Towards an Ontology for Measure of Music Aesthetics

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Abstract. Notes are entities, every note may be identified thanks to determined properties and qualities. What kind of categories gets in the play? Relationships arise between notes, between the notes and the interpreter, and between notes and the listener. And another relationship arises between notes, those agents and the composer even across time. This position paper is our very first applying the methods of ontological analysis to the music 'matter'. In our research path, a first step towards the way of understanding the music score is to proceed with a reading of the written music according to Birkhoff's (1933) formula for the aesthetics measure. This effort would be useful to implement a computer tool that helps to apply Birkhoff's aesthetic measure.

Keywords. Music, ontology, aesthetics, Birkhoff, analysis

1. Introduction

Studies in particle physics identify the existence in our space and matter of harmonic geometries [1]. Other examples about this internal relation for the all, from 'vibrations to physics' geometry' comes from the inspiration of the mathematician Alexander Grothendieck (1928-2014) the idea of a theory of motives, was understood as a musical idea of fundamental mathematical structures that act like musical motives in the great symphony of mathematics, as referred in their work by [2]. As reported in [3] Newton's grand scheme was an assumption that had always been taken for granted and thus rarely given much thought: the existence of a universal frame of reference, a kind of invisible coordinate system to which the position and motion of every particle in the universe could be referred [3].

Georges Birkhoff [4] gave a first structured answer to the possibility of a quantitative aesthetic measure about musical listening. Several researchers reasoned about internal relations in music, among them Johnson-Laird [5] in his work about the functioning of mind and creativity. Being a musician himself he tried to investigate how the musical knowledge is applied during jazz improvisation, he hypothesizes that it is a matter of memory, patterns and mathematical ratios.

Our wider purpose is to explore if it is possible to elicit the deep structure 'hidden' inside the music produced by composers. Our principal effort here is to cope with these topics looking at them from the side of the perception: of the music and the pleasant sensations that music offers to the listener.

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After the introduction (i) a brief survey about music and its relationship with mathematics and the relations between musicians and mathematics (ii), we have a reading of Birkhoff's work (iii). Then we (iv) explore how ontologies can support music analysis. In the end, we offer a (v) discussion and our (vi) conclusions: the investigation in this work closes with a focus on the possibility of elicitation of any recursion during the composition action.

2. About music and mathematics

Mathematics and music both depend on an efficient system of notation, a set of written symbols that convey a precise and unambiguous meaning to its practitioners [3]. Both mathematical notation and musical notation point to universes quite different from the one in which ordinary language functions so well. In both the human genius during centuries has developed an effective notation for giving representation to ideas that seem to lie beyond ordinary language [6].

Mathematics and music share many terms, as an example the word harmonic. Other examples of common terms are: inversion (of a musical interval; of a point with respect to a circle), root (of a musical chord; of a number or an equation), progression (of notes; of numbers), and series (in music, Arnold Schoenberg's twelve-tone system of composition; in mathematics, an infinite sum of terms) [3][7]. This indicates how the various elements stand in relation to one another, how sound creates a space and it shows how different musical voices move against and through each other. The notation in both subjects can make visible the hidden connections within each subject that reveal hidden connections among outside phenomena [6][7].

The aesthetics of musical composition have much in common with the best pieces of mathematics, where themes are established, then mutate and interweave until the listeners find themselves transformed at the end of the piece to a new place [8].
Mathematical structures, formulas, and methodologies have always played a crucial role in the creative construction of music. Bach used the symmetries of retrograde and inversion; Mozart invented the musical dice game; Bartók applied Fibonacci numbers to organize time, Messiaen, Boulez, Pousseur, Eimert, Stockhausen, and others applied group theory to create their serial compositions, extending the thoroughly mathematical dodecaphonic ideas of Schönberg and Hauer, Xenakis used graphs, many searches have been conducted to explore ways to compose music via mathematical tools [2].

3. Birkhoff: Aesthetics Measure

In 1933, George David Birkhoff wrote the first quantitative theory of aesthetics in his book Aesthetic Measure [4]. Since it involves computational methods, this work is often regarded as the beginning of Computational Aesthetics. His work showed some interesting thoughts as well as a good explanation of an attempt to formalize aesthetic measure by:

$$M = \text{Order}/\text{Complexity} \qquad (1)$$

The value of the aesthetic measure (M) is the result of a ratio between order (O) and complexity (C). This formula represents the reward one experiences when putting effort by focusing the attention (complexity) but then realizing a certain pleasant experience (order) [9]. Birkhoff intends complexity as the amount of effort the human brain has to put into the processing of an object. An effort is necessary for the experience of an aesthetic reward. Anyway, his work seems to be empirical and hardly represents scientifically how the human perceiving system reads complexity. After the Shannon complexity approach became a sophisticated measure in the theory of communication, there was hope to build more objective measures. Several researchers wove aesthetic theories, using the term information aesthetics; trying to add up to Birkhoff's work a new approach towards complexity [9].

We referred to Birkhoff because his theorization has been a starting point in the attempt of measuring the aesthetics perception quantitatively, in his own words: "measure arose in my mind, and the riddle of melody took on the aspect of a quasi-mathematical problem" [4]. His reasoning about it started from the attention to the remarkable phenomenon of melody. Did it depend mainly upon the orderly arrangement of musical notes or the pleasant perception at the ear?

In his argumentation, Birkhoff affirms that the ordinary person, by merely hearing without analysis a certain amount of music, learns rapidly to appreciate harmony even in its more complicated forms. This unanimity concerning harmony, and music generally, must rest on some rational basis [4]. We suppose it could rest on a different basis to something about a physic realm that could be understood as latent in a composition conception and sound, but this is something that has to be demonstrated. This thesis aims to be a step along this path.

Following Birkhoff's reasoning, we can affirm with him that the aesthetic measure of perception provides a systematic means of analysis in the simple form of the relation between a work of art and the analysis of the factors which constitute it. The aesthetic object is created by an artist and comes into existence as the free expression of aesthetic ideals. About the aesthetics measure many auditory and visual perceptions are accompanied by a certain intuitive feeling of value, which is separable from sensuous, emotional, moral, or intellectual feeling. Birkhoff [4] inferred that for a successful affirmation of aesthetics in its scientific aims, it had to be provided by some rational basis for such intuitive comparisons.

3.1. Birkhoff's mathematical formulation of the problem

Aiming at defining the mathematical formulation of the problem, Birkhoff [4] posed a typical aesthetic experience as compounded of three successive phases, as follows in his own words:

(1) a preliminary effort of attention, which is necessary for the act of perception, and which increases in proportion to what we shall call the complexity (C) of the object; (2) the feeling of value or aesthetic measure (M) which rewards this effort; and finally (3) a realization that the object is characterized by a certain harmony, symmetry, or order (O), more or less concealed, which seems necessary to the aesthetic effect.

To embody in a basic formula the conjecture that the aesthetic measure is determined by the density of order relations in the aesthetic object, this led Birkhoff to define the following formula:

$$M = O/C \quad (2)$$

that could be written differently too:

$$M = f(O/C) \quad (3)$$

Where the amount of order O inherent in the aesthetic object, as compared with its complexity C, from which arises the derivative feeling of the aesthetic measure M of the different objects of the class considered.

The magnitude of *f* is important but only the relative magnitude when we order according to aesthetic measure, and since M must increase with O/C, we can properly define M as equal to the ratio of O to C.

All types of associations induced by the artistic objects (as an example a melody or a complex musical composition), whether formal or connotative stay out of this analysis the order O: "unfortunately the connotative elements of order cannot be so treated, since they are of inconceivable variety and lie beyond the range of precise analysis [4]. New unheard musical passages relate to previously heard material giving rise to meaningful musical units (such as motives, themes, rhythm patterns, harmonic progressions) [10].

3.2. Steps and features for the definition of O, C, and M for a simple melody

Birkhoff's effort for the definition of the aesthetic measure of music follows an attentive attempt to individuate the possible and adequate features useful to characterize and measure the Order.

We report here the list of the features and the definitions of the elements of order that become units for the final sum to divide for the Complexity, as told previously, it is the sum of the notes that form the melody. The Aesthetic Measure will be the ratio of Order and Complexity. Every one of the following features is individuated by Birkhoff as useful to pose time by time the various unities that will give the final sum for the Order. We report here the list of features with the modalities proposed by Birkhoff [4] to elicit the unities to add to the count:

I. Tonal start and close
- there is a count of 1 for each note at the beginning as long as these lie in the tonic chord and are in the first measure;
- there is a like count of 1 for each note at the end as long as these lie in the tonic chord and are in the last measure;
- there is a further count of 1 if the last note is the tonic itself;

II. Cadence
- if there is a passage from dominant to tonic at the close of a phrase (that is, the final change of chord is from dominant to tonic) there is a count of 1 for each note involved and so of 1 in all;
- if the final chord is the dominant (half cadence) there is a count of 1 for the final note

III. Repetition of Accented Notes
- according to as one or both accented notes of a measure reappear in the

following measure there is a count of 1 or 2 as the case may be, provided this is not caused by a mere repetition of the first measure. If both accented notes of the first measure are the same, there is a count of 1 only of course;

IV. Direct Repetition

- if a single note, or a pair of two notes of which the first is accented, be directly repeated, or a measure, or a larger part not the half of a phrase, be repeated within a phrase, there is a count of 1 for each note of the first repetition;
- moreover, the approximate repetition of parts as large as a measure is counted provided there is at most one exceptional note for each measure, the count being 1 for each non-exceptional note; in this case, the second repetition is also counted;

V. Repetition in Comparable Phrases

- if a part of one phrase is repeated in the corresponding position in a later comparable phrase, or if corresponding notes in such a phrase are repeated, there is a count of 1 for each repeated note;

VI. Transposition

- an exact transposition within a phrase, of at least a measure in length but not all in one direct melodic sequence counts 1 for each note of the first transposition;
- if the transposition be repeated a second time within a phrase, there is a count of 1 for each note of the second transposition, provided that the successive transpositions differ by an equal number of degrees;

VII. Inversion

- in a direct inversion of a rising or falling sequence of at least two notes, there is a count of 1 for each repeated note, up to a count of 4;

VIII. Melodic Sequence

- in a rising or falling melodic sequence of at least three notes, there is a count of 1 for each note after the first, up to a count of 4;

IX. Harmonic Sequence

- a harmonic sequence of at least three notes lying in the same consonant chord is counted 1 for each note after the first, up to a count of 4;

X. Melodic Contrast

- if a part B is compared with an earlier part A, there is a count of 1 for each note of B which either differs by step from the corresponding note of A or which is different from any note found in A;
- a sustained note is counted as double here;
- a phrase B will only be said to contrast with an earlier comparable phrase A in case the count for melodic contrast is at least one half the number of notes in B differing from the corresponding notes of A;

XI. Harmonic Contrast

- if all but one of the notes of a measure fall in a consonant major chord, there is a count of 1 each for the last two notes;

XII. Secondary Melody

- a complete count of the elements of the order of the above types is to be made for the secondary melody formed by the alternate accented notes.

So once this kind of reading of the musical text is done and this count is complete, following Birkhoff's writing [4] we have the elements to proceed with the computation of the aesthetic measure:

The order O is the total count of all the elements of order O of the above types.

The complexity C is the total number of notes of the melody.

The aesthetic measure M is then the ratio O/C, as written above.

Anyway, Birkhoff [4] lists some further conditions which must be fulfilled if a satisfactory form is to be achieved in the analysis. Even though he supposes that the further ones he individuated were almost certain are incomplete he gave an adjunctive list of them:

i. Ease in Singing
ii. Regularity of Pattern
iii. Continuity
iv. Freedom from Obvious Formal Blemishes
v. Treatment of the Leading Note
vi. The Secondary Melody
vii. Rhythmic and Melodic Embellishment

4. How and why the ontological analysis could help

Music composition understood as generating new music from rules [11], has been the object of study and application in Computer Science and Artificial Intelligence (AI) during the last decades [12].

Ontology offers a classification system theoretically applicable to any area [13]. And what if the terms managed in an applied ontology would be pitches? In literature, we can retrieve various examples of ontologies built about/for music. An example is *music ontology* [14]. The Music Ontology Specification provides main concepts and properties for describing music (i.e. artists, albums and tracks) on the Semantic Web. Another example is by Presutti and Gangemi [15] they reason about how to extract and describe emerging content ontology design patterns, and how to compose, specialize and expand them for ontology design, with particular focus on Semantic Web technologies. They exemplify the described techniques concerning the extraction of two content ontology design patterns from the DOLCE+DnS Ultra Lite ontology [16], and by showing the design of a simplified ontology for the music industry [14][16]. In a musical score pattern matching/extraction techniques are crucial in discovering salient recurring musical patterns [10]. As an example describing a classical masterpiece in all its form (the composition, the score, the various publications, a performance, a recording, the derivative works, etc.) is a complex activity [17].

4.1. Applied ontologies for music, a literary review

Today a certain number of ontologies have been built about the specific purpose of music. Among others, we can list some, as the already indicated Music Ontology [14] and others as DOREMUS [17], Chord Ontology [18], Music Theory Ontology [19], Temperament Ontology [20] and Music Note [21]. Many existing music ontologies have focused on expressing metadata related to performances or recordings [19].

As said one of the most known examples is the Music Ontology [14] that provides a set of music-specific classes and properties for describing musical works [14][15]. Another example is the DOREMUS model. DOREMUS research develops tools and methods to manage music catalogues on the web using semantic web technologies [17]. It is an ontology for the description of music catalogs [22]. DOREMUS imports the Work-Expression-Event triple pattern of FRBR. The abstract intention of the composer (Work) exists only through an Event (i.e. the composition) that realizes it in a distinct series of choices called Expression(s). This pattern ensures that each step of the life of a musical work can be modeled separately, following the same triplet structure [22].

Music Theory Ontology [19] intends to expand on existing work by including theoretical concepts that were absent from previous music ontologies [19]. Their effort is to include: classes for musical notation, such as accidentals or time signatures;

duration, describing how long a note is or is not played; intervals, representing the tonal difference between pitches; and progressions, which involve the succession of notes or chords [19].

We think about an ontology that in some way can open the scores jumping inside the music reading so giving a kind of identity to the 'distance' between each of them (in frequency and duration) up to elicit the intrinsic path of a score.

5. Discussion

In this work we applied Birkhoff's rules to five pieces for a length of eight measures; we would like to implement in number and length our experimental objects to verify the aesthetic measure he proposed. This would allow us to elicit a recursive rule in pieces with an elevated value of the aesthetic measure.

The thesis is that an ontological analysis and the following ontological tool could help to enter the scores and understanding them from their inside. Here our work stops with the effort to individuate the right form of rationalization of the value of the differences between frequencies and between notes duration.

To better 'get inside' the score we hypothesized an investigation inside the (apparently) voids in the score among notes. As we saw previously, we posed highlighted the absolute difference of frequency between the preceding and following notes. We aim to identify the recursive distances between notes and the 'rhythm' that this recursion has in the score. In this case, we consider:

C – Difference among frequencies

O – Distance from a medium value

But this is a not trivial attempt because to give a stronger consistency to this analysis it is important to consider the distance in time, i.e. about the duration of a single pitch.

Music perception and composition seem to be influenced not only by convention or culture, manifested by musical styles or composers but also by the neuro-acoustics and psychophysics of tone perception [23][24][25].

Birkhoff's aesthetic measure analysis for a search of a quantitative value of the aesthetic fruition gave the start to a long research path that opened interesting fields of deepening relation between perception, information, and computation. We observe that his approach has been in some sense sharp and focalized on simple melodies and as he reported excluded rhythmic reading (apart from the accented notes) and the modulation matter (apart from the cadence).

Otherwise, there is a great mathematical fineness in Birkhoff's construction for the aesthetic measure formula, as we already described that in M=O/C the count of Order has an elaborated basis. Even though it is about a summary of the arithmetic of units, how these units are considered for the summation is not trivial.

The counting of a unit for each note involved in a phrasal dialogue between, as an example, two different themes that make a melody. Inside a simple arithmetic mechanism, this offers a kind of incremental value to the Order counting. And this offers a kind of 'being inside the score' even though measuring something that is outside it (although caused by it).

What we found with our analysis (that stays here at a very first step) is that:

- Birkhoff's measure gives better results (next to attended ones) with very simple melodies we could say didactic in internal building and rhythm disposition
- Birkhoff's measure gives better results applying it to a melody's length that comprises thesis-antithesis-synthesis of the musical phrasing
- Birkhoff's measure can be applied with consistent results to a musical composition written several decades after his writing (this is important because of the perception and ways of the conception of music change across time).

6. Conclusion

The purpose of this research is to offer a first small insight regarding the search for useful tools to identify that recursive harmony that represents the intrinsic value of musical composition and that is perceived even unconsciously through the ear and the fibers of our body.

We hypothesize that this search work could lead to a larger and more correct application of Birkhoff's aesthetic measure formula, so to 'build' a consistent value of the order and the complexity in more complex scores**.** This paper is explorative and intends to propose the methodology of the applied ontology for managing the reading of scores according to the rules proposed by Birkhoff.

We think to an ontology that in some way can open the score and jump inside the music score identifying to the 'distance' between each of note (in frequency and duration) up to elicit the harmonic latent path intrinsic to the score. Successively, by generalizing an approach that combines the ontological analysis and the methods of the applied ontology with the Birkhoff's proposal [4] for valuing the aesthetic measure of a piece of music, after a long research process, it could be possible to conceive a tool to organize knowledge and to support decision in composing music.

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