Interval valued intuitionistic fuzzy generalized nets of second and fourth types

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Abstract. Generalized Nets (GNs, see [1, 2, 3]) are extensions of Petri nets and the other their modifications and extensions.

Now, there are more than 20 extensions of the GNs and all of them are conservative ones. Four of these extensions are called Intuitionistic Fuzzy GNs (IFGNs) of first, second, third and fourth types. Interval Valued IFGNs (IVIFGNs) of first and third types are defined and studied in [4]. In Intuitionistic Fuzzy GNs of second type (IFGN2, see [2, 4, 6]), the tokens are replaced by some "quantities of matter" that "flow" inside the net. This fact generates some differences between the definitions of IFGN1s and IFGN2s. The IFGNs of type 4 (IFGN4s) are extensions of IFGN2s. The characteristics of the places there are estimated in the intuitionistic fuzzy sense, i.e., they obtain values (intuitionistic fuzzy pairs), that represent the degrees of validity and non-validity of the characteristics. Therefore, the two types of GNs allow describing situations where the model determines its status with degrees of validity and non-validity.

In the present research, IVIFGNs of second and fourth types are defined and studied. The possibility to use IVIFGNs from the four types as tools for describing Data Mining and Big Data processes are discussed.

Keywords: Generalized Nets, Intuitionistic fuzzy sets.

1 Introduction

The concept of a Generalized Net (GN, see [1, 4, 5]) was introduced in 1982 as an extension of the Petri nets and their extensions and modifications.

Intuitionistic Fuzzy GNs of first and second types (IFGN1s and IFGN2s) were defined in 1985 in [2]. In 2001 in [11], the Intuitionistic Fuzzy GNs of third and fourth types (IFGN3s and IFGN4s) were introduced.

Two extensions of the GNs, called Interval Valued IFGN1 and Interval

Valued IFGN3 (IFGN1s and IFGN3s) were introduced in [14].

In the present paper, two new extensions of the GNs, called Interval Valued IFGN2 and Interval Valued IFGN4 (IFGN2s and IFGN4s) will be introduced.

The IVIFGNs are based on the Intuitionistic Fuzzy Sets and Interval Valued Intuitionistic Fuzzy Sets Theory [5, 7]. Short notes related to Intuitionistic Fuzziness and especially to Interval Valued Intuitionistic Fuzzy Pairs (IVIFPs) are given in the Appendix.

2 Interval Valued Intuitionistic Fuzzy Generalized Nets of second type

In the present paper we will extend the IFGN2s and IFGN4s.

As it is mentioned in [14], similarly to Petri nets, the GNs in all their modifications contain transitions, places and tokens. The GN-places are depicted as the symbol ○ and the GN-transitions (more precisely, their graphical structure) are depicted as the symbol , which indicates the transition's conditions.

A GN, like other nets, contains tokens which transfer from place to place. Unlike all other nets, however, every GN-token enters the net with an initial characteristic and it receives new characteristics during each transfer. Each place has at most one arc entering and one arc exiting. The places with no entering arcs are called *input* places of the net and those without exiting arcs are *output* places of the net. The places situated to the left of the transition are the *transition's input places* while the rest are the *transition's output places*. Each transition has at least one input and one output place. When tokens enter the input places of a transition, it becomes potentially fired (activated) and at the moment of their transfer towards the transition's output places, it is fired.

The tokens of the new type of GNs are called "quantities" in [2, 4, 5, 6, 11, 14]. But this word here has another sense. For this reason, in the present and in future papers, the word "quantity" will be changed with the word "fluid" (cf. point (h) of Definition 2). Of course, the word "token" will be used instead of "fluid", wherever this is possible.

Definition 1. Every IVIFGN2-transition is given by a seven-tuple (see Fig. 1):

$$Z = \langle L', L'', t_1, t_2, r, M, \square \rangle \qquad (1)$$

where

a) L' and L'' are finite, non-empty sets of places (the transition's input and output places, respectively);

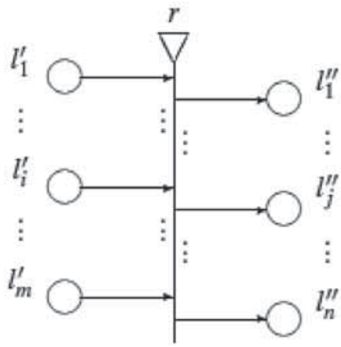

Fig. 1. An IVIFGN2-transition.

b) t_1 is the current time-moment of the transition's firing;

c) t_2 is the current value of the duration of its activity;

d) r is the transition's condition determining the fluid which will be transferred from the transition's inputs to its outputs. It has the form of an Index Matrix (IM, see [3, 8]):

$$r = \begin{array}{c|c} & l''_1 \ \ldots \ l''_j \ \ldots \ l''_n \\ \hline l'_1 & \\ \vdots & r_{i,j} \\ l'_i & (r_{i,j} \ - \ \text{predicates}) \\ \vdots & (1 \le i \le m, 1 \le j \le n) \\ l'_m & \end{array}$$

where $r_{i,j}$ denotes the element of the IM, which corresponds to the i-th input and j-th output places. These elements are predicates and when the truth value of the (i, j)-th element $f(r_{i,j}) = f(r_{i,j}) = \langle M_{i,j}, N_{i,j}\rangle$, which here and below is an IVIFP, is valid according to one of the 15 conditions given below, the fluid from i-th input place can be transferred to j-th output place; otherwise, this is not possible. Here, $M_{i,j}, N_{i,j} \subseteq [0,1]$ and $\sup M_{i,j} + \sup N_{i,j} \le 1$ and the function f is defined in point e) of Definition 2.

The conditions for interval valued intuitionistic fuzzy validity of the predicates are the following (they are the same, as in the cases of IVIFGN1s and IVIFGN3s from [14]):

- **(C1)** inf $M = 1$ (therefore sup $M = 1$ and inf N = sup $N = 0$),
- **(C2)** sup $M = 1$ (therefore inf N = sup $N = 0$),
- **(C3)** $\frac{1}{2} <$ inf $M \leq 1$ and sup $N = 0$,
- **(C4)** inf $M > \frac{1}{2} >$ sup N,
- **(C5)** inf $M \geq \frac{1}{2} \geq$ sup N,
- **(C6)** sup $M > \frac{1}{2} >$ sup N,
- **(C7)** sup $M \geq \frac{1}{2} \geq$ sup N,
- **(C8)** inf $M >$ sup N,
- **(C9)** inf $M \geq$ sup N,
- **(C10)** sup $M >$ sup N,
- **(C11)** sup $M \geq$ sup N,
- **(C12)** sup $M > 0$,
- **(C13)** inf $M > 0$,
- **(C14)** sup $N < 1$,
- **(C15)** inf $N < 1$.

Therefore, the fluid's transfer from i-th input to j-th output places is possible for a fixed criterion, when the components of $\langle M_{i,j}, N_{i,j}\rangle$ satisfy the respective criterion.

In the case of IVIFGN2s and IVIFGN4s, the following additional conditions must be valid. If the fluid in the input place l_i splits into s parts which have to enter the output places $l_{j1}, l_{j2}, \ldots, l_{js}$, then

$$\sum_{k=1}^{s} \sup M_{i,j_k} \leq 1,$$

$$\sum_{k=1}^{s} \inf N_{i,j_k} \geq 0;$$

e) M is an IM of the capacities of transition's arcs:

$$M = \begin{array}{c|ccccc} & l''_1 & \ldots & l''_j & \ldots & l''_n \\ \hline l'_1 & & & & & \\ \vdots & & & m_{i,j} & & \\ l'_i & & & (m_{i,j} \in \mathscr{R}^+, & & \\ \vdots & & & 1 \leq i \leq m, 1 \leq j \leq n) & & \\ l'_m & & & & & \end{array},$$

where R^+ is the set of real non-negative numbers.

f) □ is an object whose form is a Boolean expression that contains identifiers of the transition's input places as variables, and the Boolean operations ∧ and ∨. We assign the following semantics to this formula:

∧ $(l_{i1}, l_{i2}, \ldots, l_{iu})$ – every place $l_{i1}, l_{i2}, \ldots, l_{iu}$ must contain fluid,

∨$(l_{i1}, l_{i2}, \ldots, l_{iu})$ – there must be at least one fluid in all places $l_{i1}, l_{i2}, \ldots, l_{iu}$, where $\{l_{i1}, l_{i2}, \ldots, l_{iu}\} \subset L'$.

When the value of a type, evaluated as a Boolean expression, is *true*, the transition can become active, otherwise it cannot.

Definition 2. The ordered four-tuple:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K\rangle, \langle T, t^0, t^*\rangle, \langle X, \Phi\rangle\rangle \tag{2}$$

is called an *Interval Valued Intuitionistic Fuzzy Generalized Net of second type* (IVIFGN2) if:

a) A is a set of the transitions;

b) π_A is a function giving the priorities of the transitions, i. e., $\pi_A: A \to \mathcal{N}$, where $\mathcal{N} = \{0,1,2,\ldots\} \cup \{\infty\}$;

c) π_L is a function giving the priorities of the places, i.e., $\pi_L: L \to \mathcal{N}$, where $L = pr_1 A \cup pr_2 A$, and $pr_i X$ is the i-th projection of the n-dimensional set, where $n \in \mathcal{N}$, $n \geq 1$ and $1 \leq i \leq n$, i.e. L is the set of all GN-places;

d) c is a function giving the capacities of the places, i.e., c : $L \to \mathcal{R}^+$;

e) f is a function which evaluates the truth values of the transition's conditions predicates $r_{i,j}$ in the form $\langle M_{i,j}, N_{i,j}\rangle$, where $M_{i,j}, N_{i,j} \subseteq [0,1]$ are closed sets, and $\sup M_{i,j} + \sup N_{i,j} \leq 1$.

f) θ_1 is a function giving the next time-moment when a transition can become active, i.e., $\theta_1(t) = t'$, where $t, t' \in [T; T + t^*]$ and $t \leq t'$. The value of this function is calculated at the moment when a transition terminates its active state;

g) θ_2 is a function giving the duration of the active state of a transition, i.e., $\theta_2(t) = t'$, where $t \in [T; T + t^*]$ and $t' \geq 0$. The value of this function is calculated at the moment of activation;

h) K is the set of the fluids that enter and flow through the GN. Similarly to IFGN2s and IFGN4s, the IVIFGN2-fluids have only initial characteristics with the form "type and quantity of the fluids" (element of set X, defined below) and do not have current characteristics;

i) π_K is a function giving the priorities of the fluids, i.e., $\pi_K: K \to N$;

j) θ_K is a function giving the time-moment when fluid can enter the net, i.e., $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T; T + t^*]$;

k) T is the time-moment when the GN starts functioning. This moment is determined with respect to a fixed (global) time-scale;

l) t^o is an elementary time-step related to the fixed (global) time-scale;

m) t is the duration of the functioning of the net;

n) X is the set of all initial characteristics that the fluids can receive when they enter the net;

o) Φ is a characteristic function which gives a characteristic to each place when the fluid enters it. Now, places collect all their characteristics and therefore, an analogous of function b from the GN-definition is superfluous.

We shall illustrate the idea for the IVIFGN2 by the following example, showing on Fig 2.

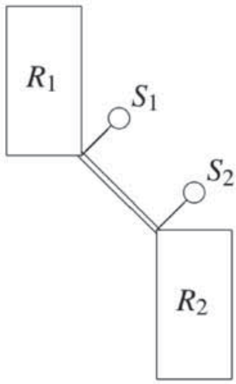

Fig. 2. An example.

Let us have two reservoirs R_1 and R_2 connected with a tube that has two sensors S_1 and S_2 on its two ends. Let reservoir R_1 contains quantity Q of some fluid.

When the predicate P = "the fluid from reservoir R_1 flows to reservoir R_2" has an evaluation

$$f(P) = \langle M_{1,2}, N_{1,2} \rangle \tag{3}$$

then this corresponds to the case in which quantity $Q \inf N_{1,2}$ of the fluid stays in the reservoir R_1, quantity $Q \inf M_{1,2}$ of the fluid enters the reservoir R_2, and quantity $Q(1 - \sup M_{1,2} - \sup N_{1,2})$ of the fluid stays in the tube. The sensor S_1 has a tolerance $\sup N_{1,2} - \inf N_{1,2}$ and the sensor S_2 has a tolerance $\sup M_{1,2} - \inf M_{1,2}$.

Two algorithms for the fluids transfer were given in [4, 6, 12]. They are modified here.

The algorithm for fluid's transfer after the time moment t_1 = TIME (here and below, we denote by TIME the current time moment of the GN), *denoted by* *algorithm A*, takes into consideration the possibility of merging and splitting of the fluids.

When the criterion *Cn* for $1 \le n \le 15$ is fixed, the *algorithm A* is described in 12 steps, as follows:

- **(A01)** Sort the input and output places of the transitions by their priorities.
- **(A02)** If the transition contains input places that are inputs for the IVIFGN2, the fluids that can enter the respective places enter them and the places obtain as characteristics the initial fluid characteristics.
- **(A03)** Generate an empty IM *R* that corresponds to the IM of the predicates *r*. Assign the value $\langle [0,0], [1,1] \rangle$, to all elements $R_{i,j}$ of *R* which:
  - **(A03a)** are in a row that corresponds to an empty input place, i.e. there are no fluids in the input place that can be transferred to an output place of the current transition;
  - **(A03b)** are in a column that corresponds to a full output place to which no fluids can be transferred;
  - **(A03c)** are placed in (*i, j*) place for which the predicate $r_{i,j}$ is set as *false* or $m_{i,j} = 0$, i.e. the current capacity of the arc between the *i*-th input place and the *j*-th output place is 0.

Assign the value $\langle [1,1], [0,0] \rangle$, to those elements $R_{i,j}$ of *R* which in a place (*i, j*) for which the predicate $r_{i,j}$ is set as *true*.

- **(A04)** Iterate through the input places in the order set by their priorities, starting with the place with highest priority for which no fluid has been transferred during the current time step and which has fluid in it. Perform consequently the following steps in order to determine if and where to transfer the current fluid.
  - **(A04a)** Check the current $R_{i,j}$ value of *R*. If the value of $R_{i,j}$ has not been set yet, go to step **(A04b)**. If $R_{i,j} = \langle [1,1], [0,0] \rangle$, go to step **(A04c)**. Otherwise, go to step **(A04d)**.
  - **(A04b)** Evaluate the truth value of the corresponding predicate $r_{i,j}$ of the IM *r*. If $r_{i,j}$ satisfies the criterion *Cn*, set the $R_{i,j}$ value of *R* to $\langle M_{i,j}, N_{i,j} \rangle$ and go to step **(A04c)**. Otherwise, set the $R_{i,j}$ value to $\langle [0,0], [1,1] \rangle$ and go to step **(A04d)**.
  - **(A04c)** The current fluid is transferred from input place l_i to the corresponding output (one or more) place l_j. The fluid is merged with specified parts in the output place, if there are such. After this, the characteristic function of the output place is evaluated, adding to it the type of the fluid in the place and its quantity $q_{i,j}$ that is determined by formula

    $$q_{i,j} = \min\left(m_{i,j}, \bar{c}(l_i, \mathrm{TIME}) \inf M_{i,j}, \mathrm{c}(l_j) - \bar{c}(l_j, \mathrm{TIME})\right),$$

    where $\bar{c}(l, \mathrm{TIME})$ is the quantity of the fluid in the place *l* at the current moment TIME.
  - **(A04d)** If a transfer has been made on the current step and the current fluid cannot be split, or all the predicates on the corresponding row are checked, go to step **(A05)**. Otherwise, go to step **(A04a)**.

If the splitting of the current fluid is not allowed, the evaluation of the predicates stops with the first one which evaluation satisfies the condition *Cn* for the fixed number *n*. The fluid then will be moved to the highest priority output place amongst those to which the fluid can be transferred.

If the splitting of the current fluid is allowed, then the fluid is split into as many parts as the number of the $R_{i,j}$ elements with evaluations satisfying criterion *Cn*. These new fluids are transferred to the corresponding output places. The characteristic functions of the output places are evaluated as above. The new characteristics are then assigned to the corresponding output places. Unlike the ordinary GNs, the fluids' characteristics of the IVIFGN2s contain not only the values evaluated by the respective characteristic functions, but also the IVIFPs that are evaluations of the respective predicates.

- **(A05)** If the highest priority fluid cannot be transferred during the current time step, it continues to stay in the input place.
- **(A06)** Increase by $q_{i,j}$ the current quantity of the fluid in each output place to which fluid has been transferred.
- **(A07)** Decrease by $(\sum_{k=1}^{s} 1 - \inf N_{i,j})\bar{c}(l_i, TIME - t^0)$ the quantity of the fluid in place l_i from which fluid has been transferred to *s* output places. If the quantity of the fluid in an input place becomes 0, set to $\langle[0,0],[1,1]\rangle$ all the elements in the corresponding row of the IM *R*.
- **(A08)** Decrease by $\bar{c}(l_i, \mathrm{TIME}) \sum_{k=1}^{s}(1 - \sup M_{i,j} - \sup N_{i,j})$ the capacities of all the arc through which a fluid has been transferred. If the current capacity of an arc becomes 0, assign $\langle[0,0],[1,1]\rangle$ to this element of the IM *R* that corresponds to the arc.
- **(A09)** If there are more input places with lower priority from which no fluid has been transferred to an output place, go to step **(A04)**. Otherwise, go to step **(A10)**.
- **(A10)** Add t^0 to the current model time.
- **(A11)** If the value of the current time is less than or equals $t_1 + t_2$ (the time components of the considered transition), go to **(A04)**. Otherwise, go to step **(A12)**.
- **(A12)** End of the transition's functioning.

The general algorithm for the GN's functioning, denoted by *algorithm B*, is described next. The concept of an *Abstract Transition (AT)* is introduced for the purpose of this algorithm as the union of all GN-transitions that are active at a given time moment.

The *algorithm B* can be described as follows:

- **(B01)** Put all α fluids for which $\theta_K(\alpha) \leq \mathrm{T}$ into the corresponding input places of the net.
- **(B02)** Construct the GN's AT. Initially it is empty.
- **(B03)** Check if the value of the current time is less or equal to $T + t^*$.
- **(B04)** If the answer to the question in **(B03)** is "no", go to step **(B12)**. Otherwise, go to step **(B05)**.

- **(B05)** Find those transitions for which t_1 is greater than or equal to the current time.
- **(B06)** Check the transitions' types of all transitions determined on step **(B05)**. The method used for the evaluation of the transitions' types is as follows:
  - **(B06a)** replace the names of all places used as variables in the Boolean expression of the transition type with the value 0, if the corresponding place has no fluid in it at the current moment, and with the value 1, otherwise;
  - **(B06b)** calculate the truth value of the Boolean expression, result of **(B06a)**.
- **(B07)** Add to the AT those transitions, the transition types of which are evaluated as *true* on step **(B06b)**.
- **(B08)** Apply *algorithm A* over the AT.
- **(B09)** Remove from the AT those transitions which are inactive at the current time moment.
- **(B10)** Increase the current time with t^0.
- **(B11)** Go to step **(B03)**.
- **(B12)** End of the GN's functioning.

By analogy with Theorem 4.1.1, [6], we have proved the following

Theorem 1. The functioning and the results of the work of every IVIFGN2 can be described by some ordinary GN.

Proof. Let the IVIFGN2 E be given. We shall construct a new, this time ordinary, GN G and we shall prove that both nets function equally. The validity of Theorem 5.3.1 [4] is used in the proof. This gives us the opportunity to research the functioning and the results of the works of the corresponding transitions in both nets (E and G).

Let the components of IVIFGN2 E be marked by index E and these of GN G – by index G.

Let initially, GN G have the same graphical structure as GN E and let all its other components, without the function f_G and the characteristic function Φ_G, be the same in both nets. For example, the sets X_E and X_G are equal, i.e., the (equal) tokens in both nets will have equal initial characteristics and the transition condition predicates are also equal in both nets. We set $b(\alpha) = \infty$ for each G-token α, because the function b does not exist in the definition of E.

Below, we shall construct the new G-components.

Let for the real number x:

$$sg(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x \le 0 \end{cases}$$

and

$$\overline{sg}(x) = \begin{cases} 0, & if\ x > 0 \\ 1, & if\ x \le 0 \end{cases}$$

Function f_G is defined in respect of the choice of criterion Cn, as follows:

- in case **(C1)**: $f_G(r_{i,j}) = sg(\inf M_{i,j}) \,.\, \overline{sg}\,(\sup N_{i,j})$;
- in case **(C2)**: $f_G(r_{i,j}) = sg(\sup M_{i,j}) \,.\, \overline{sg}\,(\sup N_{i,j})$;
- in case **(C3)**: $f_G(r_{i,j}) = sg(\inf M_{i,j} - \frac{1}{2}) \,.\, \overline{sg}\,(\sup N_{i,j})$;
- in case **(C4)**: $f_G(r_{i,j}) = sg(\inf M_{i,j} - \frac{1}{2}) \,.\, sg(\frac{1}{2} - \sup N_{i,j})$;
- in case **(C5)**: $f_G(r_{i,j}) = \overline{sg}(\frac{1}{2} - \inf M_{i,j}) \,.\, \overline{sg}\,(\sup N_{i,j} - \frac{1}{2})$;
- in case **(C6)**: $f_G(r_{i,j}) = sg(\sup M_{i,j} - \frac{1}{2}) \,.\, sg(\frac{1}{2} - \sup N_{i,j})$;
- in case **(C7)**: $f_G(r_{i,j}) = \overline{sg}(\frac{1}{2} - \sup M_{i,j}) \,.\, \overline{sg}\,(\sup N_{i,j} - \frac{1}{2})$;
- in case **(C8)**: $f_G(r_{i,j}) = sg(\inf M_{i,j} - \sup N_{i,j})$;
- in case **(C9)**: $f_G(r_{i,j}) = \overline{sg}(\sup N_{i,j} - \inf M_{i,j})$;
- in case **(C10)**: $f_G(r_{i,j}) = sg(\sup M_{i,j} - \sup N_{i,j})$;;
- in case **(C11)**: $f_G(r_{i,j}) = \overline{sg}(\sup N_{i,j} - \sup M_{i,j})$;
- in case **(C12)**: $f_G(r_{i,j}) = sg(\sup M_{i,j})$;
- in case **(C13)**: $f_G(r_{i,j}) = sg(\inf M_{i,j})$;
- in case **(C14)**: $f_G(r_{i,j}) = sg(1 - \sup N_{i,j})$;
- in case **(C15)**: $f_G(r_{i,j}) = sg(1 - \inf M_{i,j})$.

Therefore, function f_G gives as a result value

$$f_G(r_{i,j}) = \begin{cases} 1, & if\ criterion\ C_n\ is\ satisfied \\ 0, & otherwise \end{cases}$$

Let the characteristic function Φ_G be defined by

$$\Phi_G(\alpha) = \text{“}\langle \Phi_E(l_j), f_G(r_{i,j})\rangle\text{”}.$$

Thus, we described the new components of the GN G. Now, we must show that both nets function equally.

Let Z and Z^* be corresponding transitions in E and G nets, respectively. First, we will change the form of the transition Z (without restriction, we can assume that it has the from Fig. 1) to the form from Fig. 3. The places $l'_1, \ldots, l'_m, l''_1, \ldots, l''_n$ in Fig. 3 are the same input and output places as in the transition Z.

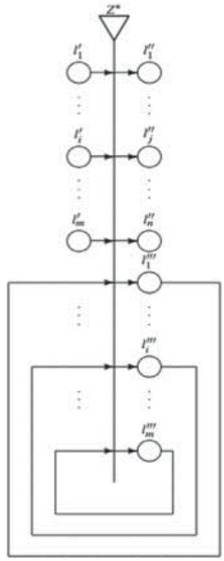

Fig. 3. GN-transition Z^*.

We shall show that both transitions function equally. When some fluid enters some Z-input place for the first time, a token will enter the respective Z^*- input place. If this fluid splits into parts that enter some (one or more) output places and there is still a part
that remains in the input place l'_i of the transition Z, this token will split to one or more tokens that will enter the respective output places of Z^* and one token will enter the place l'''_i. Therefore, the transition type of Z^* must be $\square_{Z^*} = \square_Z \vee \vee(l'''_1, \ldots, l'''_m)$ and all elements of IM M_{Z^*} are ∞. The predicates $r^{Z^*}_{l'_i,l''_j}$ of r_{Z^*} have the forms:\

$$r^{Z^*}_{l'_i,l''_j} = r^{Z^*}_{l'''_i,l''_j} = r^{Z}_{l'_i,l''_j}$$

$r^{Z*}_{l'_i,l'''_i}$ = "a part of the fluid from place l'_i cannot flow outside it".

$r^{Z*}_{l'''_i l'''_i}$ = "a part of the fluid from place l''_i cannot flow outside it".

$r^{Z*}_{l'_i,l'''_j} = r^{Z*}_{l'''_i,l'''_j} =$ *false* for $i \neq j$

The fact that both transitions have equal tokens in equal input places is obvious, if they are transitions from the first level in the sense of Theorem 5.3.1 [4]. By induction, we can admit that they have equal tokens with equal current characteristics in equal time moments t_1^Z and t_1^{Z*}. Then, following the sequential steps of the *Algorithm A* for token's transfer and its IVIFGN2's modification, we see that in the case of the IVIFGN2 E the IM is constructed

$$
\begin{array}{c|c}
 & l''_1 \ \dots \ l''_j \ \dots \ l''_n \\
\hline
l'_1 & \\
\vdots & f_E(r_{i,j}) \\
l'_i & (r_{i,j} - \text{predicates}) \\
\vdots & (1 \le i \le m, 1 \le j \le n) \\
l'_m &
\end{array}
$$

where $f_E(r_{i,j}) \in \mathcal{P}([0,1]) \times \mathcal{P}([0,1])$ and for GN G – the IM is

$$
\begin{array}{c|c}
 & l''_1 \ \dots \ l''_j \ \dots \ l''_n \ l'''_1 \ \dots \ l'''_m \\
\hline
l'_1 & \\
\vdots & \\
l'_m & f_G(r_{i,j}) \\
l'''_1 & (r_{i,j} - \text{predicates}) \\
\vdots & (1 \le i \le m, 1 \le j \le n) \\
l'''_i & \\
\vdots & \\
l'''_m &
\end{array}
,
$$

where for every set Y, $\mathcal{P}(Y)$ is the set of the subsets of set Y.

Therefore, from the above, we see that the corresponding elements of both IMs are equal. Hence, the tokens in both transitions will make equal transitions from input to output transition places. There, they will receive similar new characteristics. The tokens from the new net will receive more values, but for the proof, it is important that they will receive all the values which the tokens will receive from the first net.

Hence, both transitions really function equally.

3 Interval Valued Intuitionistic Fuzzy Generalized Nets of fourth type

Here, we introduce the following new

Definition 3. The ordered four-tuple:

$$E = \langle\langle A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \rangle, \langle K, \pi_K, \theta_K \rangle, \langle T, t^0, t^* \rangle, \langle X, \Phi \rangle\rangle \qquad (3)$$

is called an *Interval Valued Intuitionistic Fuzzy Generalized Net of fourth type* (IVIFGN4) if:

a) A is a set of the transitions defined in Definition 1;

Points b) – n) and p) from Definition 2 are valid again, while point o) has the form

o') Φ is a characteristic function which gives a new characteristic to every place when fluid enters in it and its interval valued intuitionistic fuzzy evaluation $\langle M^*_{i,j}, N^*_{i,j}\rangle$, where $M^*_{i,j}, N^*_{i,j} \subseteq [0,1]$ are closed sets, and $\sup M^*_{i,j} + \sup N^*_{i,j} \leq 1$.

There exists a small addition in the first algorithm for the fluid transfer, described in previous Section, related to the new form of place characteristics.

The following assertion holds.

Theorem 2. For every IVIFGN4 there exists a standard GN which represents it.

4 Conclusion

Following the ideas from [9, 10, 15], in future, we will discuss the possible applications of the IVIFGN2s and IVIFGN4s as tools in the Data Mining instruments. Also, different processes, described by GNs will be remodeled by IVIFGN2s and/or IVIFGN4s and especially in the cases, when the processes flow in uncertainty. Such example is a GN-model of the organization of the firefighting process described in [13]. This process flows in conditions of a high degree of uncertainty. So, in near future, we will rewrite the model from [13] using an IVIFGN2.

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Appendix

Interval Valued Intuitionistic Fuzzy Pair (IVIFP) is an object with the form $\langle M, N \rangle$, where $M, N \subseteq [0,1]$ are closed sets, $M = [\inf M, \sup M]$, $N = [\inf N, \sup N]$, and $\sup M + \sup N \leq 1$, that is used as an evaluation of some object or process and which components (*M* and *N*) are interpreted as intervals of degrees of membership and non-membership, or intervals of degrees of validity and non-validity, or intervals of degrees of correctness and non-correctness, etc. One of the basic geometrical interpretations of an IVIFP is shown on Fig. 4

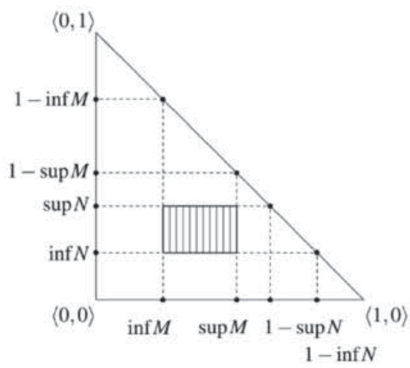

Figure 4: Geometrical interpretation of an IVIFP

It is suitable to mention that the logical *true* and *false* are represented by the IVIFP $\langle [1,1], [0,0] \rangle$ and $\langle [0,0], [1,1] \rangle$, respectively. Obviously, these values correspond to the intuitionistic fuzzy points $\langle 1,0 \rangle$ and $\langle 0,1 \rangle$.