Mathematical Modeling of the Contact Interaction of Two Nanobeams Timoshenko S.P.

V.A. Krysko-jr.[1], T.V. Yakovleva.[1,2], O.A. Saltykova[1,2], V.S. Kruzhilin[1]
vadimakrysko@gmail.com |yan-tan1987@mail.ru| olga_a_saltykova@mail.ru |mrkruzhilin@mail.ru
[1]Yuri Gagarin State Technical University of Saratov, Saratov, Russia;
[2]Tomsk Polytechnic University, Tomsk, Russia;

The mathematical model of the contact interaction of two nanobeams obeying the kinematic hypothesis of the second approximation S.P. Timoshenko is constructed. There is a small gap between the nanobeams; an external alternating transverse load acts on the upper nanobeam. Nanobeams are isotropic, elastic, and they are connected through boundary conditions. Modified couple stress theory has been applied to describe the size-dependent effects of a beam nanostructure. Contact interaction is accounted for by the model B.Ya. Cantor. The paper studies the effect of the size-dependent coefficient. The system of differential equations is reduced to the Cauchy problem by the finite-difference method with an approximation of 0(h2) in the spatial coordinate. Further, the solution was carried out by the Runge-Kutta methods of the 4th order of accuracy in time. The convergence of numerical methods is investigated. The visualization of the results obtained by the methods of nonlinear dynamics and using wavelet transforms.

Keywords: *contact interaction of nanobeams Timoshenko S.P., modified couple stress theory, nonlinear oscillations, finite difference method, Runge-Kutta method.*

1. Introduction

Nanobeams are the components of structures and devices that are subject to external dynamic effects of the most diverse nature. Therefore, the nature of their oscillations will largely depend on control parameters, such as the size-dependent coefficient and the type of load [1]. That is why the study of the nonlinear dynamics of beams and their contact interaction was devoted to a vast amount of scientific work - from the first approximation models of Bernoulli-Euler to the models of the third approximation Peleh-Sheremetyev-Reddy. We will choose the second approximation model developed by the famous scientist-mechanic S.P. Timoshenko in the first half of the XX-th century. It allows to take into account the lateral shear deformation together with the inertia of rotation.

The aim of this investigation is to study the chaotic dynamics and the contact interaction of two nanobeams with a small gap between them, described by the model of S.P.Timoshenko, under the influence of an external transverse alternating distributed load. In the scientific literature there is a huge amount of work devoted to the study of full-length beams by S.P. Timoshenko [2, 4, 9], full-size and nanoscale beams of Euler-Bernoulli [4-6]. In [8], nonlinear oscillations of Euler-Bernoulli nanoscale beams are studied according to the non-local theory of elasticity. The beams described by the Timoshenko model are used in scientific works for the aviation industry [7]. An important issue is the methods of scientific visualization of the results.

2. Mathematical model of Timoshenko S.P. nanobeams

A mathematical model of a two-layer nanostructure has been constructed, which consists of two parallel nanobeams. The beam nanostructure is under the action of an external transverse alternating load $q = q_0 \sin \omega_p t$. The beam structure is shown in Fig. 1.

Nanobeams are described by the kinematic model of the second approximation - S.P. Timoshenko. To account for the size-dependent coefficients, a modified Yang's theory of elasticity was applied [3]. The contact interaction of elements of a beam's nanostructure is taken into account according to the Winkler model according to the theory of B.Ya. Cantor.

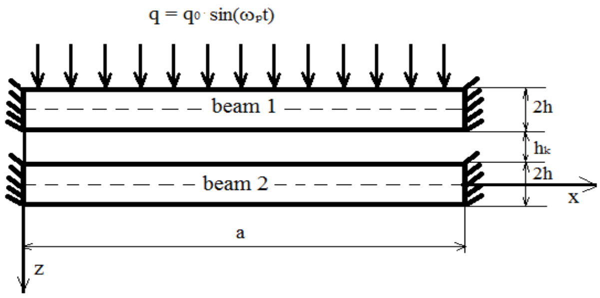


Fig. 1. The settlement scheme

The system of ordinary differential equations in displacements describing the movement of beams with allowance for energy dissipation, in a dimensionless form, is given below:

$$\lambda^2 \frac{\partial}{\partial x}\left[J_0\left(\frac{\partial u_i}{\partial x}+\frac{1}{2}\left(\frac{\partial w_i}{\partial x}\right)^2\right)+J_1\frac{\partial \varphi_i}{\partial x}\right]+f=I_0\frac{\partial^2 u_i}{\partial t^2}+I_1\frac{\partial^2 \varphi_i}{\partial t^2}, \tag{1}$$

$$\lambda^2 \frac{\partial}{\partial x}\left[J_1\left(\frac{\partial u_i}{\partial x}+\frac{1}{2}\left(\frac{\partial w_i}{\partial x}\right)^2\right)+J_2\frac{\partial \varphi_i}{\partial x}\right]-\lambda^4 A_0\left(\varphi_i+\frac{\partial w_i}{\partial x}\right)+\frac{\lambda^2}{4}\frac{\partial}{\partial x}\left(B_0\left(\frac{\partial \varphi_i}{\partial x}-\frac{\partial^2 w_i}{\partial x^2}\right)\right)+$$

$$+\lambda^2\frac{C_0}{2}=I_1\frac{\partial^2 u_i}{\partial t^2}+I_2\frac{\partial^2 \varphi_i}{\partial t^2},$$

$$\lambda^2\frac{\partial}{\partial x}\left[A_0\left(\varphi_i+\frac{\partial w_i}{\partial x}\right)\right]+$$

$$+\frac{\partial}{\partial x}\left\{\left[J_0\left(\frac{\partial u_i}{\partial x}+\frac{1}{2}\left(\frac{\partial w_i}{\partial x}\right)^2\right)+J_1\frac{\partial \varphi_i}{\partial x}\right]\left(\frac{\partial w}{\partial x}\right)\right\}+\frac{1}{4}\frac{\partial}{\partial x}\left\{\frac{\partial}{\partial x}\left[B_0\left(\frac{\partial \varphi_i}{\partial x}-\frac{\partial^2 w_i}{\partial x^2}\right)\right]+\frac{k^2 B_0}{\lambda^2}\frac{\partial w_i}{\partial x}\right\}+$$

$$+\frac{1}{2}\frac{\partial C_0}{\partial x}+q+(-1)^i K(w_1-w_2-h_k)\Psi=I_0\frac{\partial^2 w_i}{\partial t^2},$$

where $i = 1,2$ – is number of nanobeam,

$$\left(\bar{J}_0,\bar{J}_1,\bar{J}_2\right)=\int_{-1/2}^{1/2}\bar{E}\left(1,\bar{z},\bar{z}^2\right)d\bar{z}=\frac{1}{AE_0}\left(\frac{J_0}{1},\frac{J_1}{h},\frac{J_2}{h^2}\right),$$

$$\left(\bar{I}_0,\bar{I}_1,\bar{I}_2\right)=\int_{-1/2}^{1/2}\bar{\rho}\left(1,\bar{z},\bar{z}^2\right)d\bar{z}=\frac{1}{A\rho_0}\left(\frac{I_0}{1},\frac{I_1}{h},\frac{I_2}{h^2}\right),$$

$$\bar{A}_0=\int_{-\frac{1}{2}}^{\frac{1}{2}}\bar{G}d\bar{z}=\frac{A_0}{AE_0},\quad \bar{B}_0=\int_{-\frac{1}{2}}^{\frac{1}{2}}\bar{G}\left(\frac{l}{h}\right)^2 d\bar{z}=\frac{B_0}{AE_0 h^2},$$

$$\Psi=\frac{1}{2}\left[1+sign\left(w_1-h_k-w_2\right)\right]$$

Dimensionless variables (with a dash above) are:

$\bar{x} = x/L$, $\bar{z} = z/h$, $\bar{w} = w/h$, $\bar{\varphi} = \varphi L/h$, $\bar{u} = uL/h^2$, $\bar{E} = E/E_0$, $\bar{G} = G/E_0$,
$\bar{\rho} = \rho/\rho_0$, $\lambda = L/h$, $\bar{t} = \omega_0 t$, $\omega_0 = \sqrt{E_0/\rho_0 \lambda^2 L^2}$,
$\bar{q} = \lambda^3 Lq/AE_0$, $\bar{C}_0 = \lambda^3 C_0/AE_0$.

In system (1), dashes over dimensionless parameters are omitted for ease of recording. To the system of differential equations (1), one should add the boundary conditions (clamped-clamped):

$$J_0\left(\frac{\partial u_i}{\partial x} + \frac{1}{2}\left(\frac{\partial w_i}{\partial x}\right)^2\right) + J_1 \frac{\partial \varphi_i}{\partial x}\Bigg|_{x=0}^{x=1} = \bar{N} \text{ or } u_i\Big|_{x=0}^{x=1} = \bar{u}_i,$$

and initial conditions:

$$w_i(x,0) = 0; \quad \frac{\partial w_i(x,0)}{\partial t} = 0,$$

$$u_i(x,0) = 0; \quad \frac{\partial u_i(x,0)}{\partial t} = 0,$$

$$\varphi_i(x,0) = 0; \quad \frac{\partial \varphi_i(x,0)}{\partial t} = 0.$$

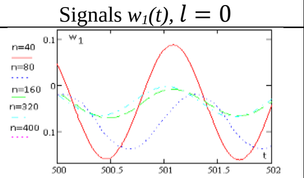
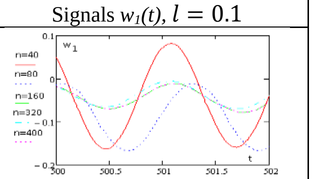
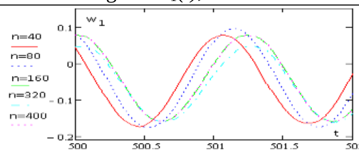
$i = 1{,}2$ – is number of nanobeam.

3. Solution methods

An infinite-dimensional problem using the finite-difference method with an approximation of $0(h^2)$ is reduced to a finite-dimensional system of ordinary differential equations. Next, the Cauchy problem was solved by the Runge-Kutta method of the fourth order of accuracy in time. The convergence of numerical methods is investigated: the finite differences method depending on the number of partitions along the length of the beams and the Runge-Kutta method depending on the step. In the finite difference method, the number of split points was taken to be n = 40, 80, 160, 320, 400 for each of the values of the size-dependent coefficient $l = 0, 0.1, 0.3, 0.5$. In table 1 shows the signals of deflection nanobeam 1 (w_1) for the $l = 0, 0.1, 0.3, 0.5$.

Table 1.

Convergence of the finite difference method.

Signals $w_1(t)$, $l = 0$	Signals $w_1(t)$, $l = 0.1$
	
Signals $w_1(t)$, $l = 0.3$	
	

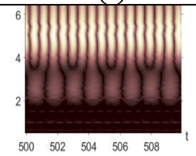
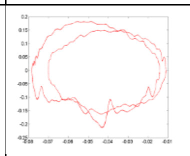
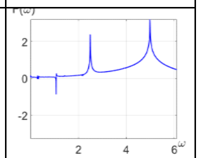
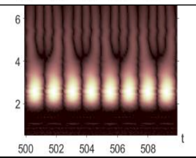
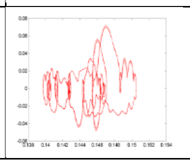
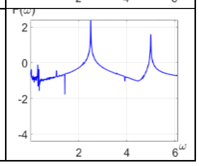
The convergence of the finite difference method for the problem in question occurs when the number of partitions is n = 160. Scientific visualization of convergence results obtained using mathcad. A further visualization of the study of the contact interaction and the nature of the oscillations of the two nanobeams was carried out by nonlinear dynamics methods with the construction of signals, phase portraits, Fourier power spectra and using wavelet analysis. For reliable visualization of the results, the Morlet, Gauss 8 - Gauss 32, and Haar wavelets were used as the mother wavelet.

4. Numerical experiment

We present the results of a numerical experiment for the contact interaction of two nanobeams fixed along the edges, described by the S.P. Timoshenko model. The amplitude of the external transverse load $q_0 = 5000$, the frequency of external excitation $\omega_p = 5.1$, the size-dependent coefficient $l = 0.1$. The initial contact interaction of two nanobeams occurs not in the central point, but in the quarters. In this case, a change occurs in the nature of the oscillations of nanobeams; two Hopf bifurcations are observed. Table 2 shows the 2D Morlet wavelet spectra $\omega(t)$, phase portraits $w(\dot{w})$ and Fourier power spectra for the upper (w_1) and lower ($w2$) nanobeams.

Table 2.

Dynamic characteristics of nanobeams.

	the 2D Morlet wavelet spectra $\omega(t)$	phase portraits $w(\dot{w})$	Fourier power spectra
Nanobeam 1			
Nanobeam 2			

5. Conclusion

A mathematical model of the contact interaction of two nanobeams with a small gap between them was constructed, taking into account the kinematic hypothesis of S.P. Timoshenko. The convergence of numerical methods used to solve the problem is investigated. It is established that the convergence of the finite difference method for the problem in question occurs when the number of partitions is n = 160. The scientific visualization of the results is based on the construction of signals, phase portraits, Fourier power spectra and the use of wavelet transforms. It has been established that the Morlet wavelet is the most informative for this class of problems, since it gives the best frequency localization at every moment in time. It is worth noting that the Fourier power spectrum gives a general picture of the nature of the oscillations of nanobeams over the entire time interval. The proposed approach allows us to study the nonlinear dynamics of the contact interaction of two nanobeams, with a gap between them, under the influence of an external alternating load, depending on the size-dependent coefficient. As a result of contact interaction, two Hopf bifurcations occur.

6. Acknowledgements

The reported study was funded by RFBR according to the research project № 18-38-00878 mol_a and № 18-41-700001 r_a.

7. References

[1] Awrejcewicz, J., Krysko, V.A., Yakovleva, T.V., Pavlov, S.P., Krysko, V.A. Nonlinear dynamics of contact interaction of a size-dependent plate supported by a size-dependent beam. Chaos. 28, 053102 (2018).
[2] Chen X., Lu, Y., Zhu B., Zhang X., Li Y. Nonlinear resonant behaviors of bi-directional functionally graded material microbeams: One-/two-parameter bifurcation analyses. 2019. Composite Structures. V. 223. №110896.

[3] Jia X.L., Yang J., Kitipornchai S. Pull-in instability of geometrically nonlinear micro-switches under electrostatic and Casimir forces. // Acta Mech. 218, (2011). P. 161–174.
[4] Khasawneh F.A, Segalman D. Exact and numerically stable expressions for Euler-Bernoulli and Timoshenko beam modes - 2019 - Applied Acoustics - V.151, P. 215-228.
[5] Krysko A. V., Awrejcewicz J., Pavlov S. P., Bodyagina K. S., Zhigalov M. V., Krysko V. A. Non-linear dynamics of size-dependent Euler–Bernoulli beams with topologically optimized microstructure and subjected to temperature field // International Journal of Non-Linear Mechanics 2018.
[6] Migda J., Migda M., Zdanowicz M. Asymptotic properties of solutions to fourth order difference equations - 2019 - Journal of Computational and Applied Mathematics - V.362, P. 68-82.
[7] Qian Y.J., Yang X.D., Zhang W., Liang F., Yang T.Z., Ren Y. Flutter Mechanism of Timoshenko Beams in Supersonic Flow. 2019. Journal of Aerospace Engineering. V.32. I.4. №04019033.
[8] Togun N. Nonlocal beam theory for nonlinear vibrations of a nanobeam resting on elastic foundation. 2016. Boundary Value Problems. V.1.
[9] Zhao B., Chen J., Liu T., Song W., Zhang J. A new Timoshenko beam model based on modified gradient elasticity: Shearing effect and size effect of micro-beam. 2019. Composite Structures. V.223. №110946.