Visualization of Transition's Scenarios from Harmonic to Chaotic Flexible Nonlinear-elastic Nano Beam's Oscillations

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In this study, a mathematical model of the nonlinear vibrations of a nano-beam under the action of a sign-variable load and an additive white noise was constructed and visualized. The beam is heterogeneous, isotropic, elastic. The physical nonlinearity of the nano-beam was taken into account. The dependence of stress intensity on deformations intensity for aluminum was taken into account. Geometric non-linearity according to Theodore von Karman's theory was applied. The equations of motion, the boundary and initial conditions of the Hamilton-Ostrogradski principle with regard to the modified couple stress theory were obtained. The system of nonlinear partial differential equations to the Cauchy problem by the method of finite differences was reduced. The Cauchy problem by the finite-difference method in the time coordinate was solved. The Birger variable method was used. Data visualization is carried out from the standpoint of the qualitative theory of differential equations and nonlinear dynamics were carried out. Using a wide range of tools visualization allowed to established that the transition from ordered vibrations to chaos is carried out according to the scenario of Ruelle-Takens-Newhouse. With an increase of the size-dependent parameter, the zone of steady and regular vibrations increases. The transition from regular to chaotic vibrations is accompanied by a tough dynamic loss of stability. The proposed method is universal and can be extended to solve a wide class of various problems of mechanics of shells.

Keywords: *visualization of scenarios of transition of vibrations into chaos, geometric nonlinearity, nano-beam, inhomogeneous material, micropolar theory, Euler-Bernulli model.*

1. Introduction

Visualization behavior of elements of microelectromechanical systems (MEMS) in the form of beams, plates and shells under the action of various kinds of loads currently is an actual scientific problem [1,2,7,8,9,18]. One of the ways of MEMS evolution is to reduce their mechanical components to the nanoscale and to reduce their mass. Already now about 10% of GDP in European countries is directly related to micro and nanoengineering. It is expected that the potential development of traditional microelectronics will be exhausted in the coming decade. Further development of electronics is associated with the development of nanotechnology. The scope of the NEMS is very wide. Nanosensors (cantilevers, nano-suspensions, resonators, etc.) and nanoactuators (nano-motors) are used in physics, biology, chemistry, medicine (diagnostics, cellular nano- and microsurgery, drug delivery, the affected area of the body), electronic industry, criminology. Despite all the advances in nanotechnology, any work at the molecular level remains extremely complex scientific problem. NEMS operate on the basis of other physical laws than MEMS.

The classical theory of continuum mechanics is not ideal for analyzing the dynamic properties of nanostructures since size effects significantly affect the dynamic behavior of nanostructures. For objects with ultra-small dimensions, it is necessary to use more complex theories, for example, modified couple stress theory, nonlocal theory of elasticity, surface theory of elasticity, etc. A very important scientific problem is taking into account the heterogeneity of the material when designing the NEMS element, i.e. Dependencies of physical properties of a material on deformation, spatial coordinates and time There are a large number of studies devoted to the size-dependent behavior of beams. Longitudinal vibrations of heterogeneous rods at nano-dimensional levels using , nonlocal theory of elasticity were studied [6]. It was shown that the heterogeneity of the material can strongly influence the longitudinal vibrations of nano-rods. Depending on the value of the coefficient of elastic modulus, the natural frequency of nano-rods may decrease or increase with an increase in the number of degrees of freedom [19]. In addition to the longitudinal vibrations of nano-rods, wave propagation was also investigated [4]. It was found that the scale parameter strongly affects the propagation of waves in nano-rods.

Various modified models of nano-beams were proposed for the study of bending using non-local mechanics of solid [3, 17, 16, 20]. More recently, a beam model based on a nonlocal gradient stress theory was proposed in [10, 11] to study the mechanical behavior of inhomogeneous nano-beams.

The article [15] is devoted to the development of a linear theory for the analysis of the behavior of beams based on the mechanics of a micropolar continuum. The nature of bending and longitudinal waves in a micropolar beam of infinite length was investigated. The deformation of a cantilever beam under the action of a transverse concentrated load on the free edge was also studied. In [13], the size-dependent behavior of Timoshenko beams using a combination of micropolar theory with nonlocal elasticity was modeled. The authors of [14] proposed a new numerical approach to the analysis of the bending of Euler – Bernoulli nano-beams in the context of integral non-local models. The authors of [12] studied the nonlinear vibrations of functionally graded nano-beams based on an elastic base and subjected to a uniform increase in temperature. The effect of small size, which plays a significant role in the dynamic behavior of nano-beams, is considered here using an innovative non-local integral model. The main partial differential equations of the theory of Bernoulli-Euler beams using von Karman relations were obtained.

The review of articles confirms the need to visualize the behavior of NEMS components using methods of nonlinear dynamics and taking into account the specifics of small sizes of NEMS objects and the operating conditions of NEMS. Basically, the visualization of the frequency spectrum of the signal is carried out only using the Fourier spectrum. Use for visualization of modal, phase portraits, cross-section, Poincare mapping, spectrum of Lyapunov exponents, etc. will allow you to explore the phenomenon of determinate chaos, to determine its truth. For a qualitative assessment of the transition of the oscillations of the nano-system into chaos, it is necessary to use tools like Fourier analysis and wavelet analysis, based on the strengths of each of them, when visualizing. The methods proposed in this work for visualizing the behavior of NEMS elements can be useful in non-destructive testing systems, as well as for designing NEMS.

1. Problem statement

In the modified couple stress theory [21], the stored strain energy Π for an elastic body with infinitely small strains is written as

$$\Pi = \frac{1}{2}\int_A\int_\Omega\left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij}\right)d\Omega dA, \qquad (1)$$

where: ε_{ij} – components of the strain tensor and χ_{ij} – components of the symmetric curvature gradient tensor, which are defined as follows:

$$\sigma_{ij} = \lambda\varepsilon_{mm}\delta_{ij} + 2\mu\varepsilon_{ij}, \qquad (2)$$

$$\varepsilon_{ij} = \frac{1}{2}\nabla u, \qquad \chi_{ij} = \frac{1}{2}\nabla\theta,\ \theta_{ij} = \frac{1}{2}\left(rot(u)\right)_i,\ m_{ij} = 2\mu l^2\chi_{ij}, \qquad (3)$$

here $\mathbf{u}$ - displacement vector with components u_i, $i = \{x, y, z\}$, $\boldsymbol{\theta}$ is an infinitely small rotation vector with components θ_i. Denote σ_{ij}, ε_{ij}, m_{ij} and χ_{ij} respectively components: classical stress tensor $\boldsymbol{\sigma}$, strain tensor $\boldsymbol{\varepsilon}$, deviator part of a symmetric tensor moment higher order $\mathbf{m}$ and the symmetric part of the curvature tensor $\boldsymbol{\chi}$; $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$, $\mu = \frac{E}{2(1+\nu)}$ - Lame parameters, which also depend on the coordinates and intensity of deformations; δ_{ij} - Kronecker symbol. The parameter l appearing at the moment of higher order m_{ij} is an additional independent material parameter of length. It is connected with the symmetric tensor of a rotation gradient. In this model, in addition to the usual Lame parameters, one more scale parameter of length l [7] must be taken into account. This is a direct consequence of the fact that in couple stress theory, the strain energy density function is a function of the strain tensor and the symmetric curvature tensor. It does not explicitly depend on rotation (the asymmetric part of the strain gradient) and the asymmetric part of the curvature tensor.

Consider a beam of length a, constant thickness h. The beam occupies the area $\Omega = \left\{(x,z)\,|\,0 \le x \le a, -\frac{h}{2} \le z \le \frac{h}{2}\right\}$. We introduce the notation: h_0 – beam thickness in the center, b_0 - beam width in the center, $u_{30}(x,t)$ – bend deflection, $u_{10}(x,t)$ – midline displacement. Beam designed from isotropic but heterogeneous material $E(x,z,e_i)$ and $\nu(x,z,e_i)$ – module elasticity and Poisson's ratio, depending on the coordinates and intensity of deformations e_i, according to the deformation theory of plasticity, ε – coefficient damping, γ – unit weight of the material, g – acceleration gravity. Euler-Bernoulli hypothesis was applied.

Geometric nonlinearity is taken into account according to the Karman model.

Nano dimension is taken into account by the modified couple stress theory.

To account for the physical nonlinearity of the material of beams, the deformation theory of plasticity and the method of variable parameters of elasticity are used [1]. Diagram of deformations material $\sigma_i(\varepsilon_i)$ can be arbitrary, but in numerical examples it is accepted for pure aluminum in the form:

$$\sigma_i = \sigma_s\left[1 - \exp\left(-\frac{e_i}{e_s}\right)\right]$$

e_s and σ_s – strain intensity and stress yields depending from longitudinal (x) and transverse (z) coordinates.

Equations of motion, the boundary and initial conditions of the beam follow from the Hamilton – Ostrogradski variational principle.

$$k_1 = \int_A(\lambda + 2\mu)dA,\ k_3 = \int_A\left[(\lambda+2\mu)z^2 + \frac{1}{2}\mu l^2\right]dA,\ b_1 = \int_A\rho\, dA$$

$$\frac{\partial^2}{\partial x^2}\left[\int_{-\frac{h}{2}}^{\frac{h}{2}}(\lambda+2\mu)zdz\left(\frac{\partial u_{10}}{\partial x} + \frac{1}{2}\left(\frac{\partial u_{30}}{\partial x}\right)^2\right) - \left(\int_{-\frac{h}{2}}^{\frac{h}{2}}\left((\lambda+2\mu)z^2 + \frac{1}{2}\mu l^2\right)dz\right)\frac{\partial^2 u_{30}}{\partial x^2}\right] + q +$$
$$+\frac{\partial}{\partial x}\left[\frac{\partial u_{10}}{\partial x}\left(\int_{-\frac{h}{2}}^{\frac{h}{2}}(\lambda+2\mu)dz\left(\frac{\partial u_{10}}{\partial x} + \frac{1}{2}\left(\frac{\partial u_{30}}{\partial x}\right)^2\right) - \int_{-\frac{h}{2}}^{\frac{h}{2}}(\lambda+2\mu)zdz\frac{\partial^2 u_{30}}{\partial x^2}\right)\right] = \qquad (4)$$
$$= h\frac{g}{\gamma}\frac{\partial^2 u_{30}}{\partial t^2} + h\frac{g}{\gamma}\varepsilon\frac{\partial u_{30}}{\partial t};$$
$$\frac{\partial}{\partial x}\left[\int_{-\frac{h}{2}}^{\frac{h}{2}}(\lambda+2\mu)dz\left(\frac{\partial u_{10}}{\partial x} + \frac{1}{2}\left(\frac{\partial u_{30}}{\partial x}\right)^2\right) - \int_{-\frac{h}{2}}^{\frac{h}{2}}(\lambda+2\mu)zdz\frac{\partial^2 u_{30}}{\partial x^2}\right] = h\frac{g}{\gamma}\frac{\partial^2 u_{10}}{\partial t^2}.$$

The system (4) is reduced to dimensionless form using the following dimensionless parameters: $\bar{x} = \frac{x}{a}$, $\bar{z} = \frac{z}{h_0}$, $\bar{u}_{30} = \frac{u_{30}}{h_0}$, $\bar{q} = \frac{a^4}{G(h_0)^4}q$, $\bar{h} = \frac{h}{h_0}$, $\bar{b} = \frac{b}{b_0}$, $\bar{u}_{10} = \frac{au_{10}}{h_0^2}$, $\bar{E} = \frac{E}{G}$, $\bar{t} = \frac{h_0}{a^2}\sqrt{\frac{Gb_0 g}{\gamma}}t$, $\bar{\varepsilon} = \frac{h_0^2}{a^2}\sqrt{\frac{Gb_0 g}{\gamma}}\varepsilon$, where $q = q_0\,\mathrm{Sin}(\omega_p t)$ - load acting on the beam, q_0 and ω_p - amplitude and load frequency, respectively.

To system (4) are added boundary conditions rigid fixation

$$u_{30}(x,t) = u_{10}(x,t) = \frac{\partial u_{30}(x,t)}{\partial x} = 0, \quad x = 0; a \qquad (5)$$

and initial conditions

$$u_{30}(x,t) = \frac{\partial u_{30}(x,t)}{\partial t} = u_{10}(x,t) = \frac{\partial u_{10}(x,t)}{\partial t} = 0,\ t = 0 \qquad (6)$$

2. Solution methods

The integration of equations (4) with the boundary (5) and initial (6) conditions is carried out by the finite difference method. To improve accuracy, central difference schemes for derivatives have been applied.

The convergence of the method along the spatial coordinate was studied. To obtain results with the required degree of accuracy, it suffices to split the integration interval [0, 1] into 120 parts. It is necessary to solve an extensive system of equations. At each time layer, the iterative method procedure variable parameters Birger's elasticity [5] was built. The value of the Young's modulus in the spatial grid $\{x, z\}$ was refined. The stability of the solution in time, i.e. the choice of the time step is carried out according to the Runge principle.

The reliability of the numerical results is proved by the complete coincidence of the solutions obtained by the method described above with the results obtained using the Bubnov form finite element method in the spatial coordinate. Then the Cauchy problems were solved using the Runge – Kutta type method from the second to the eighth order of accuracy.

3. Numerical results

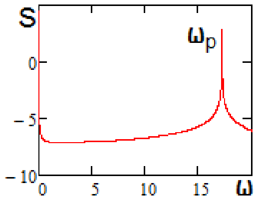
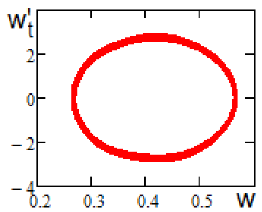
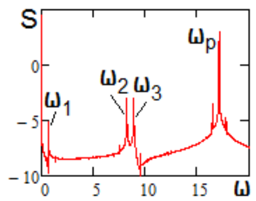
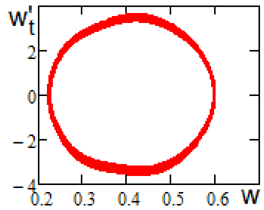
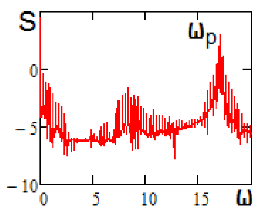
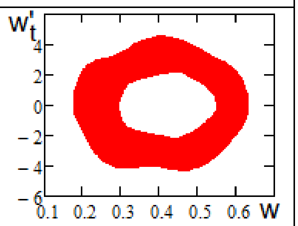
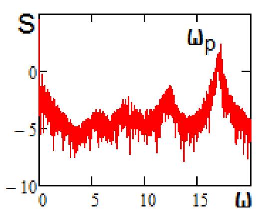
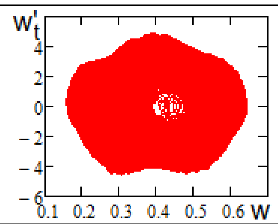
Visualization of the calculation results for a rigidly clamped geometrically and physically nonlinear beam, to which a uniformly distributed alternating load with a frequency $\omega_p = 5.1$ is applied, which coincides with the frequency of linear natural vibrations, was based on the methods of nonlinear dynamics. The parameters of the experiment: the material is aluminum, the ratio of length to thickness λ=50.

The aim of the study was to visualize the scenarios for the transition of vibrations of nano-beams from harmonic to chaotic, depending on load changes. As well as visualizing the impact on accounting scenarios moment stresses, i.e. the value of an additional parameter l related to the tensor gradient curvature χ .

To achieve this goal, the following visualization tools were used: signals, phase and modal portraits, Poincaré sections and maps, autocorrelation function, Fourier spectra, 2D and 3D wavelet spectra constructed on the Morlet mother wavelet were analyzed. Also changes the sign of the the largest Lyapunov exponent (LLE) in time, depending on the value of the additional independent length parameter l , was studied.

At the table shows the most informative results for $l = 0$.

Table 1. Scenario of transition to chaos at $l = 0$.

q_0	Fourier spectrum	Phase portrait
84		
85		
87		
88		

The transition from harmonic vibrations to chaotic is obtained according to the scenario of Ruel-Takens-Newhouse with $l = 0;\ 0.1;\ 0.3;\ 0.5$. Thus, an independent frequency and linear combinations of the excitation frequency and independent frequency appear.

Visualization of the nature of vibrations at the amplitude of the load $q_0 = 84$ (Table 1), based on the power spectrum, it shows harmonic vibrations. However, visualization based on a phase portrait demonstrates the presence of additional frequencies, since the phase portrait has a thickening. For a qualitative analysis of the behavior of the system, several visualization tools must be considered together. With increasing load amplitude, the Fourier spectrum reflects the appearance of harmonics in the signal at independent frequencies. With a further increase in the amplitude, the power spectrum shows a continuous pedestal, and the phase portrait shows a solid spot. Which indicates the chaotic state of the system.

With increasing l the load value at which the transition to chaotic vibrations takes place, because flexural rigidity of the system increases.

4. Conclusion

This study presents a visualization of nonlinear vibrations of a flexible, inhomogeneous, rigidly clamped at the ends of a nano-beam under the action of a uniformly distributed alternating load. Using the Fourier spectrum and phase portrait as a means of visualization, it was possible to determine the transition from harmonic to chaotic vibrations according to the scenario of Ruel-Tackens-Newhouse. Accounting moment stresses and an increase in the value associated with this parameter does not change the scenario of the transition of system vibrations to chaos.

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6. References

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