The Simultaneous Use of Excel and GeoGebra to Training the Basics of Mathematical Modeling

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Abstract. The main objective of this study is testing the hypothesis that the visualization of simulation results creates the conditions for improving students' knowledge, taking into account the specifics of their professional training. In this article we are exploring how the simultaneous use of Excel and GeoGebra can improve the learning outcomes of engineering students in agricultural universities when learning the basics of mathematical modeling (using as example the mathematical model of mechanical movement of two bodies with their elastic collision). We worked with our students as follows. First, we build and interpret the mathematical model. Then we obtain two alternative computer models: we use Excel spreadsheets for numerical modeling as well as GeoGebra software for analytical-geometrical modeling. By this models we visualize and explore the processes closely related to topics that plays an important role in the training of agricultural production engineers, in particular, with study the movement and interaction of particles during the loading / unloading of seeds, the description of industrial processes of seed scarification, with study of the movement of particles of yeast suspension in a plate separators, modeling the processes of shot-blast cleaning of metal surfaces etc. We have tested this approach in teaching the 163 students enrolled in the specialties "Agroengineering" and "Industry Engineering" in Poltava State Agrarian Academy. According to data we collected our students demonstrated a better understanding of the conceptual issues of mathematical modeling and acquired significant abilities in using this method to solve real problems.

Keywords: Basics of mathematical modeling, computer simulation of mechanical movement, software used in learning of modeling, computer simulation and visualization, simultaneous use Excel and GeoGebra.

1 Introduction

Learning for basics of mathematical modeling (BMM) is an important component in the training of modern agrarian production engineers. Its practical value is due to the fact that learning based on mathematical modeling can be an effective strategy in modern realities [13].

Currently, the entrants to the engineering specialties of agricultural institutions of higher education in Ukraine (IHEs) have a relatively low level of training in mathematics, physics and other related disciplines, as compared with entrants of other technical specialties. In most cases, in the further education, they also demonstrate little progress in the study of mathematical disciplines. Additionally, it should be noted that among students of engineering specialties in agrarian IHEs, the male population is traditionally dominated. However, the consideration of gender characteristics in the training of BMM is not critical for students of the student age, as evidenced in particular by the results of research on gender stereotypes in mathematics teaching [16].

In the learning for mathematical modeling, computer software is considered as an integral component of a three-point didactic model (student – teacher – information and communication pedagogical environment (ICPE)) [18]. By choosing Excel spreadsheets and GeoGebra dynamic geometry system for training of BMM, we were guided by the certain criteria for selecting educational software applications proposed in [10, p. 32-42].

The use of Excel spreadsheets in the study of the basics of mathematical modeling is devoted to a fairly large number of scientific publications covering a variety of aspects of this scientific discipline. Their review can be found, in particular, in works [15], [17], [19], [20], [21], [22], [23] by Serhiy O. Semerikov and Illia O. Teplytskyi, where the most significant, fundamental achievements in this direction are presented.

At the same time, to the system of dynamic mathematics GeoGebra are mainly devoted the articles that describe the capabilities of this program and examples of its use in teaching.

In particular, some publications by Olena Hrybiuk [10] devoted to modeling in the GeoGebra environment during the training of mathematical and chemical-biological cycles, mathematical disciplines and mathematical fundamentals of informatics.

Marta Caligaris, María Schivo and María Romiti as a result of their research came to the conclusion that the incorporation of the GeoGebra Applets, and the teaching situations arising therefrom, is a much more effective teaching methodology than traditional one [4].

Nazihatulhasanah Arbain, Nurbiha A. Shukor, the authors of the study devoted to investigates the effectiveness of using GeoGebra software on Mathematics learning among students in Malaysia, are notes that results show that students have positive perception towards learning and have better learning achievement using GeoGebra [1].

In the article by Esperanza G. Valdés y Medina, Leilani Medina Valdés is shown an example of use the GeoGebra to change educational methodology, which confirms the conclusion that the software is introduced as a friendly model that can be used to exemplify the mathematical concepts ranging from the basic ones to complex applications like angular velocity [12].

Also, the big number of practical examples of using GeoGebra software for teaching mathematics and science made by other authors are presented on the GeoGebra web resource [8].

As we know, MS Excel has built-in tools for visualizing changes in values, and GeoGebra has a built-in SpreadSheet component that allows you to partially accomplish the tasks set in this study. However, the use of these opportunities is based on the results of previous teaching of information technology, which are not provided for by the training plans of training in agricultural universities.

Thus, the analysis of modern publications was shown the practical absence of well-known scientific works devoted to the problem of the simultaneous use of Excel and GeoGebra for teaching the basics of mathematical modeling and studying their influence on the learning outcomes of the basics of mathematical modeling.

The reason for this research was the negative dynamics the final indicators of learning outcomes in the discipline BMM of the students of engineering and technical specialties of the Poltava State Agrarian Academy, Poltava, Ukraine (PDAA): the average score was in 2014 – 75.8; 2015 – 75.6; 2016 – 72.6; 2017 – 70.1.

Among the factors that have an impact on the learning outcomes, we have chosen to study the individual style of coding educational information. The published scientific data on this subject indicates that for the students of applied training areas the dominant is the visual style of coding educational information: visual learning style preferred by 61% of the students; whereas, auditory (33%) and kinesthetic (6%) [2].

On this basis we have formulated the hypothesis of the investigation that the BMM learning outcomes can be improved by the use of appropriate computer (software) visualization tools in practical and laboratory learning performances. The main objective of this study is testing the hypothesis that the visualization of simulation results creates the conditions for improving students' knowledge, taking into account the specifics of their professional training.

2 Experiment description

2.1 General design

This article based on the results of the experiment that we performed in September-November 2018 in PDAA. The experiment was attended by 163 students of the Faculty of Engineering and Technology, who studied the discipline of BMM in a single program. Main hypothesis of the experiment: GeoGebra software, as a computer visualization tool, will increase the level of learning outcomes of students with BMM.

The participants of the experiment were divided into three groups, each of which was offered the same learning task, which has a direct connection with the topics important in the training of engineers of agrarian production, in particular: studying the movement and interaction of particles during loading and unloading of seeds from the vehicle, description of industrial processes of seed scarification, research of motion of particles of yeast suspension in a container separator, simulation of processes of blast-blast cleaning of metal surfaces, etc.

The provided sample size makes it possible with ANOVA method to establish significant differences between group averages at the level of 1 point: at a significance level of 0.05, number of groups 3 and a power of 80%, the required sample size for groups is at least 50 units.

The first group (E) used Excel spreadsheets during the training of (traditional course). The second group (G) – used the GeoGebra dynamic geometry program (updated base course). Third group (EG) used simultaneously Excel and GeoGebra (experimental course). The training time in all three groups was the same. Learning outcomes of the students of groups E, G and EG were evaluated based on the results of a set of typical tasks for individual independent work.

The learning task was: simulate the movement of two spherical bodies, which were thrown at an angle to the horizon towards each other, without taking into account air resistance, gravitational, electrostatic interaction of bodies, etc.; investigate the conditions of bodies' collision; body collision is considered absolutely elastic.

Some variants of the methodology for solving similar problem using Excel were considered in papers [23], [9] and MathCAD [6]. A similar technique was also used for implementation a differentiated approach while training future agroengineers [7].

Algorithm for solving this learning problem has the following steps:

1. Build a mathematical model and find its solution.
2. Create a computer implementation of a mathematical model.
3. Using a mathematical model:
   (a) Calculate the coordinates and the speeds of the bodies at given moments of time (before the collision of bodies).
   (b) Build trajectories of body movement before their collision.
   (c) Determine the initial conditions of motion, in which there is a collision of bodies.
   (d) Determine: the moment of bodies' contact, coordinates of centers and speeds of bodies at the moment of their collision.
   (e) Determine the initial velocity of bodies after moment of their collision.
   (f) Determine trajectories of body movement after their collision.

The learning outcomes of students were assessed on a 100-point scale (Table 1) on the basis of the performance results of an individual independent learning tasks.

Table 1. Methodology for assessing the performance results of an individual independent learning tasks by the students

Final score	**Points**	**The achieving level**	**The correctness task implementation**	**Self-support the performing of the learning task**
A	90-100	The task is full complete	without remarks and errors	by yourself
B	82-89	The task is full complete	with minor comments and / or inaccuracies that did not affect the result	by yourself
C	74-81	The task is full complete	with remarks and / or inaccuracies corrected by the student	by yourself, with little help from the teacher
D	64-73	The task is not full complete	with remarks and / or inaccuracies that affected the result and were completely corrected by the student	with help from the teacher
E	60-63	The task is not full complete	with significant comments and / or inaccuracies affecting the result and	with help from the teacher

Final score	Points	The achieving level	The correctness task implementation	Self-support the performing of the learning task
			were partially corrected by the student	
FX	35-59	The task is uncompleted	There were errors that affected the result and were only partially corrected by the student	with help from the teacher
F	0-34	The task is uncompleted	There were errors that affected the result and were not corrected by the student	with help from the teacher

2.2 Mathematical model building

For better understanding what follows, the authors found it necessary to provide here some basic information related to the construction of a mathematical model of motion and collision of two bodies thrown at an angle to the horizon.

Mathematical description the mechanical motion of a body was thrown at an angle to the horizon is based on the mechanical sense of the derivative and the laws of Newton’s dynamics. As a result, we have a Cauchy problem for a system of four ordinary first order differential equations [5, p. 253]:

$$\frac{dx}{dt}=v_x,\ \frac{dy}{dt}=v_y,\ \frac{dv_x}{dt}=0,\ \frac{dv_y}{dt}=-g,$$
$$x(0)=x_0,\ y(0)=y_0,\ v(0)=v_0,\ \alpha(0)=\alpha_0, \tag{1}$$

where $x=x(t)$, $y=y(t)$ – coordinates of body centre, $\left(v_x(t),v_y(t)\right)=v(t)=v$ – instant body velocity, $\alpha=\alpha(t)$ – angle of inclination the body trajectory to the horizon. Then,

$v_x(t)=v\cos\alpha=v_x(v,\alpha)$, $v_y(t)=v\sin\alpha=v_y(v,\alpha)$, $v=\sqrt{v_x^2+v_y^2}$, $\alpha=arctg\frac{v_y}{v_x}$.

System (1) has a simple analytical solution:

$$x(t)=x_0+v_{0x}t,\ y(t)=y_0+v_{0y}t-\frac{gt^2}{2},\ v_x=v_{0x},\ v_y=v_{0y}-gt. \tag{2}$$

From where, excluding time t, we get the equation of body motion trajectory:

$$y(x)=y_0+\frac{v_{0y}}{v_{0x}}(x-x_0)-\frac{g}{2v_{0x}^2}(x-x_0)^2 \tag{3}$$

For visualize the trajectory of the movement of body centre, we build a graph of function (3) on a segment $t\in[t_0,t_M]$, where $t_M=t_k-t_0$ is the time of simulation (the time of virtual observation of the movement of each body in each phase of flight), which begins at the moment t_0 and ends at the moment of the body's fall to the ground without collision t_k. From (3) for $y_k=r$ we get:

$$t_k\left(y_0, v_{0y}, r\right) = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2g\left(r - y_0\right)}}{g} \quad (4)$$

The collision of bodies occurs at the point $D\left(\frac{x_1 + \lambda x_2}{1+\lambda}, \frac{y_1 + \lambda y_2}{1+\lambda}\right)$ which divides the segment O_1O_2 with ratio $\lambda = \frac{r_1}{r_2}$, r_1 and r_2 – effective radii of bodies 1 and 2 (Fig. 7). The moment of collision time t_D is:

$$t_D\left(x_{01}, y_{01}, x_{02}, y_{21}, v_{01}, v_{02}, \alpha_{01}, \alpha_{02}, r_1, r_2\right) = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad (5)$$

where $a = \left(v_{0x2} - v_{0x1}\right)^2 + \left(v_{0y2} - v_{0y1}\right)^2$,

$b = 2\left(\left(y_{02} - y_{01}\right)\left(v_{0y2} - v_{0y1}\right) + \left(x_{02} - x_{01}\right)\left(v_{0x2} - v_{0x1}\right)\right)$,

$c = \left(x_{02} - x_{01}\right)^2 + \left(y_{02} - y_{01}\right)^2 - \left(r_1 + r_2\right)^2$.

Further movement of bodies 1 and 2 after their collision is described by the equations (2) with the new initial coordinates $x_{0i} = x_{iD}$, $y_{0i} = y_{iD}$ and velocities $u_{01} = \left(u_{01x}, u_{01y}\right)$, $u_{02} = \left(u_{02x}, u_{02y}\right)$, which are determined from the laws of conservation of energy and momentum:

$$u_{01x} = \frac{\left(m_1 - m_2\right)v_{1xD} + 2m_2v_{2xD}}{m_1 + m_2},\ u_{01y} = \frac{\left(m_1 - m_2\right)v_{1yD} + 2m_2v_{2yD}}{m_1 + m_2};$$
$$u_{02x} = \frac{\left(m_2 - m_1\right)v_{2xD} + 2m_1v_{1xD}}{m_1 + m_2},\ u_{02y} = \frac{\left(m_2 - m_1\right)v_{2yD} + 2m_1v_{1yD}}{m_1 + m_2}. \quad (6)$$

The common formulas obtained here are used further to perform the next calculations. However, their implementation in Excel spreadsheets and in the GeoGebra software has its own nuances, which are reflected in the relevant sections of the article.

2.3 Performing the learning task with Excel (Group E)

In the Excel environment, this mathematical model is implemented according to the algorithm presented in [7].

The figure (Fig. 1) shows the input data block: the input parameters of the model in Excel are manually changed.

Final calculations are performed using standard Excel tools. The results of calculations are presented in the table (Fig. 2).

To visualize the trajectories of the movement of body centre, diagrams are constructed of the points (x_i, y_i) – the graph of function (3) on the segment $t \in \left[t_0, t_M\right]$. The corresponding calculation formulas have the form:

$$n = 100\,,\ \Delta t = \frac{t_M}{n}\,,\ t_i = t_0 + i \cdot \Delta t\,,\ x_i = x_0 + v_{0x} t_i\,,\ y_i = y_0 + v_{0y} t_i - \frac{g t_i^2}{2}\,,\ i = \overline{0, n}\,. \tag{7}$$

	A	B	C	D	E	F	G	H	I	J	K	L
1	Simulation of the collision of two bodies								Time of collision of bodies			
2									0,4653	s		
3	*General parameters of the model*											
4	g=	9,81	m/s2	n=	100		t0=	0	s			
5	tM=	0,4653	s	deltaT=	0,00465	s						
6												
7	Body 1. Initial data						Body 2. Initial data					
8	d1=	1	m				d2=	0,7	m			
9	m1=	2	kg				m2=	1	kg			
10	x01=	0	m	v01x=	14,4168	m/s	x02=	10	m	v02x=	-5,44789	m/s
11	y01=	2	m	v01y=	9,00863	m/s	y02=	1	m	v02y=	10,6921	m/s
12	v01=	17	m/s				v02=	12	m/s			
13	alpha01=	32	degrees	0,55851	radians		alpha02=	117	degrees	2,04204	radians	

Fig. 1. The input data block of the model in Excel

	A	B	C	D	E	F	G	H	I	J	K	L
16	i	ti	x1i	y1i	x1i-	y1i-	x1i+	y1i+	v1xi	v1yi	v1i	alpha1i, рад
17	0	0	0	2	0,26496	1,575976	-0,26496	2,424024	14,41682	9,008627	17	0,558505361
18	1	0,004653	0,067081	2,041811	0,331073	1,617184	-0,19691	2,466438	14,41682	8,962982	16,97586	0,556225065
19	2	0,009306	0,134163	2,08341	0,397183	1,65818	-0,12886	2,508639	14,41682	8,917336	16,9518	0,553938285
20	3	0,013959	0,201244	2,124796	0,463289	1,698964	-0,0608	2,550627	14,41682	8,87169	16,92783	0,551645017
21	4	0,018612	0,268326	2,165969	0,52939	1,739536	0,007261	2,592403	14,41682	8,826044	16,90395	0,549345259
22	5	0,023265	0,335407	2,206931	0,595488	1,779897	0,075327	2,633965	14,41682	8,780398	16,88017	0,547039007
23	6	0,027918	0,402489	2,24768	0,661581	1,820045	0,143397	2,675314	14,41682	8,734752	16,85647	0,544726258
24	7	0,032571	0,46957	2,288216	0,72767	1,859982	0,211471	2,716451	14,41682	8,689106	16,83286	0,542407009
25	8	0,037224	0,536652	2,328541	0,793754	1,899707	0,279549	2,757374	14,41682	8,64346	16,80934	0,540081258
26	9	0,041877	0,603733	2,368652	0,859835	1,939221	0,347631	2,798084	14,41682	8,597814	16,78592	0,537749003
27	10	0,04653	0,670815	2,408552	0,925911	1,978522	0,415718	2,838582	14,41682	8,552168	16,76258	0,535410242
28	11	0,051183	0,737896	2,448239	0,991984	2,017612	0,483808	2,878866	14,41682	8,506522	16,73934	0,533064974
29	12	0,055836	0,804977	2,487714	1,058052	2,05649	0,551903	2,918937	14,41682	8,460876	16,71619	0,530713197
30	13	0,060489	0,872059	2,526976	1,124116	2,095157	0,620002	2,958795	14,41682	8,41523	16,69313	0,52835491
31	14	0,065142	0,93914	2,566026	1,190175	2,133612	0,688106	2,998439	14,41682	8,369584	16,67017	0,525990112
32	15	0,069795	1,006222	2,604863	1,25623	2,171856	0,756213	3,037871	14,41682	8,323939	16,6473	0,523618804

Fig. 2. The final calculations results in Excel (fragment of table)

The movement of the extreme points of the bodies most distant from the trajectory of the centre of the body (these points are located at the ends of the effective diameter of the body perpendicular to the direction of motion), is described by the formulas:

$$x_{i-} = x_i + r \cdot \sin\alpha_i\,,\ y_{i-} = y_i - r \cdot \cos\alpha_i\ \text{– bottom point},$$
$$x_{i+} = x_i - r \cdot \sin\alpha_i\,,\ y_{i+} = y_i + r \cdot \cos\alpha_i\ \text{– top point}. \tag{8}$$

The following figure shows the trajectories of the movement of centers and extreme points of two bodies before their collision (Fig. 3).

The time moment the collision of bodies is determined by the computer simulation, according to the data presented in the calculation table (Fig. 2). New initial conditions for the movement of bodies after their collision are calculated on the basis of the laws of conservation of energy and momentum by the formulas (2), (6) (Fig. 4).

Trajectories of bodies' movement after the collision are determined similarly: the calculation scheme is copied to a new spreadsheet; new initial conditions are entered automatically; the trajectories of the bodies after the collision are added to the trajectories graphs before collision (Fig. 5).

In addition, Excel allows us to check the implementation of the laws of energy conservation and impulse according to the formulas: $E_{ki} = \frac{m \cdot v_i^2}{2}$, $E_{pi} = m \cdot g \cdot y_i$,

$E_i = E_{ki} + E_{pi}$; $p_{xi} = m \cdot v_{xi}$, $p_{yi} = m \cdot v_{yi}$, $p_i = \sqrt{p_{xi}^2 + p_{yi}^2}$ or as an alternative, $p_i = m \cdot v_i$, $p_{xi} = p_i \cdot \cos\alpha_i$, $p_{yi} = p_i \cdot \sin\alpha_i$.

The calculations are presented in the Fig. 6.

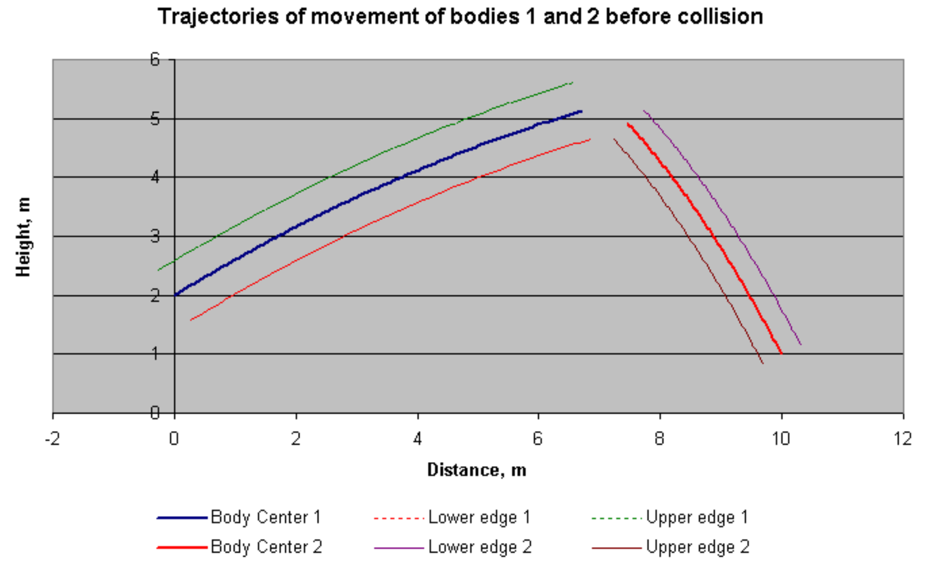

Fig. 3. Trajectories the centers and extreme points of bodies 1 and 2 before collision

	Q	R	S	T	U	V	W	X
3	**Parameters of movement of bodies at the moment of collision**							
4	**t=**	0,4653	s	Time of collision of bodies (moment of touch of the				
5	**x1=**	6,708145	m	Coordinates of body centers at the moment of the				
6	**y1=**	5,129762	m					
7	**x2=**	7,465099	m					
8	**y2=**	4,913071	m	**Speed of bodies at the moment of a collision**				
9	**v1=**	15,08622	m/s	**v1x=**	14,41682	m/s		
10	**alpha1=**	0,299012	radians	**v1y=**	4,444034	m/s		
11	**v2=**	8,199118	m/s	**v2x=**	-5,44789	m/s		
12	**alpha2=**	2,297551	radians	**v2y=**	6,127485	m/s		
13								
14	**New speeds (defined by conservation laws)**							
15	Initial parameters of motion of bodies after a collision							
16	**u01x=**	1,173682	m/s	**u02x=**	21,03839	m/s		
17	**u01y=**	5,566335	m/s	**u02y=**	3,882884	m/s		
18	**u1=**	5,688727	m/s	**u2=**	21,3937	m/s		
19	**alpha1=**	1,362987	radians	**alpha2=**	0,182508	radians		
20		78,09339	degrees		10,45695	degrees		

Fig. 4. Calculations the bodies collision moment and initial conditions of movement the bodies after their collision

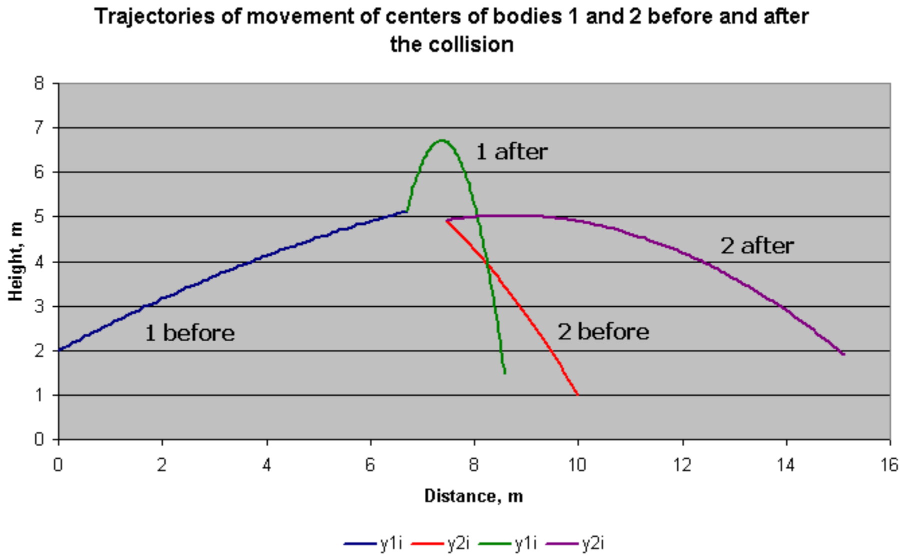

Fig. 5. Trajectories of centers the bodies 1 and 2 before and after their collision

	Q	R	S	T	U	V	W
22	**Checking the law of energy conservation**						
23	**Before collision**			**After the collision**			
24	**Ek1=**	227,5941	J	**Ek1=**	32,36161	J	
25	**Ek2=**	33,61277	J	**Ek2=**	228,8452	J	
26	**Сума=**	261,2068	J	**Сума=**	261,2068	J	
27							
28	**Check the law of conservation of momentum**						
29	**Before collision**			**After the collision**			
30	**px1=**	28,83364	kg*m/s	**px1'=**	2,347364	kg*m/s	
31	**py1=**	8,888069	kg*m/s	**py1'=**	11,13267	kg*m/s	
32	**p1=**	30,17244	kg*m/s	**p1'=**	11,37745	kg*m/s	
33	**px2=**	-5,44789	kg*m/s	**px2'=**	21,03839	kg*m/s	
34	**py2=**	6,127485	kg*m/s	**py2'=**	3,882884	kg*m/s	
35	**p2=**	8,199118	kg*m/s	**p2'=**	21,3937	kg*m/s	
36	**phi=**	1,99854	radians	**phi'=**	-1,18048	radians	
37	**p=**	27,79137	kg*m/s	**p'=**	27,79137	kg*m/s	
38	**2nd way**						
39	**px=**	23,38575	kg*m/s	**px'=**	23,38575	kg*m/s	
40	**py=**	15,01555	kg*m/s	**py'=**	15,01555	kg*m/s	
41	**p=**	27,79137	kg*m/s	**p'=**	27,79137	kg*m/s	

Fig. 6. Checking the laws of energy conservation and impulse in Excel

Thus, using the constructed mathematical model and changing the input parameters of the model in the input block (Fig. 1), the students of the group E were able to perform various virtual experiments in Excel. The analysis of the obtained numerical results and graphs allowed them to comprehensively study the patterns of motion of bodies 1 and 2 before and after the collision, to establish the initial conditions of the movement of bodies in which their collisions occur, to investigate the behaviors of bodies after the collision.

2.4 Performing the learning task with GeoGebra (Group G)

The implementation of mathematical models in the GeoGebra environment is performed on the basis of their algebraic-geometric description [11]: the program sequentially, step by step, creates a dynamic visualization of basic mathematical objects such as a point, angle, segment, vector, circle, line. It is advisable to rely on the geometric interpretation of objects of linear algebra [3].

First, GeoGebra creates a description of the initial values of the main objects of the model.

$O_{01}\left(x_{01}, y_{01}\right)$, $O_{02}\left(x_{02}, y_{02}\right)$ – starting points of trajectories of bodies 1 and 2;

Vectors of initial velocity:

$\vec{v}_{01x} = \overrightarrow{O_{01}A_{01}}$, $\vec{v}_{01y} = \overrightarrow{O_{01}B_{01}}$, $\vec{v}_{01} = \overrightarrow{O_{01}C_{01}}$; $\vec{v}_{02x} = \overrightarrow{O_{02}A_{02}}$, $\vec{v}_{02y} = \overrightarrow{O_{02}B_{02}}$, $\vec{v}_{02} = \overrightarrow{O_{02}C_{02}}$, where $C_{01}\left(x\left(O_{01}\right)+v_{01x}\cdot mst, y\left(O_{01}\right)+v_{01y}\cdot mst\right)$, $C_{02}\left(x\left(O_{02}\right)+v_{02x}\cdot mst, y\left(O_{02}\right)+v_{02y}\cdot mst\right)$, $A_{01}\left(x\left(C_{01}\right), y\left(O_{01}\right)\right)$, $A_{02}\left(x\left(C_{02}\right), y\left(O_{02}\right)\right)$, $B_{01}\left(x\left(O_{01}\right), y\left(C_{01}\right)\right)$, $B_{02}\left(x\left(O_{02}\right), y\left(C_{02}\right)\right)$, *mst* – scale factor that allows interactively to resize the individual objects in the GeoGebra software.

The next step is a geometric description of the dynamic characteristics of the movement of bodies:

$O_1\left(x_1, y_1\right)$, $O_2\left(x_2, y_2\right)$ – current coordinates the canters of bodies 1 and 2; vectors of instant velocity:

$\vec{v}_{1x} = \overrightarrow{O_1A_1}$, $\vec{v}_{1y} = \overrightarrow{O_1B_1}$, $\vec{v}_1 = \overrightarrow{O_1C_1}$; $\vec{v}_{2x} = \overrightarrow{O_2A_2}$, $\vec{v}_{2y} = \overrightarrow{O_2B_2}$, $\vec{v}_2 = \overrightarrow{O_2C_2}$, where $C_1\left(x\left(O_1\right)+v_{1x}\cdot mst, y\left(O_1\right)+v_{1y}\cdot mst\right)$, $C_2\left(x\left(O_2\right)+v_{2x}\cdot mst, y\left(O_2\right)+v_{2y}\cdot mst\right)$, $A_1\left(x\left(C_1\right), y\left(O_1\right)\right)$, $A_2\left(x\left(C_2\right), y\left(O_2\right)\right)$, $B_1\left(x\left(O_1\right), y\left(C_1\right)\right)$, $B_2\left(x\left(O_2\right), y\left(C_2\right)\right)$.

Vectors of forces acting on bodies 1 and 2: $\overrightarrow{F_{G1}} = \overrightarrow{O_1F_1}$, $\overrightarrow{F_{G2}} = \overrightarrow{O_2F_2}$, where $F_1 = \left(x\left(O_1\right), y\left(O_1\right)-m_1\cdot g\cdot mst\right)$, $F_2 = \left(x\left(O_2\right), y\left(O_2\right)-m_2\cdot g\cdot mst\right)$.

Visualizations the bodies 1 and 2 motion trajectories are constructed according to equation (3). Changing the input parameters of the model in the GeoGebra environment is carried out interactively with the help of sliders created when you first enter the corresponding numeric values. The results of the visual representation of this model and the control block are shown in the figure in GeoGebra (Fig. 7).

In GeoGebra, the moment of bodies' collision is determined by formula (5). After the collision the body 1 and 2 took new initial velocities (6) are represented, which are represented by vectors $\vec{u}_{1x} = \overrightarrow{O_1A_1'}$, $\vec{u}_{1y} = \overrightarrow{O_1B_1'}$, $\vec{u}_1 = \overrightarrow{O_1C_1'}$; $\vec{u}_{2x} = \overrightarrow{O_2A_2'}$, $\vec{u}_{2y} = \overrightarrow{O_2B_2'}$, $\vec{u}_2 = \overrightarrow{O_2C_2'}$.

Next figure (Fig. 8) shows the vectors of the instant velocity of bodies 1 and 2 at the moment of their collision ($t = 0,46$ s) at given values of the model parameters (Fig. 1). Ibid, in the Algebra panel, the numerical values of the coordinates of the velocities are presented.

By changing the model parameters in the input block (Fig. 8), students in group G performed virtual experiments in the GeoGebra environment. The analysis of interactive graphs in the Graphics area and the numerical results presented in the Algebra panel allowed them to investigate the movement of bodies 1 and 2 before and after their collision, to determine the initial conditions under which bodies collide and the characteristics the movement of bodies after their collision.

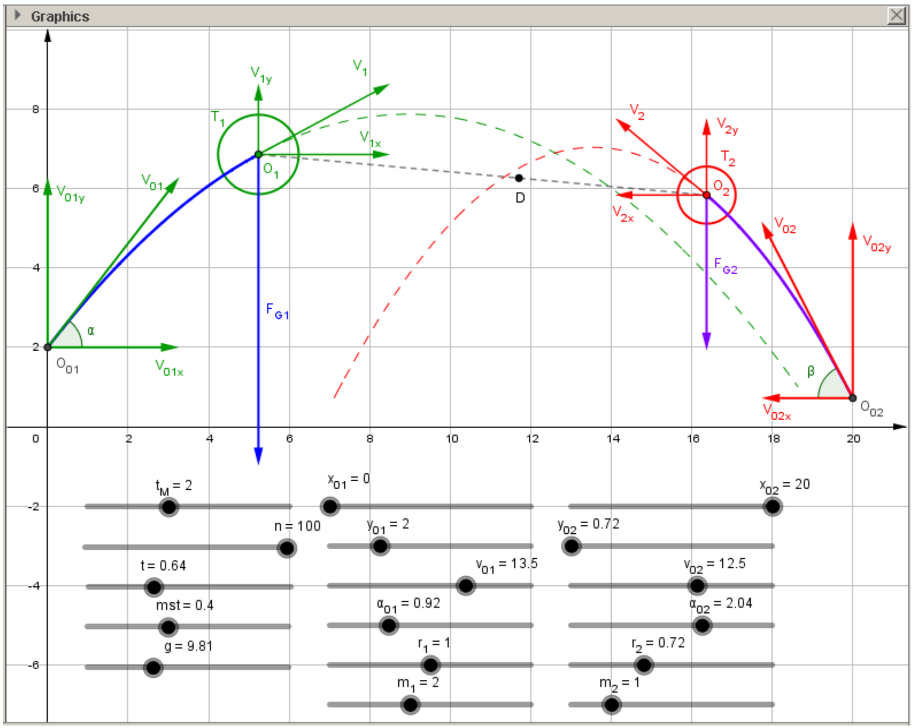

Fig. 7. Interactive model of motion and collision of two bodies in GeoGebra

To our regret, the limited scope of the article does not allow the authors to give a complete description of the methodology for using the proposed models in the educational process.

2.5 Performing the learning task with Excel and GeoGebra (Group EG)

The third group of students (EG) performed the training task using both Excel and GeoGebra in accordance with the methodology described above. Excel program was used mainly for numerical calculations and representation of numerical results in the form of tables. The GeoGebra program was used mainly for visual representation and analysis of dynamic motion characteristics.

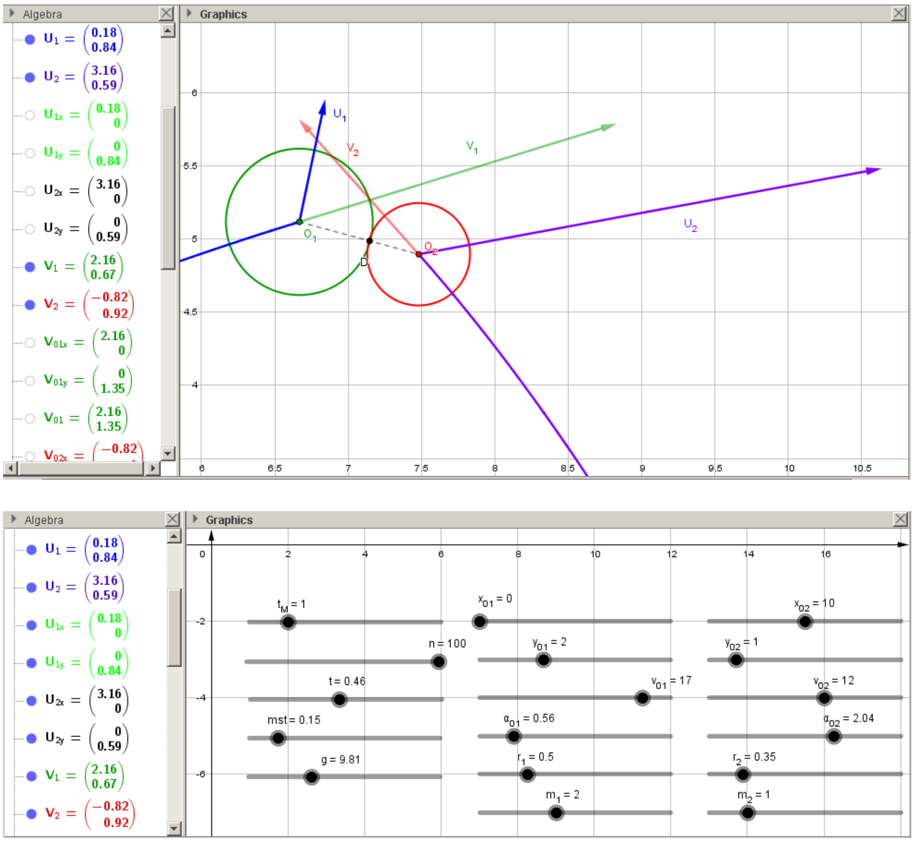

Fig. 8. Vectors the instant velocity of bodies 1 and 2 at the moment of their collision (t = 0.44624 s with the specified initial conditions)

3 Results

The learning outcomes of students after the experiment are shown on a 100-point scale (Table 2).

Table 2. Final learning outcomes of students after the experiment

Score	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95
Group E	1	2	6	9	14	10	7	4	-
Group G	-	3	6	10	13	15	7	3	1
Group EG	-	1	3	5	10	15	9	6	3

4 Discussion

The prime statistical data processing results of the experiment (Table 3) are showed that the average scores in all groups (mean) exceed the corresponding figures for the two previous years (it presented at the beginning of the article). In this case the means for groups E and G were close to each other (73.4 and 73.7) but both lower than the mean of group EG (77.4). Shapiro-Wilk test result [14] is demonstrating the true of hypothesis about normal data distribution in all groups (Table 3). Analysis of variance (ANOVA) showed a statistically significant difference in average values of learning out-comes (Score/Point/Bal) in all groups ($F = 4.1632$; $p = 0.0177$). This case, the post-hoc comparison for means of groups E vs. G, E vs. EG, G vs. EG demonstrated that the difference between the means of groups E and G is within the statistical error (Table 4). The pair-wise post-hoc comparisons results presented in Table 5 indicate the statistical significance of the difference between the mean for group EG and the mean groups E and G.

Table 3. The primary statistical data processing results

Group	Valid N	Mean	Conf. -95%	Conf. +95%	Median
E	53	73,4	71,2	75,6	74
G	58	73,7	71,6	75,8	74
EG	52	77,4	75,7	79,6	77

Group	Mode	Frequency Mode	Min	Max	SD	Shapiro-Wilk test	
E	Multiple		55	90	7,95	W=0,98823	p=0,87840
G	76	5	56	91	8,06	W=0,98859	p=0,86117
EG	76	6	60	95	7,96	W=0,98956	p=0,92711

Table 4. The ANOVA results

	SS	df	MS	SS	df	MS	F	p
Point	528,7585	2	264,3793	10226,91	160	63,91819	4,136213	0,017724

Table 5. The pair-wise post-hoc comparisons results

Pair-wise post-hoc comparisons of means	E vs. G	E vs. EG	G vs. EG
LSD-test	p>0,8277	p<0,0108	p<0,0166
Duncan-test	p>0,8293	p<0,0120	p<0,0161
Tukey HSD for unequal N	p>0,9753	p<0,0276	p<0,0161

There are several probable reasons for this result need to point out. In our opinion, the visualization trajectories the mechanical movement of bodies in the form of Excel graphs contributes to the formation of "engineering thinking", in particular, intuitive ideas about the movement of bodies and the conditions of their collision from initial conditions and other parameters. This allows students to acquire skills in meaningful adjustments to process input parameters in order to achieve the desired simulation result. However, the standard features of Excel do not allow to visualize changes in the values of instantaneous velocities and directions of movement of bodies along their trajectories. Therefore, the use of Excel in teaching does not create sufficient conditions for the formation of skills of an «intuitive» analysis of dynamic characteristics of motion by numerical data (Fig. 2), requires developed abstract thinking, formed skills of "physical thinking", and takes more time than visual analysis.

Instead, the use of GeoGebra provides faster formation of intuitive spatial representations of students in the analysis of dynamic characteristics of motion. This was manifested, in particular, during discussion with students of a qualitative picture of the characteristics of the movement of two bodies before and after their collision and interaction of bodies at the time of the collision. In our opinion, this has become possible thanks to the dynamic visualization of vector motion characteristics. After completing the proposed study with GeoGebra, students easily formulated meaningful answers to questions such as "How will the shape of the trajectory and the direction of movement of bodies change before and after their collision, depending on the initial characteristics of motion, mass and body size?". Most students of group G during computer experiments with the model, showed interest and creative approach, independently put forward and tested their own hypotheses, based mainly on geometric representations. After a brief discussion, guided by the instructional materials, they were able to describe and implement the corresponding mathematical model in the GeoGebra environment independently.

The disadvantage of using GeoGebra to study the basics of modeling is the inconvenience of evaluating simulation results in a numerical dimension. The use of GeoGebra during the training of OMM promotes the formation of intuitive spatial representations important for specialists in the engineering field, but does not provide sufficient level of formation of the skills of numerical evaluation of the characteristics of the phenomenon or simulated process. That is why, in our opinion, there is no significant difference between the learning outcomes in groups E and G. Simultaneous use of Excel and GeoGebra compensates for these shortcomings, and therefore provides better learning outcomes.

5 Conclusion

Results of the study get conclusion that the simultaneous use of Excel and GeoGebra improved the academic achievement of students with BMM. This is indicated by the statistically significant difference between the average results of students' academic achievement, shown in Table 3 confirmed with the results of Table 4 and Table 5. Consequently, as our results show, the hypothesis that the visualization of modeling results improves students' knowledge of the basics of mathematical modeling has been confirmed.

In the next we plan to reproduce this study by offering students more sophisticated learning tasks, in particular, to construct models of inelastic and semi-elastic collision of two bodies, which should take into account air resistance, shape and rotation of the bodies, their gravitational and / or electrostatic interactions, etc.

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