th series of the consecutive numbers in the vector $\mathbf{V}3$; n_c – the total number of the identified series. Using the presented structure and values of the elements of the vector $\mathbf{V}3$, it is possible to determine the number of sampling intervals that fit between adjacent pairs of mutually correlated segments of signal measurements, i.e. a period of "bursts" of correlation:

$$Num1 = \frac{\left(v3_{k_{(n_c-1)}} - v3_{k_2}\right) - \left(v3_{n_{(n_c-1)}} - v3_{n_2}\right)}{2\left(n_c - 2\right)}. \tag{13}$$

Using the obtained value $Num1$, we can determine the average value of the modulation interval T_p and, therefore, the average modulation rate W:

$$T_p = \frac{Num1}{f_\partial},\ W = T_p^{-1}. \tag{14}$$

For the final determination of the time-frequency structure of the signal, we must find the number of carrier frequencies n_f and the vector of their nominal values $\mathbf{F} = \left\{f_0, \ldots, f_{n_f-1}\right\}$. This can be done on the basis of previously obtained values $T_p, T, \Delta f$ when the position of the element of the array $\mathbf{Q}$ corresponding to the beginning of the first full modulation interval is determined correctly. The beginning of a reliably identified clock interval could be most correctly associated with the beginning of the second series of maximal responses of correlators in the vector $\mathbf{V}3$, since, due to the randomness of the beginning of the observation, the first series may be incomplete. It should be taken into account that the beginning of a series of maximal responses of correlation of the segments from K samples must appear before the next modulation interval actually begins. Therefore, to fall within the interval with the duration T_p (taking into account that demodulation can be performed on any segment T within T_p) it is necessary to add the number $K/2$ to the starting sample, at least. Then the beginning of the modulation interval can be assumed to coincide with the next element number in the sequence

$$n_T^0 = round\left\{\left[v3_{n_2} \bmod\left(\frac{f_\partial}{W}\right)\right] + \frac{K}{2}\right\}. \tag{15}$$

Here $round\left(x\right)$ – the rounding function to the nearest integer. The lowest possible frequency f_0 in the group of carrier frequencies is determined by the value of function (3) and the fulfillment of the corresponding condition in (10):

$$f_0 = \begin{cases} \frac{1}{2}\Delta f, & \max(\mathbf{H}1) \geq \max(\mathbf{H}2); \\ \Delta f, & \max(\mathbf{H}1) < \max(\mathbf{H}2). \end{cases} \quad (16)$$

The maximal number of subcarrier frequencies (or half the number of quadrature components) that can fit in the channel band F_{ef} is

$$n_{f\,\max} = round\left(\frac{F_{ef} - f_0}{\Delta f} + 1\right). \quad (17)$$

3.2 Determining the amount and nominal values of subcarrier frequencies of OFDM signals

The correlation method, considered above, allows making a reliable assessment of the main structural parameters of OFDM – T and T_P. The value of $\Delta f = T^{-1}$ uniquely defines the spacing of adjacent subcarrier frequencies. The minimal value of the subcarrier frequency and the maximal possible number of subcarriers placed within the signal bandwidth $n_{f\,\max}$ are determined from (16) and (17).

The number of samples N taken into account when analyzing a signal on one modulation interval, as well as, the harmonic quadrature $\left(2 \cdot n_{f\,\max}\right)$ define the dimensions of the matrix of the linear algebraic equations system (SLAE) which can be compiled and solved to estimate the frequency range. Depending on the ratio of the vertical and horizontal dimensions of the matrix of coefficients, the system of equations can be overdetermined $\left(N > 2 \cdot n_{f\,\max}\right)$, determined $\left(N = 2 \cdot n_{f\,\max}\right)$ or underdetermined $\left(N < 2 \cdot n_{f\,\max}\right)$. The simplest one is the $\left(N = 2 \cdot n_{f\,\max}\right)$ case because then the SLAE is a joint one almost every time. The number of equations that matches the number of used elements of the digital sample $\mathbf{Q}$ equals to the number of unknowns $\left(2 \cdot n_{f\,\max}\right)$ determining the amplitudes of quadrature components in the spectrum of carrier frequencies OFDM. For the correct solution of SLAU $\left(2 \cdot n_{f\,\max}\right)$ uniformly spaced sample counts $\mathbf{Q}$ starting from the point of beginning of the observation of the first complete clock interval of signal n_T^0 should be selected on i -m modulation interval. For this the following rule is used:

$$n_T^i = n_T^0 + round\left(i \cdot \Delta\right) \text{ where } \Delta = T_P / t_\partial \,. \quad (18)$$

Square matrix of coefficients for unknown SLAE with size $\left(2 \cdot n_{f\,\max}\right) \times \left(2 \cdot n_{f\,\max}\right)$ composed for quadrature components of subcarrier frequencies is formed according to the rule:

$$\mathbf{A}_1 = \left\| a_{i,j} \right\|, \quad i, j = 0, \ldots, \left(2 \cdot n_{f\,\max} - 1\right);$$
$$a_{i,j} = \mathrm{Sin}\left[\omega_j \cdot t_i\right], \quad 0 \le j \le n_{f\,\max} - 1; \qquad (19)$$
$$a_{i,j} = \mathrm{Cos}\left[\omega_j \cdot t_i\right], \quad n_f \le j \le 2 \cdot n_{f\,\max} - 1;$$

where

$$\omega_j = \omega_{\left(j + n_{f\,\max}\right)} = 2\pi\left(f_0 + j \cdot \Delta f\right), \quad j = 0, \ldots, n_{f\,\max}, \quad t_i = n_T^i + i \cdot t_\partial .$$

The column-matrix of free members is formed as a vector of signal measurements on the duration of one orthogonality interval:

$$\mathbf{B}_1 = \left\{ b_0, \ldots, b_{\left(2 \cdot n_{f\,\max} - 1\right)} \right\}; \; b_i = q_i, \; i \cdot t_\partial \in T_P{}^i . \qquad (20)$$

Normal solution of normally defined SLAE

$$\mathbf{A}_1 \cdot \mathbf{X}_1 = \mathbf{B}_1 \quad \Rightarrow \quad \mathbf{X}_1 = \mathbf{A}_1^{-1} \cdot \mathbf{B}_1 \qquad (21)$$

gives an estimation of the amplitude vector of quadrature components $\mathbf{X}_1 = \left\{ x_0^1, \ldots, x_{\left(2 \cdot n_{f\,\max} - 1\right)}^1 \right\}$ which corresponds to the permissible values of carrier frequencies.

On the basis of this solution, it is possible to determine the power distribution vector of the signal between the harmonic oscillations of the carrier frequencies:

$$\mathbf{Y} = \left\{ y_0, \ldots, y_{\left(n_{f\,\max} - 1\right)} \right\}, \; y_i = \left(x_i^1\right)^2 + \left(x_{\left(i + n_{f\,\max}\right)}^1\right)^2, \; i = 0, \ldots, \left(n_{f\,\max} - 1\right). \qquad (22)$$

The case of insufficiently defined SLAE $\left(N < 2 \cdot n_{f\,\max}\right)$ is interesting for analyzing small samples of the signal. To solve such a SLAE, the pseudo inverse matrix method Moore-Penrose can be used. It is known that there is the normal solution of an underdetermined SLAE and is the only one. It is found by: $\mathbf{X}_1 = \mathbf{A}^+{}_1 \cdot \mathbf{B}_1$ where $\mathbf{A}^+{}_1$ – Moore-Penrose pseudo inverse matrix of size $2 \cdot n_{f\,\max} \times 2 \cdot n_{f\,\max}$ which is determined by the ratio: $\mathbf{A}_1 \cdot \mathbf{A}^+{}_1 \cdot A_1 = \mathbf{A}_1$. In practice $\mathbf{A}^+{}_1$ can be found by the formula:

$\mathbf{A}_1^+ = \mathbf{C}^+ \cdot \mathbf{D}^+ = \mathbf{C}^* \cdot (\mathbf{C} \cdot \mathbf{C}^*)^{-1} \cdot (\mathbf{D} \cdot \mathbf{D}^*)^{-1} \cdot \mathbf{D}^*$.

The representation of the matrix $\mathbf{A}_1^+$ in the form of a product of two matrices with the size of $N \times r$ and $r \times N$ is used:

$$\mathbf{A}_1 = \mathbf{D} \cdot \mathbf{C} = \begin{pmatrix} d_{1,1} & \cdots & d_{1,r} \\ \vdots & \ddots & \vdots \\ d_{N,1} & \cdots & a_{N,r} \end{pmatrix} \cdot \begin{pmatrix} c_{1,1} & \cdots & c_{1,N} \\ \vdots & \ddots & \vdots \\ c_{r,1} & \cdots & c_{r,N} \end{pmatrix}.$$

With various skeletal decompositions of the matrix A the same solution for A^+ which can be written in the form: $\mathbf{X}_1 = \mathbf{A}^+{}_1 \cdot \mathbf{B}_1$ is derived. It is a pseudo solution giving a zero residual: $\left\| \mathbf{X}_1 - \mathbf{A}^+{}_1 \cdot \mathbf{B}_1 \right\| = 0$.

The case of $\left(N > 2 \cdot n_{f\,\max}\right)$ is the most advantageous for the maximal recording of signal information on the modulation interval. Due to using additional signal measurements from the sample $\mathbf{Q}$ the system which contains more equations with the same number of unknowns is formed. To form the matrix $\mathbf{A}_2$ and the vector $\mathbf{B}_2$ the maximal number of signal measurements determined by $Num \approx T_P / t_\partial$ on the duration T_P is used:

$$\begin{aligned} &\mathbf{A}_2 = \left\| a_{i,j} \right\|, \quad i = 0, \ldots, (Num - 1), \;\; j = 0 \ldots \left(2 \cdot n_{f\,\max} - 1\right); \\ &a_{i,j} = \mathrm{Sin}\left[\omega_j \cdot t_i\right], \quad 0 \le j \le n_{f\,\max} - 1; \\ &a_{i,j} = \mathrm{Cos}\left[\omega_j \cdot t_i\right], \quad n_{\max} \le j \le 2 \cdot n_{f\,\max} - 1; \end{aligned} \tag{23}$$

$$\mathbf{B}_2 = \left\{ b_0, \ldots, b_{(Num-1)} \right\}, \quad b_k = q_k, \quad k = 0, \ldots, \left(Num - 1\right). \tag{24}$$

SLAE has the form:

$$\mathbf{A}_2 \cdot \mathbf{X}_2 = \mathbf{B}_2\,, \tag{25}$$

and as a rule, has many solutions. To select the only one we need to use some criteria. In practice, the maximum likelihood criterion is used more often. In the case of a normal distribution of the vector $\mathbf{B}_2$ it is equivalent of the least square's criterion:

$$\mathbf{X}^*_2 = (\mathbf{A}_2{}^T \cdot \mathbf{A}_2)^{-1} \mathbf{A}_2{}^T \cdot \mathbf{B}_2\,. \tag{26}$$

An approximate solution of system (26) gives a more accurate result than a strict solution of (25). The noise immunity of the solution is achieved by averaging the disturbing effect of interference when the number of signal measurements exceeds the required minimum. The obtained vector of amplitudes of the quadrature components $\mathbf{X}^*_2$, as well as $\mathbf{X}^*_1$ gives a possibility to calculate the power distribution signal in carrier frequencies using expression (22), wherein x_i^2 is used instead of x_i^1.

For any type of SLAE determining the actual list of carrier frequencies in the OFDM spectrum is performed by comparing the elements of power distribution histograms with a threshold value. The obtained nominal values of frequencies determine the last structural time-frequency parameter of the analyzed signal – the vector of working subcarrier frequencies $\mathbf{F}$.

Thus, the previously obtained signal parameters $T_p, T, \Delta f$, W and the obtained in this subsection vector $\mathbf{F}$ identifies completely its structural properties and makes signal demodulation possible.

4 Conclusions

The considered statistical method for analyzing the structure and demodulation of OFDM signals under conditions of a priori uncertainty of solving radio monitoring tasks has been practically tested. It has demonstrated the high accuracy of parameter identification. The relatively low computational complexity of correlation and algebraic analysis makes it possible to identify the structure and the parameters of signals practically in seconds.

The noise immunity of the analysis is achieved by solving a SLAE with rectangular overdetermined matrixes of coefficients. To eliminate phase errors generated by asynchronous, with respect to the clock intervals of modulation, sampling the method for calculating phase corrections which takes into account the time-frequency parameters of the signal structure is proposed. The application of the phase correction method provides ideal conditions for identifying the modulation type of subcarrier oscillations. Mathematical formalization of solving the problem of determining the modulation multiplicity, based on generating the multimodal reference functions and sequential calculating the degree of mutual correlation, allows us completely automate the process of identifying the secondary parameters which are necessary for demodulating the signals of subcarrier frequencies.

The further researches can be focused on the generalization of the method for any structures of mono and poly frequency signals including those with a linear frequency modulation and also proposed author's method can be used in some different other areas [19-23].

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