# Numerical Study of the Stability of the Steam-Water Flow in Pipelines of Geothermal Gathering System

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#### Abstract

A mathematical model of the steam-water flow in the geothermal fluid transportation system has been developed and, on its basis, the macroscopic temporal flow instability due to gravitational forces has been investigated.

# 1 Introduction

The use of geothermal resources is a perspective and developing area of energy [1, 2]. The scale of development is already beyond the scope of investment projects, and the development of geothermal resources is increasingly carried out on a commercial basis. Therefore, along with new prospective studies, for example, the creating improved geothermal systems [3–5] and extracting geothermal energy without raising the deep fluids to the surface [6–8], increased attention to the already existing technologies is paid.

In the world practice of development of steam-water fields, which are the basis of modern geothermal power engineering, about 20 years ago, the two-phase transportation of thermal fluid through surface pipelines from wells to group separators began to use. This technology was used at the Mutnovsky field (Kamchatka) among the first. The main problem of two-phase transportation is the choice of an efficient diameter of the pipelines. The small diameter creates large pressure losses, reducing the energy potential (flow rate and temperature) of the thermal fluid. The large diameter creates the risk of instable operate of the surface pipeline and the well–pipeline system.

Hydraulic calculation of steam-water pipelines at domestic geothermal power facilities was carried out using the computer program MODEL, in which the instability problem was solved on the basis of empirically justified criterion. This experience has shown the possibility of liberalization of the criterion used for pipelines without ascending sections. Specification of criteria of stability transportation can answer many modern questions of the technology of two-phase transportation of thermal fluid [9].

The flow in the surface pipeline is similar to the flow in wells, which has a rich history of study [10-13]. For the flow in the well, the criterion is used [14], which relates to the macroscopic temporal instability. This criterion is due to the influence of gravitational force in the well and it can be applied to surface pipelines as well. In this paper, the stability of steam-water mixture in surface pipelines of geothermal fluid gathering system is studied on the basis of the theoretically justified condition of the absence of macroscopic temporal instability.

### 2 Mathematical Model

As well as the well [14], for the steam-water mixture pipeline the flow stability condition is determined by the relationship

$$\frac{\partial \Delta p}{\partial G} + \frac{\partial p_w}{\partial G} > 0 , \qquad (1)$$

where  $\Delta p$  is the internal pressure drop in the pipeline (Pa), G is mass flow rate (kg/s),  $p_w$  is the outlet pipeline pressure (Pa).

As a rule, the outlet pipeline pressure does not have a significant dependence on the flow rate. For example, steamwater mixture pipelines at the Mutnovsky field are connected to group separators, in which the pressure does not have a significant dependence on the flow rate of a particular well. Given the proximity to zero of the second term, the stability condition can be represented as

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$$\frac{\partial \Delta p}{\partial G} > 0 , \qquad (2)$$

The internal pressure drop in the pipeline is expressed in terms of friction, acceleration, gravity and local resistances

$$\Delta p = \Delta p_f + \Delta p_m + \Delta p_a + \Delta p_g , \qquad (3)$$

where  $\Delta p_f$ ,  $\Delta p_m$ ,  $\Delta p_a$ ,  $\Delta p_g$  are the terms of internal pressure drop for friction, local resistances, acceleration and gravity (Pa).

For this type of instability, the main component is the gravity term. Since the mathematical model, which is used in the program MODEL, does not take into account the specified term, the solution of the problem requires the development of a new model.

Recommendations for the practical determination of the individual terms of the formula (3) for two-phase fluids depend significantly on the parameters of the mixture and the conditions of its transportation. The steam-water mixture transported through pipelines is characterized by dominant volume content of the steam phase. For example, at the Pauzhetskoye field, separation is usually carried out near the wellhead. The water level in the separator is set by selecting the degree of throttling on the valve at water line entrance. As a result of throttling, saturated water boils, and a steam-water mixture enters the main pipeline.

Steam-water mixture pipelines in the fields are characterized by a large diameter (0.3 m and more) and low pressure (to 15 bar), except in uncial cases. In a saturated media, is associated with pressure. In order to maintain the temperature that determines the energy potential of the thermal fluid, during transportation, pressure losses are minimized, that is, as a rule, pressure drops are small, and the composition of the mixture at the beginning and at the end of the pipeline differs little. Such conditions make it possible to neglect the component of the differential pressure on the acceleration.

According to the classical concepts, the friction component is determined by the formula obtained from the analysis of forces acting on the selected element of the pipe

$$dp_f = \frac{4\tau}{D} dL , \qquad (4)$$

where  $\tau$  is the shearing stress on the pipe wall (Pa), D is the pipe diameter (m), dL is the length of the element (m).

The total friction pressure drop is determined by integration (4) along the entire length of the pipeline. This procedure is not difficult when the pipe diameter and shearing stress can be considered constant. Difficulties are appeared in the case of not constant shearing stress. There are many recommendations for determining the shearing stress or the friction pressure drop in two-phase flows. These recommendations are suitable for certain conditions and justified experimentally. If there is a shortage of relevant experimental data, there is doubt in true of recommended formulas. It is not surprising that for the conditions under consideration the calculation by the simplest homogeneous model shows a better agreement with the experimental data in comparison with some of the most well-known recommendations [15]. A reasonable solution in this case is the compilation of known formulas that have shown good agreement with the experimental data in considered ones.

The procedure for calculating the friction pressure drop in the MODEL program, which is close to the calculation by the homogeneous model [9], has shown successful application in practice. However, this procedure is initially focused on high transport velocities and unacceptable to describe flows with low velocities characteristic of the instability.

The search for the most appropriate equation for the shearing stress for the flow in the upper section of the geothermal well [10], where friction plays a significant role and where the flow parameters are close to the ground transportation conditions, revealed the good agreement with the experimental data in case the use of some formula. The agreement was better than the homogeneous model. This formula represents the shearing stress of the mixture as the sum of the stresses generated by the gas and liquid taking into account their averaged in cross section the velocities and the fraction in the volume under consideration. It can be represented as

$$\tau = (\rho_l v_l^2 (1 - \alpha) + \rho_g v_g^2 \alpha) \lambda / 8 , \qquad (5)$$

where  $\rho_l$  and  $\rho_g$  are the density of water and steam (kg/m<sup>3</sup>),  $v_l$  and  $v_g$  are the averaged velocities of water and steam (m/s),  $\alpha$  is the void fraction (ratio of volume of the gas phase to the total volume of the considered element, dimensionless),  $\lambda$  is the friction coefficient (dimensionless).

To determine friction coefficient it is recommended that the well-known formula of Shifrinson [16]

$$\lambda = 0.11(\delta/D)^{0.25},$$
 (6)

where  $\delta$  is the absolute equivalent roughness of the internal pipe wall (m).

A homogeneous model gives satisfactory agreement with the experimental data for pipes with large diameter [15]. This model in the operating velocity range is close to the formula used in the program MODEL [9]. Shearing stress by homogeneous model is determined as

$$\tau = \lambda \rho_w w^2 / 8 , \qquad (7)$$

where  $\rho_w \mu w$  are the density (kg/m<sup>3</sup>) and velocity (m/s) of the mixture according to homogeneous model. The resulting expression is obtained by compiling formulas (5) and (7)

$$\tau = (\rho_l v_l^{\ 2} (1 - \alpha) + \rho_g v_g^{\ 2} \alpha + \rho_w w^2) \lambda / 16 , \qquad (8)$$

To determine the pressure drop at the local resistance, the MODEL program used a formula based on a homogeneous model with the introduction of a correction coefficient (1.4) and not limited to a range of velocities. Numerous calculations to justify the reconstruction of steam-water mixture pipelines at the Mutnovsky field by of the elimination of non-functional local resistances, confirmed by practical results, allow us to recommend the same formula

$$\Delta p_m = 1.4 \zeta \rho_w w^2 / 2 , \qquad (9)$$

where  $\zeta$  is the local resistance coefficient for a single-phase flow (dimensionless).

The expression for the gravitational component of the pressure change in the flow direction is well known

$$dp_{g} = -\rho g \sin \theta dL , \qquad (10)$$

where g is module of gravitational acceleration (m/s<sup>2</sup>),  $\theta$  is the angle of inclination of the pipe axis relative to the horizon plane (rad),  $\rho$  is the mixture density (kg/m<sup>3</sup>,  $\rho = \rho_g \alpha + \rho_l (1 - \alpha)$ ).

The desired component of the pressure drop is determined by integration (10). The main difficulty lies in determining the density of the mixture, which depends on the void fraction. The value of the void fraction depends on many factors, including those that do not have an accurate theoretical description. Therefore, there is no universal theoretical formula for its determination on the basis of flow parameters. The corresponding practical problems are solved with the help of correlations containing empirically justified interrelations and coefficients.

Many correlations are known to determine the void fraction [17-20]. Among them, there are correlations that stream the maximum coverage of the conditions that were encountered in the experiments. But it should be noted the lack of experimental data corresponding to the conditions of transportation of steam-water geothermal fluids for such an important parameter as the diameter of the pipes. In experiments to determine the void fraction, a pipe diameter of 0.15 m is considered as a large diameter [21], while the pipelines of the steam-water mixture at the steam-water fields have a diameter of 0.3 m and more.

In the absence of experimental data to determine the void fraction in the appropriate conditions, the only way to obtain the necessary correlations is to analyze the main determining factors with the use of indirect experimental data. It is important to choose the general type of the required correlations. The most successful type is considered to be based on the drift flux model [19-21], according to which the average velocity of the gas phase is determined by the formula

$$v_g = C_0 w + v_{gm} \,, \tag{11}$$

where  $v_g$  is the averaged (over the cross-section) velocity of the gas phase (m/s),  $C_0$  is the distribution parameter (dimensionless), w is the superficial velocity of the mixture equal to the homogeneous velocity (m/s),  $v_{gm}$  is the drift velocity (m/s).

In formula (11), the drift velocity characterizes the actual velocity difference of phases. The distribution parameter takes into account the unevenness of the phase distribution in an uneven field of velocities. In a result of averaging over the cross-section these factors leads to difference in the velocities of phases even in the case of equality of their actual local values.

Void fraction is related to steam velocity

$$\alpha = w_g / v_g , \qquad (12)$$

where  $w_g$  is the superficial steam velocity (m/s), defined as the ratio of the volume gas flow to the total area of the pipe section.

The search for the correlation for the drift velocity and the distribution parameter was probably so fascinating that the void fraction itself began to be included in the circle of the parameters determining it [20, 21]. Note that the introduction of the void fraction into the correlations can make it difficult to find it, and can also lead to an ideal correlation that can describe any experimental data, but is not able to solve practical problems, for example  $C_0 = 1.1$ ;  $v_{gm} = w_g/\alpha - 1.1w$ . Substitution of these expressions in (11) and (12) leads to the identity. That is, substitution of experimental data in the formula for the drift velocity and then determination of the calculated value of  $\alpha$  give, in the absence of errors in the calculations, a complete coincidence with the experimental value of  $\alpha$ .

Among the parameters that determine the value of the void fraction, it should be noted, first of all, the two-phase flow quality. In the conditions under consideration, the approach of quality to 1 should lead to the same approach of void fraction.

Experiments on the critical expiration of steam-water mixtures under high velocities indicate the presence of complete homogeneity of the mixture [22]. For homogeneous flow  $C_0 = 1$ ,  $v_{gm} = 0$ . For velocities approaching the critical, there must be a corresponding tendency for the values in formula (11). It is reason to introduce the Mach number into the range of parameters under consideration as the ratio of the superficial velocity of the mixture to the critical flow velocity. According the formula corresponding to the homogeneous metastable model [22], the critical flow velocity is determined as

$$M = w_{g} (kxp/\rho_{g})^{-0.5},$$
(13)

where M is the Mach number (dimensionless), k is the adiabatic coefficient of saturated steam (dimensionless, in our case it is recommended to take 1.1), x is the quality (dimensionless, ratio of mass flow rate of the gas phase to the total mass flow rate of mixture), p is the pressure (Pa).

Experiments on the measurement of local characteristics of the horizontal flow in a pipe with internal diameter of 0.31 m at quality of 0.49 and Mach number of 0.7 showed that the local phase slip ratio is insignificant; there are

velocity and quality changes in the cross-section [22]. This indicates a small drift velocity and a difference from the unity of the distribution parameter. Note that the nominal conditions of transportation of the steam-water mixture at the steam-water fields correspond to the Mach numbers determined by the formula (13) in the range from 0.1 to 0.15.

Void fraction is depended on the two-phase flow regime (flow structure). The number of experimental works devoted to the detection of flow regimes, as well as the number of the identified regimes, is estimated by at least hundreds. It is undeniable that the flow regimes depend on the channel orientation in the gravity field. Even a small ten-degree deviation from the horizontal axis affects the flow regime map [23]. Consequently the desired correlation should take into account, along with other factors, the influence of channel orientation.

For drift velocity, correlations containing the channel diameter are often used. In the case of large diameters, such correlations give unreal values [20]. No less often and more attractive for large diameter pipes case to use the type of correlation

$$v_{gm} = Kk(g\sigma(\rho_l - \rho_g)/\rho_l^2)^{0.25}, \qquad (14)$$

where *K* and *k* are condition and correction coefficients (dimensionless),  $\sigma$  is coefficient of surface tension (N/m).

Note that for our conditions the vapor density in (14) can be neglected, since even for the maximum limit pressures (up to 15 bar) this will cause a deviation in the drift velocity of no more than 0.3 %, which is insignificant. Taking into account the noted factors and not having experimental data to detail their impact, a simple formula for the condition coefficient is proposed

$$K = (1 - x)(1 - M)(1 + \sin \theta) .$$
(15)

It is proposed to adopt a correction coefficient of 1.4 so that the maximum value of the total coefficient of proportionality in the formula (14) corresponds to 2.8. This value is close to the maximum value of the corresponding coefficient in similar correlations [19].

Maximum value of 1.1 for the distribution parameter is recommended. This value was recommended for geothermal wells [24]. Taken into consideration the noted factors the formula is proposed

$$C_0 = 1 + 0.5K . (16)$$

Formulas (3, 4, 6, 8-16) as well as the stability criterion (2) are the basis of the proposed model for the calculation of steam-water flow in ground pipelines of steam-water geothermal fluid. The model aims to calculate the pressure drop in the pipeline according to the formula (3). In addition, criterion (2) indicates flow stability. This model should not be regarded as universal, suitable for all conditions. Some of the expected effects can not be described by this model in principle. For example, it is the excess of the liquid velocity over the gas velocity in the descending pipelines, which was practically observed in other conditions [25]. As new data become available, the model is planned to be improved.

# 3 Calculation and Discussion of Results

The presented model was implemented in the simplest version – all values and their gradients were determined for one nodal point and were considered constant throughout the calculated interval. A similar approach was used when creating the MODEL program, which did not cause significant difficulties in practice, because it is always possible to split the pipeline into sections within which this simplification is acceptable.

Due to the lack of reliable experimental data, verification of the model was carried out by comparison with the program MODEL. The program MODEL was actively used in the calculations of steam-water mixture pipelines at the Mutnovsky steam-water field, while there was always a good agreement of the calculated data with the results of practical implementation. A positive characteristic of the new model should be the agreement with the calculation by program MODEL in nominal operating conditions. In this case, the comparison should be carried out for a horizontal pipeline (MODEL does not take into account the gravitational component) in the absence of local resistances (these models are identical in this component).



Figure 1: Calculated pressure drops in dependence on flow rate: 1 – the program MODEL, 2 – the proposed model

Figure 1 shows the comparison of the calculated pressure drops for a 100 m long pipeline with a nodal pressure of 7.5 bar, 0.4 m in diameter and an enthalpy of mixture 1200 kJ/kg corresponding to the typical conditions of the Mutnovsky field. Nominal operation of such a pipeline is carried out in the range of flow rate from 50 to 70 kg/s. In this range, there is a good agreement of calculations, that is, the verification of the model can be considered successful.

According to the MODEL program, transportation of a mixture with a flow rate below 40.2 kg/s is associated with the risk of instable regimes. The use of the condition (2) in the proposed model did not reveal such a risk in the entire flow-rate range.

The calculation of the stability index, according to the condition (2), by the proposed model for the ascending and descending pipelines (sin  $\theta = \pm 0.5$ ) is shown in figure 2. For an ascending pipeline, a flow rate of less than 26.6 kg/s does not meet the stability condition. For the downward pipeline the instability region is not observed.



Figure 2: Stability index, according to the condition (2), for ascending (1,  $\sin \theta = 0.5$ ) and descending (2,  $\sin \theta = -0.5$ ) pipelines

In principle the drift velocity can have negative value at extremely low flow rates. In this case the steam velocity determined by formula (11) can be less than the superficial steam velocity, which is physically impossible (the cross-section area of the steam flow should be greater than the area of the pipe cross-section). Therefore, the model adopted an expression that excludes the possibility of a negative value of drift velocity.

Without denying the principal possibility of instability of descending flows, it is necessary to pay attention to the difference in the conditions of its development. In ascending flows, a spontaneous flow increase is leads to a decrease in the density of the mixture, reducing the gravity force that prevents movement, increasing the total force that causes the movement. In this case, a spontaneous increase in the flow rate at the bottom does not lead to a decrease in the mass of the mixture in the pipe as a whole, that is, the effective development of instability is possible only from the outlet of the pipe [12].

In descending flows at extremely low flow rates, when the water velocity exceeds the steam velocity, a spontaneous increase in the flow rate can lead to an increase in the density of the mixture, increasing the gravity and the total force causing the movement. In this case, a spontaneous increase in the flow rate at the outlet of the pipeline will not increase the gravity force in the pipe as a whole, that is, the process of instability development will be different. In particular, for downward flows we should not expect analogues of the most dangerous manifestation of instability – self-kill [12]. Note also that the interest in extremely low flow rates is more theoretical than practical.

The minimum velocity, which is necessary to ensure the stability of the flow, varies depending on the inclination angle for ascending flows. Figure 3 shows this dependence. The maximum value (19.1 m/s) corresponding to the vertical pipe is close to the value prescribed by the program MODEL (19.8 m/s). This is because the MODEL program uses a criterion that guarantees stable transportation regardless of the orientation of the pipes, that is, the maximum value of the minimum required velocity. This criterion [9] is reduced to the fulfillment of the condition

$$w > 31.3\sqrt{D}$$
 (17)

The criterion (17) was justified by experimental works at the test plant of "Kamchatskenergo" [22] at the flow of a steam-water mixture in a pipeline with a diameter of 0.31 m, having a vertical part of 5.5 m. Flow in vertical part is a potential cause of instability. At the same time, the experience of pipelines operation at the Mutnovsky field shows the possibility of stable transportation at velocities lower than the prescribed by condition (17), through pipelines that do not have ascending sections. This suggests that condition (2) is a theoretical analogue of the empirical condition (17) for ascending vertical flows. The condition (2) has broader opportunities in terms of an adequate application to pipes of a different orientation.

The similarity of the dependence of the maximum permissible velocity on the diameter of the pipe also indicates the generality of these conditions. Figure 4 shows those dependencies. The similarity of dependencies is obvious.

A possible violation of condition (2) is caused by the influence of the gravitational component of the internal pressure drop. Therefore, when designing steam-water pipelines it is important to consider the orientation of the flow in the gravitational field. The proposed model allows you to entertain this task.



Figure 3: Dependence of the minimum superficial (homogeneous) velocity, which provides a stability flow rate, on the inclination angle of the pipeline



Figure 4: The dependence of the maximum permissible velocity to ensure a stable flow from the diameter of the pipe: 1 - the calculation by the condition (2) for the vertical ascending flow using the proposed model, 2 - the calculation according to (17)

#### Conclusion

1. Application of the condition (2) with the use of adequate model calculations reveals new prospects of the study of steam-water flow stability in surface pipelines at the development of geothermal fields. New prospects allow take into account the terrain in the prediction of flow instability at design and exploitation of pipelines.

2. The flow stability condition used in the MODEL program, which does not take into account the pipe incline, can be interpreted as a special case of the condition (2); this case corresponds vertical ascending flow.

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